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DNS AND MODELLING OF TURBULENT HEAT TRANSFER IN CHANNEL FLOW WITH A SPANWISE MEAN TEMPERATURE GRADIENT

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ABSTRACT

Several direct numerical simulations (DNS) have been performed for a major mean temperature gradient in a wall-normal direction. In this case, the turbulent heat fluxes in streamwise and wall-normal directions arises. On the other hand, not so many investigations have been made on the turbulent heat transfer with a spanwise mean temperature gradient. Therefore, the aim of the present study is to perform the DNS in the channel flow with the spanwise mean temperature gradient and to compare the results with the turbulent heat flux model proposed by the present authors' group.

NOMENCLATURE

C_p ; specific heat at constant pressure
C_ε, C_θ; model constants of $\varepsilon_{i\theta}$ and $\Pi_{i\theta}$
$D_{i\theta}$; molecular diffusion of $\overline{u_i\theta}$
F ; flatness factor
k, k_θ ; turbulent kinetic energy and temperature variance
p ; fluctuating pressure
Pr ; molecular Prandtl number
Pr_{ty}, Pr_{tz}; turbulent Prandtl numbers of isoflux and present conditions
q_0 ; heat flux by spanwise molecular heat conduction
R ; time scale ratio
Re_t ; turbulent Reynolds number $= k^2/\varepsilon\nu$
Re_τ ; Reynolds number $= u_\tau\delta/\nu$
$T_{i\theta}$; turbulent diffusion of $\overline{u_i\theta}$
t_0 ; representative temperature $= q_0/\rho C_p u_\tau$
U_i, $\overline{U_i}$, u_i; instantaneous, mean and fluctuating velocities
$\overline{u_iu_j}$; Reynolds stress tensor
$\overline{u_i\theta}$; turbulent heat flux tensor
u_τ ; friction velocity
x_i ; Cartesian coordinates
δ ; channel half width
ε, $\varepsilon_{i\theta}$, ε_θ; dissipation rate of k, $\overline{u_i\theta}$ and k_θ
κ ; molecular heat diffusivity
ν ; kinematic viscosity
$\Pi_{i\theta}$; temperature pressure-gradient correlation
Θ, $\overline{\Theta}$, θ; instantaneous, mean and fluctuating temperatures
ρ ; density
$()^+$; non-dimensionalization with u_τ, ν and t_0
$()^*$; non-dimensionalization with u_τ and δ

INTRODUCTION

Since the first attempt by Kim-Moin (1989), several direct numerical simulations (DNS) have been performed for the turbulent heat transfer in channel flow. These simulations adopted different kinds of heating boundary conditions; uniform internal heat source by Kim-Moin(1989), uniform heat flux heating from both walls by Kasagi et al. (1992), Kawamura-Kondoh (1996) and Kawamura et al. (1998a) and uniform temperature difference on both walls by Lyons et al. (1991). The major mean temperature gradient imposed by all of these heating conditions takes place in the wall-normal direction.

The turbulent heat flux arises in combination with the temperature and velocity fields. Thus it is also interesting to realize the temperature gradient in the direction other than the wall-normal one. Matsubara et al. (1998) performed DNS of the turbulent heat transfer in a channel flow with a spanwise mean temperature gradient for Reynolds number of Re_τ=150, which is based on the friction velocity u_τ and the channel half width δ. They obtained the distribution of the spanwise turbulent heat flux and also examined the ratio of thermal and kinematic eddy diffusivities.

As for the modelling of the turbulent scalar transport, various models have been proposed to date. They were expressed in the tensorial form; however, their validity was examined almost always only with respect to the wall-normal mean temperature gradient. The present work performs the DNS of turbluent heat transfer in the channel flow with a spanwise mean temperature gradient and discusses the effect of difference between present and isoflux condi-

Table 1: Computational conditions

Grid		Staggered grid
Coupling algorithm		Fractional step method
Time advancement	Viscous term (y)	2^{nd} Crank-Nicolson
	others0	2^{nd} Adams-Bashforth
Scheme	Nonlinear terms	2^{nd} central (consistent)
	Viscous terms	2^{nd} central
Boundary condition		Periodic (x, z-direction) Non-slip (y-direction)
Grid number (x, y, z)		$128 \times 66 \times 128$
Computational volume (x, y, z)		$6.4\delta \times 2\delta \times 3.2\delta$
Reynolds number Re_τ		180
Prandtl number Pr		0.025, 0.05, 0.4, 0.71
Spacial resolution		$\Delta x^+ = 9, \Delta z^+ = 4.5$ $\Delta y^+ = 0.4 \sim 11.5$

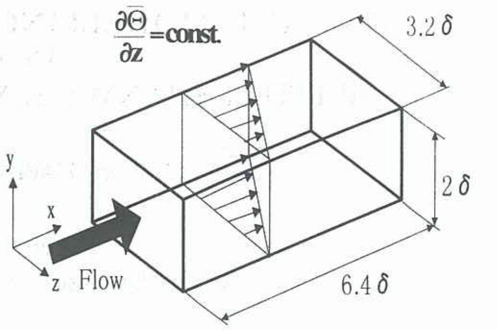

Figure 1: Configuration of calculation

tions. Then, the results by the DNS is compared with the turbulent heat flux of present authors' group (H. Kawamura et al., 1998b) .

CALCULATION PROCEDURE

The finite difference method is applied to the DNS of turbulent heat transfer in a channel flow. The configuration is shown in Fig. 1 and the calculation condition is given in Table 1. Flow is assumed to be fully developed in the streamwise direction (x) and periodic in the spanwise one (z). A linear mean temperature gradient is imposed in z direction. The Reynolds number of 180 and the Prandtl number of 0.71, 0.4, 0.05 and 0.025 are assumed.

The governing equations are

$$\frac{\partial U_i^+}{\partial x_i^*} = 0, \qquad (1)$$

$$\frac{\partial U_i^+}{\partial t^*} + U_j^+ \frac{\partial U_i^+}{\partial x_j^*} = -\frac{\partial p^+}{\partial x_i^*} + \frac{1}{Re_\tau}\frac{\partial^2 U_i^+}{\partial x_j^* \partial x_j^*} + \delta_{1i}, \qquad (2)$$

$$\frac{\partial \theta^+}{\partial t^*} + U_j^+ \frac{\partial \theta^+}{\partial x_j^*} = \frac{1}{PrRe_\tau}\frac{\partial^2 \theta^+}{\partial x_j^* \partial x_j^*} + Re_\tau Pr U_3^+, \qquad (3)$$

where $\theta^+ = \Theta^+ - \overline{\Theta^+}$. The velocity, the length and the temperature are normalized by the friction velocity u_τ, the channel half width δ and repersentative temperature t_0, respectively. The boundary conditions are periodic in x and z directions for both U_i^+ and θ^+.

RESULTS

The obtained root-mean-square temperature fluctuation θ_{rms}^+ is shown in Fig. 2. It reaches a maximum at the channel center, while that of isoflux condition reaches a maximum near the wall. This feature is due to the fact that the value of production term in the budget of temperature variance k_θ^+ is roughly constant from the position of maximum value to the channel center (see Figs. 3 and 4).

The budget of k_θ^+ is given in Figs. 3 and 4 for Pr=0.71 and 0.025, respectively. Figure 3 indicates that the effect of diffusion terms can be neglected and thus the budget is nearly the equilibrium state in whole region except near-wall. The sign of turbulent diffusion becomes negative near channel center, while under the isoflux condition, it becomes positive.

The transport equation for the turbulent heat flux consists of production, dissipation, temperature pressure-gradient correlation (TPG), molecular and turbulent diffusion terms. Firstly, the production need not be modelled. With inclusion of the near wall modification, the molecular diffusion term cahnbe modelled as

$$D_{i\theta} = \frac{\partial}{\partial x_j}\left(\frac{\kappa+\nu}{2}\frac{\partial \overline{u_i\theta}}{\partial x_j} + \frac{\nu-\kappa}{6} n_i n_k \frac{\partial \overline{u_k\theta}}{\partial x_j}\right). \qquad (4)$$

The turbulent diffusion is usually modelled based on the gradient diffusion assumption. However, the present model includes the term proportional to the mean temperature gradient in the budget of the triple correlation $\overline{u_i u_j \theta}$ as

$$T_{i\theta} = \frac{\partial}{\partial x_j}\left\{c_t \frac{k}{\tilde{\varepsilon}}\left(\overline{u_j u_k}\frac{\partial \overline{u_i\theta}}{\partial x_k} + c_t^* \overline{u_i u_j u_k}\frac{\partial \overline{\Theta}}{\partial x_k}\right)\right\} \qquad (5)$$

$$c_t = \frac{0.11}{1 - exp(-y_K/5)}, \quad c_t^* = 0.5.$$

The TPG and dissipation terms are modelled in a combined form with inclusion of the effects of mean temperature gradient as :

$$\Pi_{i\theta} - \varepsilon_{i\theta} = \Phi_{i\theta,1} + \Phi_{i\theta,2} + \Phi_{i\theta,w}, \qquad (6)$$

$$\Phi_{i\theta,1} = -\left(\frac{1+Pr}{2Pr}\frac{\varepsilon}{k}\overline{u_i\theta} + \frac{1+3Pr}{2Pr}\frac{\varepsilon}{k} n_i n_j \overline{u_j\theta}\right) f_{w1}$$
$$- \left(c_{\theta1,a}\delta_{ij} + c_{\theta1,a}^* b_{ij}\right)\frac{\tilde{\varepsilon}}{k}\overline{u_j\theta}\,(1 - f_{w1})$$
$$- \left(c_{\theta1,b}\delta_{ij} + c_{\theta1,b}^* b_{ij}\right) k \frac{\partial \overline{\Theta}}{\partial x_j}, \qquad (7)$$

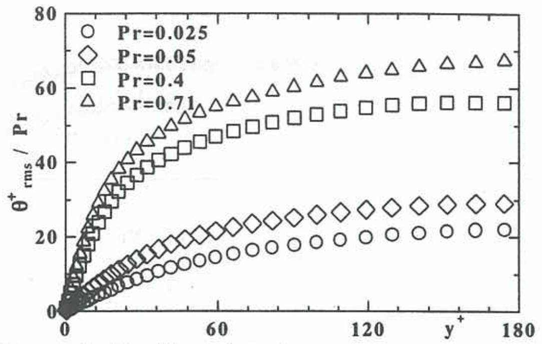

Figure 2: Profiles of root-mean-square temperature fluctuation

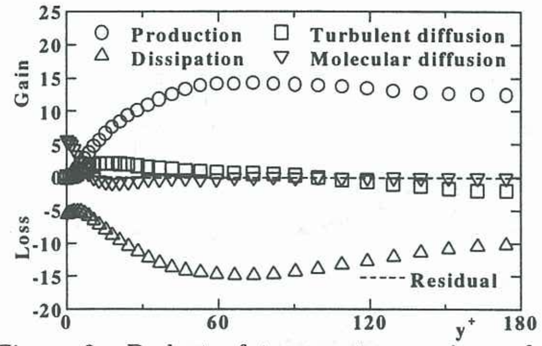

Figure 3: Budget of temperature variance for Pr=0.71

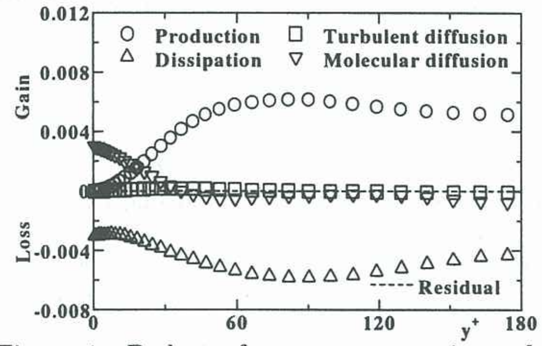

Figure 4: Budget of temperature variance for Pr=0.025

$$
\begin{aligned}
\Phi_{i\theta,2} &= \alpha_1 \overline{u_j\theta}\frac{\partial \overline{U_i}}{\partial x_j} + \alpha_2 \overline{u_j\theta}\frac{\partial \overline{U_j}}{\partial x_i} \\
&+ \left\{\alpha_3 \overline{u_iu_j}\,\overline{u_k\theta} + \alpha_4\left(\overline{u_iu_k}\,\overline{u_j\theta} + \overline{u_ju_k}\,\overline{u_i\theta}\right)\right\}\frac{1}{k}\frac{\partial \overline{U_j}}{\partial x_k} \\
&+ \beta_1 \frac{\partial \overline{\Theta}}{\partial x_j}\frac{k^2}{\varepsilon}\frac{\partial \overline{U_i}}{\partial x_j} + \beta_2 \overline{u_iu_j}\frac{\partial \overline{\Theta}}{\partial x_k}\frac{k}{\varepsilon}\frac{\partial \overline{U_j}}{\partial x_k} \\
&+ \beta_3\left(\overline{u_iu_k}\frac{\partial \overline{\Theta}}{\partial x_j} + \overline{u_ku_j}\frac{\partial \overline{\Theta}}{\partial x_i}\right)\frac{k}{\varepsilon}\frac{\partial \overline{U_j}}{\partial x_k}, \qquad (8)
\end{aligned}
$$

$$\Phi_{i\theta,w} = -c_w \frac{\tilde{\varepsilon}}{k} n_i n_j b_{jk} \overline{u_k\theta} \frac{k^{1.5}}{2.5\varepsilon y}, \qquad (9)$$

$$c_{\theta 1,a} = 3.6\left\{1 - 0.74 exp\left(-\frac{Re_t}{100}\right)\right\}\left(1 + \frac{0.05}{Pr}\right)$$

$$c^*_{\theta 1,a} = -3.5,\ c_{\theta 1,b} = 0.023\sqrt{F},\ c^*_{\theta 1,b} = 0.09\sqrt{F}$$
$$\alpha_1 = 0.1,\ \alpha_2 = 0.001,\ \alpha_3 = 0.1,\ \alpha_4 = 0.01$$
$$\beta_1 = 0.08,\ \beta_2 = -0.044,\ \beta_3 = 0.042,\ c_w = 1.5$$
$$f_{w1} = exp\left(-y^+/A\right),\ A = 15 min\,(1,\ 1/Pr)\,.$$

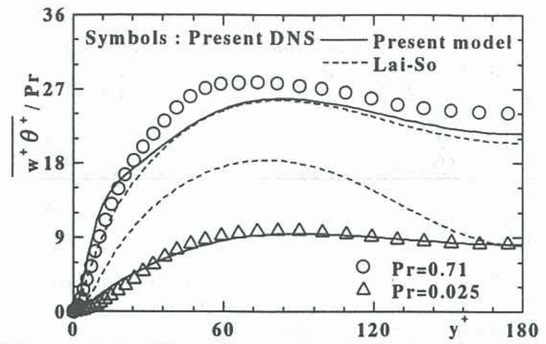

Figure 5: Profiles of spanwise turbulent heat flux

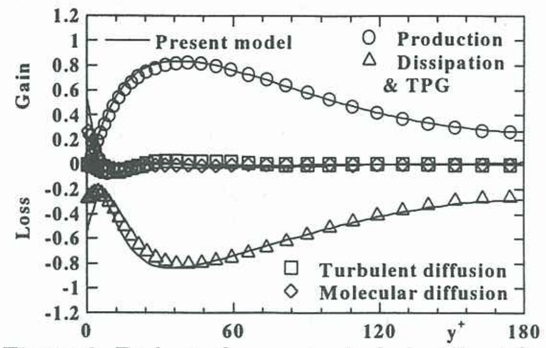

Figure 6: Budget of spanwise turbulent heat flux for Pr=0.71

Figure 5 shows the profiles of spanwise turbulent heat flux $\overline{w^+\theta^+}$. The dashed lines is the prediction by Lai-So (1990) model. The present model captures the DNS results for two Prandtl numbers well. This result indicates that this model takes the Prandtl number effect into consideration adequately.

The budget of $\overline{w^+\theta^+}$ is shown in Figs. 6 and 7 for Pr=0.71 and 0.025, respectively. The prediction given by the present model is good agreement with the DNS results for two Prandtl numbers. The molecular and turbulent diffusions are negligibly small except in the near wall region.

The turbulent Prandtl number in case of the isoflux Pr_{ty} is given as

$$Pr_{ty} = \frac{\overline{u^+v^+}\frac{\partial \overline{\Theta^+}}{\partial y^+}}{\overline{v^+\theta^+}\frac{\partial \overline{U^+}}{\partial y^+}}\,. \qquad (10)$$

In the present case of the spanwise gradient, the turbulent Prandtl number Pr_{tz} may be defined similarly as

$$Pr_{tz} = \frac{\overline{u^+v^+}\frac{\partial \overline{\Theta^+}}{\partial z^+}}{\overline{w^+\theta^+}\frac{\partial \overline{U^+}}{\partial y^+}}\,. \qquad (11)$$

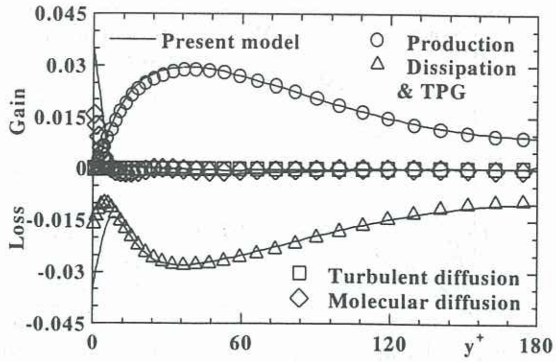

Figure 7: Budget of spanwise turbulent heat flux for Pr=0.025

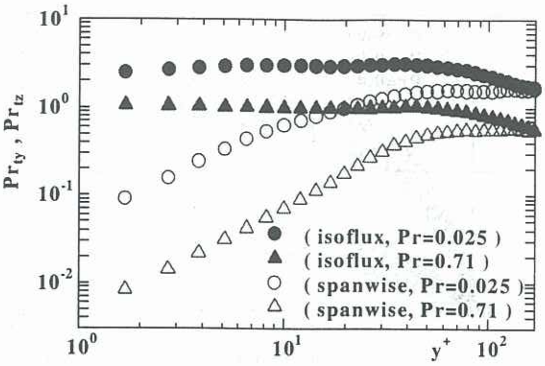

Figure 8: Profiles of turbulent Prandtl number

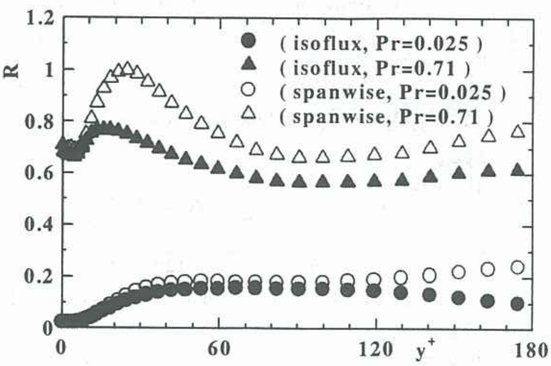

Figure 9: Profiles of timescale ratio

The wall value of Pr_{ty} is about 1.0 as discussed by Antonia-Kim (1991). On the other hand, in case of the spanwise temperature gradient, the Pr_{tz} is protional to y^+, because $\overline{w^+\theta^+}$ varies as y^{+2} while $\overline{u^+v^+}$ as y^{+3}. The turbulent Prandtl numbers Pr_{ty} and Pr_{tz} are shown in Fig. 8. Indeed Pr_{tz} behaves as y^+ in the wall vicinity. Pr_{tz} is considerably smaller than Pr_{ty} in whole of the channel.

The time scale ratio defined as

$$R = \frac{k_\theta \varepsilon}{\varepsilon_\theta k} \tag{12}$$

is shown in Fig. 9. The one by the uniform wall heating (Kawamura et al., 1998a) is also plotted and compared with the present one. The wall limiting value of R is exactly equal to the Prandtl number. R for the both cases behaves quite similarly inspite of the different direction of the imposed mean temperature gradient.

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