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STRUCTURE OF A TURBULENT BOUNDARY LAYER AND ITS DEPENDENCE ON REYNOLDS NUMBER

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ABSTRACT
A vortex row model which is composed of three layers of vortices is introduced in order to calculate the velocity profile of a turbulent boundary layer. In spite of its simplicity, the model is found to be very effective to describe the velocity profile as the function of Reynolds number. The result also gives important information on the structure of the turbulent boundary layer.

INTRODUCTION
The intermittent appearance of turbulent bulges is known to be caused by the large scale coherent structures of the outer layer, which are formed by the ensemble of randomly grouped eddies which we call "grouped vortex". They disturb the inner layer which otherwise is quite regular and cause the bursting phenomena to produce the intermediate layer which conveys the turbulent energy into the outer layer. The vortex model of three rows, each of which corresponds to the outer-, inner- and intermediate layer respectively, is proposed so that it can represent the structures and their effects on the development of a turbulent boundary layer.. The model is found to be very effective not only to describe the velocity profile of the boundary layer but also to understand the mechanism of its development with Reynolds number.

VORTEX ROW MODEL
Turbulent and Rankin Vortex
For calculating the mean characteristics such as turbulence or velocity distribution, the grouped vortex is not convenient and is better be replaced by a turbulent vortex which contains regularly arranged eddies.
The turbulent vortex does not produce any real turbulent flow field because of its regular arrangement of eddies, but it is much more convenient for calculating the mean flow characteristics. Furthermore it can be replaced by a Rankin vortex if it has the same core-size and strength or circulation. The fact makes the analysis more practical. Another merit is that the superposition of vortex cores of different radii can represent the organized motions of distributed turbulent eddies .

Vortex Equation
The velocity induced by a vortex of radius Ro which is located at (Xo,Yo) where Yo is the distance from the wall is given by

for $R \leqq Ro$

$$U=\{(Y-Yo)^2/Ro+(Y+Yo)^2/S\}Ao \qquad (1)$$

for $R \geqq Ro$

$$U=\{(Y-Yo)^2/R+(Y+Yo)^2/S\}Ao \qquad (2)$$

where

$$R=\{(X-Xo)^2+(Y-Yo)^2\}^{1/2}$$

$$S=\{(X-Xo)^2+(Y+Yo)^2\}^{1/2}$$

and Ao is the strength (circulation) of the vortex.

Inner Layer Equation
The vortex model is applicable only to the outer layer and therefore we must find a velocity formula good for the inner layer which is composed of three layers; viscous sub-layer, buffer layer and log-layer.
Basically the inner layer is a regular structure of constant shear stress, but is known to be characterized by the bursting phenomena which are agitated by the outer layer structure .
We propose a velocity formula which satisfies the characteristic feature of the layers as following:

$$U\{1-F\exp(-MY)\}\{1-\exp(-NY)\} \qquad (3)$$

where U is the outer layer velocity obtained by the vortex row model, the second and third term correspond to the log-layer velocity and that of sub- and buffer layer respectively. The parameters F, M and N are to be determined to fit the experiments.

Model Configuration

The schematic picture of the vortex row model is shown in Fig.1. Fig.1a shows the arrangement of vortex rows at some fixed time. The top row vortex is located at Y_a=0.6 where the intermittency starts and the bottom row vortex is centered at the edge of the inner layer (Y_b=0.15). The third one which plays important role to mingle the inner and outer layer is located in between (Y_c=0.375). The pitch and size of the vortices are determined from the experiments. They are X_l=5.0, X_m=3.0 and R_a=0.6, R_b=0.15 and R_c=0.225.

Because the streamwise scale of the organized motion is much larger than the boundary layer thickness, we employ elliptic vortex. E_a, E_b, E_c are the flatness factor of the vortex cores defined as the ratio of vertical axis to streamwise length such as R_a/X_m, R_b/X_m and R_c/X_m.

It moves in the flow at the velocity induced by coexisting vortices and by its own image in the wall, and as the result, the vortices of different rows move at different speeds and have a chance to pass each other.

Fig.1b shows the arrangement of vortices at different times, and in Fig.1c the T-X diagram of the top and bottom row vortices is shown, from which we can see how the vortex arrangement is changed and repeated with time.

All quantities are normalized by using the main flow speed U_0 and the boundary layer thickness δ. Reynolds number R_δ used in this model is therefore different from the usually used one R_θ which is based on the momentum thickness.

The core shape is assumed to be unchanged even when it passes near the neighboring vortices or even when it overlaps with other ones.

OUTER AND INNER LAYER PARAMETERS

The outer and inner layer parameters are determined so that they satisfy the experimental results. An example obtained at R_δ = 47,962 is shown in Figs 2a and 2d.

The friction velocity is obtained by fitting the log-law profile to Fig.2a in its inner layer range ($Y \leq Y_b$).

More than twenty experiments were used

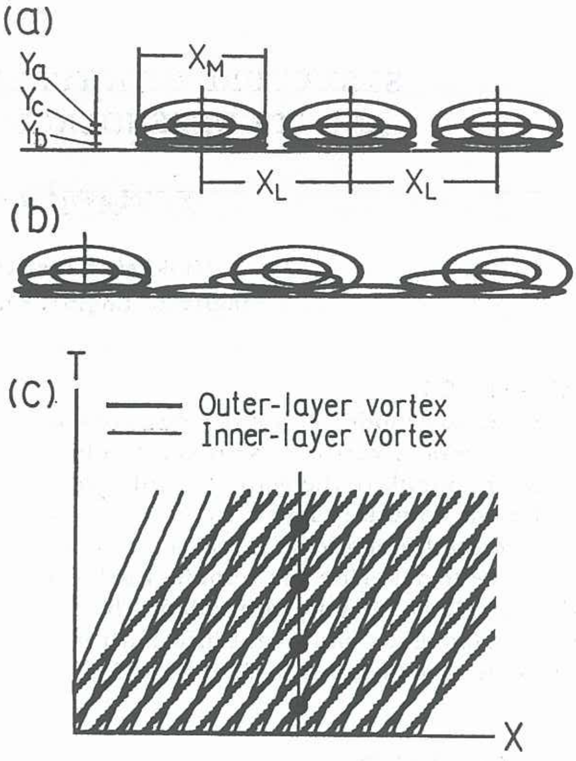

Fig.1 Schematic picture of vortex row model

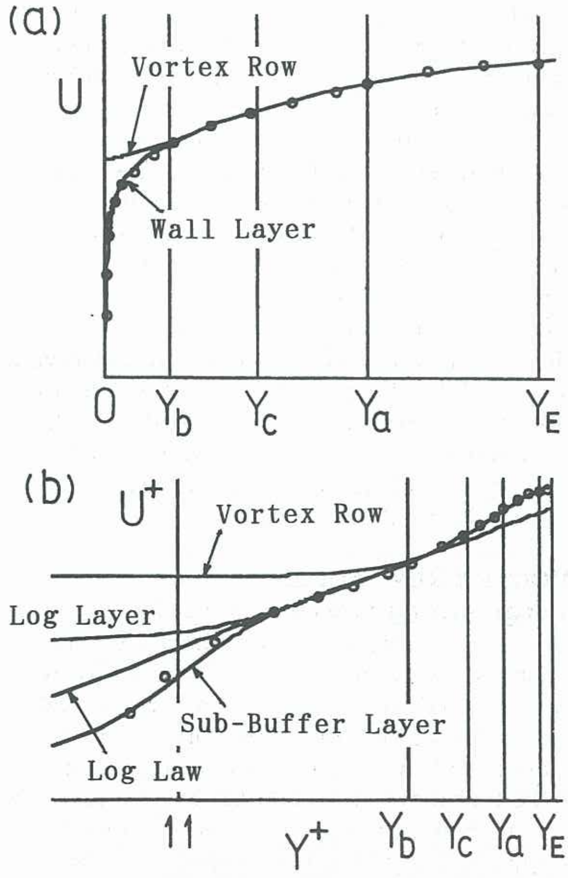

Fig 2 Velocity profiles

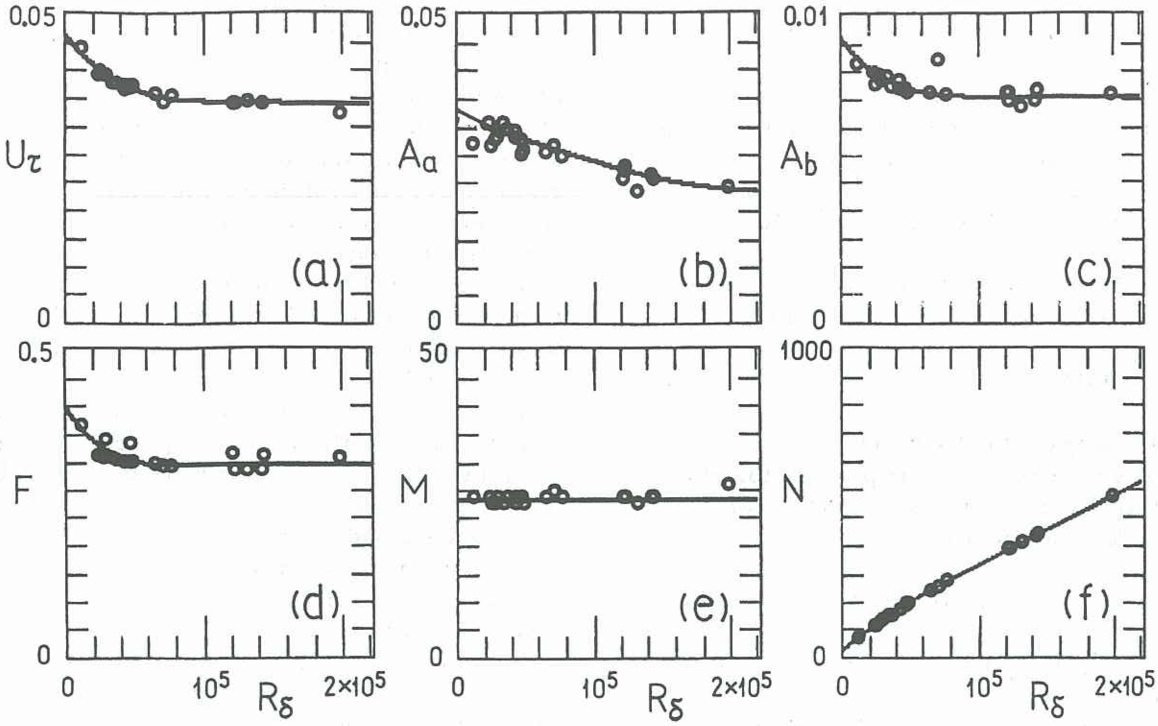

Fig.3 Velocity parameters as function of Reynolds number

to find the nature of parameters and the results are shown in Fig.3 as the function of Reynolds number.

If we use these curves we can calculate the velocity profile at any Reynolds number very easily. But it is more important to find the role of the structures on the development of the turbulent boundary layer. For this purpose it is necessary to formulate the change of parameters as the function of Reynolds number. Due to the scattering of the data obtained from the different sources, many trials of curve fitting were necessary but finally we got the following results.

Friction Velocity

The friction velocity U_τ which is the most important quantity of the boundary layer, was determined from the data of log layer. It decreases gradually and tends to some constant value suggesting that there exists an equilibrium state at $R_\delta = \infty$. If we put $RD = 10^5 R_\delta$, it is given by

$$U_\tau = 0.0305 + 0.0125 \exp(-3.5 RD) \quad (4)$$

Outer Layer Parameters

Aa which represents the strength of the outer layer structure has a similar but not exactly the same tendency as U_τ. It decreases gradually and again tends to a

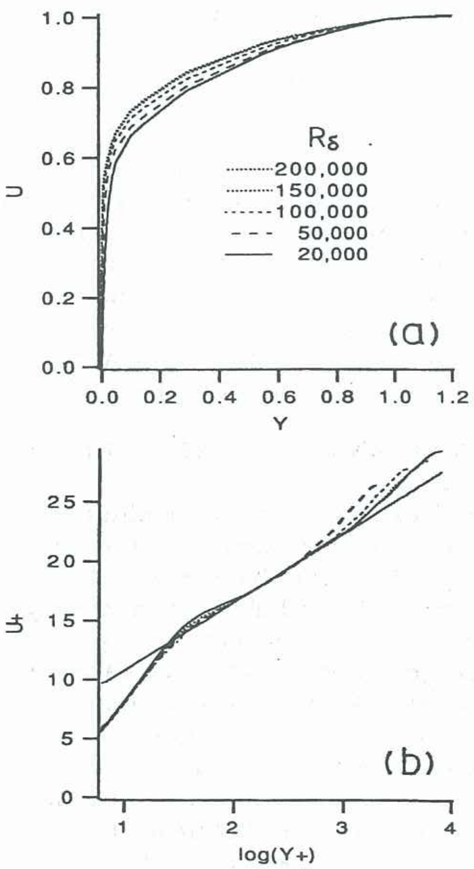

Fig.4 Velocity profiles as function of Reynolds number

constant value. It is given by

$$A_a=0.025+0.028\ EXP(-0.39\ RD) \quad (5)$$

A_b represents the wall turbulence of the inner layer and is closely related with the friction velocity and is given by

$$A_b = 0.0034+0.0014\ exp(-3.5RD) \quad (6)$$

A_c, the strength of the third row vortex keeps almost constant value and is

$$A_c=0.0785 \quad (7)$$

Inner Layer Parameters

F has a meaning of slipping factor of the wall turbulence and is proportional to U_τ/U_{m0} and is given by

$$F= 0.6+0.4\ exp(-3.5RD) \quad (8)$$

where U_{m0} is the outer layer velocity on the wall.

M is determined so that the second term of Eq.(3) fits the slope of log-layer.

$$M= 2.15 \quad (9)$$

N represents the velocity gradient at the wall, and is expressed by

$$N=U_\tau{}^2 R_\delta/\{U_{m0}(1-F)\} \quad (10)$$

It increases almost linearly with R_δ.

These formulae are good enough to understand the role of the structures on the turbulent boundary layer development. Especially the fact that both Aa and U_τ tend to constant values is very important because it suggests the existence of equilibrium state of the boundary layer when Reynolds number becomes infinite.

PROBLEM AND POSSIBILITY OF THE MODEL

The vortex row model was found to be very useful not only for calculating the mean velocity profile but also to solve the problems of the turbulent boundary layer which have not been made clear yet.

i) Large Scale Structure and the Vortex Model: It is known well that the intermittent phenomena of the turbulent boundary layer are caused by the large scale coherent structure of vortical motion, which is a laminar structure at its beginning but breaks down into randomly arranged eddies (grouped vortex) when it loses its stability. It moves with the flow and grows its size just like the structure of the free shear turbulent flows.

The only difference is that its energy is fed from the inner layer which develops along the wall.

For the purpose of representing the large scale structure one row vortex is good enough, but for explaining the energy transport mechanism we need two more rows; the one which plunders the energy from the inner layer and the one which carries it into the outer layer. This is the basic idea of the present vortex row model.

The three dimensional feature of the structure which is not discussed in this paper can be explained by superposing the streamwise vortices which were discovered by Kovasznay et al [1].

ii) Wall Turbulence and Burst : Burst is the important phenomenon of the turbulent boundary layer, because it is the source of turbulent energy. Many works have been made on its structure, but not so many on its origin. We believe that the burst appears when the wall turbulence is disturbed by the large scale structure of the outer layer, just in the same way as Blackwelder et al discovered in their ingenious experiment [2].

iii) Drag Reduction: Drag reduction is the most serious problem of the turbulent boundary layer, and many trials have been made, most of which are how to manipulate the burst structure. The present model suggests that controlling of the large scale structure is more effective way of drag reduction. We have some, though not many, experimental evidences to this supposition.

CONCLUDING REMARKS

A vortex row model is proposed in order to describe the velocity profile of the turbulent boundary layer. In spite of its simplicity, the results are found to be satisfactory and are very helpful to understand the effect of Reynolds number on the boundary layer flow. It also gives very useful information about the mechanism of boundary layer structures although there still remain many problems which should be shot as the new field of turbulence research.

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