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AN INVESTIGATION OF WAVE TRAPPING AS A CAUSE OF VORTEX BREAKDOWN IN THE TORSIONALLY DRIVEN CYLINDER

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ABSTRACT

The observation of wave trapping and focusing as a possible cause of vortex breakdown in open pipe flows has led to this study, in which the behaviour of the azimuthal vorticity in a torsionally driven cylinder is investigated numerically. Similar to the pipe studies, an amplification of an initial change in the azimuthal vorticity is observed near the axial position at which breakdown subsequently develops. An examination of the transport of this vorticity is also presented.

INTRODUCTION

Vortex breakdown has been observed in a number of swirling flows, including flows over delta wings, in open pipes, and in the torsionally driven cylinder. It is a sudden (in space) expansion of the vortex core at a certain axial location, along with flow reversal in the interior of the breakdown bubble and often turbulent flow downstream. Its presence above the delta wing brings about a reduction in lift due to the destruction of the vortex core. The flow downstream of breakdown is often turbulent, and can adversely affect airframe components. A number of studies have catalogued the various types of breakdown observed experimentally (Sarpkaya (1971), Faler and Leibovich (1977)) and reveal the diversity of forms and behaviours associated with breakdown.

Theoretical attempts to find a cause for vortex breakdown have not yet produced a complete explanation; reviews of this work have been presented by Hall (1972), Leibovich (1984), and Delery (1994).

One theory which has received some attention is that due to Benjamin (1962). Benjamin proposed that two flow states exist on either side of the breakdown - one subcritical and the other supercritical, breakdown acting as a transition between the two flow states. Hence vortex breakdown is analogous to the flow disturbance observed at a hydraulic jump between two flow states in channel flows, as described in Lamb (1932).

Darmofal and Murman (1994) proposed a variant of Benjamin's critical state theory. They suggested that long waves resulting from a perturbation originating upstream become trapped at a location beyond which the group velocity of these waves becomes negative. They compared this idea with the theory behind shock formation in a compressible flow. An oscillatory disturbance on a compressible flow results in one wave which moves downstream at velocity $(U + a)$ and another with moves upstream with velocity $(U - a)$, where U is the flow velocity and a is the speed of sound. The amplitude of the wave moving upstream varies as $(U - a)^{-\frac{1}{2}}$, so as the flow approaches the speed of sound a, the amplitude of the wave tends toward infinity. It was suggested that an equivalent wave amplification in a swirling flow brings about vortex breakdown.

Darmofal and Murman examined the onset of breakdown in an open pipe flow numerically. They started with a steady solution at a swirl at which vortex breakdown was not observed. Then the swirl was suddenly increased at the inlet to a level where vortex breakdown is observed in the steady state solution. As the flow adjusted to this jump in swirl, the perturbation azimuthal vorticity near the centreline (vorticity at each timestep (η) minus the initial condition vorticity (η_i)) was plotted as a function of both axial distance and time. The results revealed a small wave which travelled downstream from the inlet. At a certain position some wave modes became trapped while others propagated downstream. The trapped waves subsequently increased in amplitude, and this amplification corresponded in time and axial position with the onset of the recirculation bubble.

Darmofal and Murman suggested that this study indicates that there is a cause and effect link between this wave trapping and the onset of vortex breakdown in a pipe.

THIS STUDY

As part of a study into the relationship between wave trapping and vortex breakdown, this paper re-

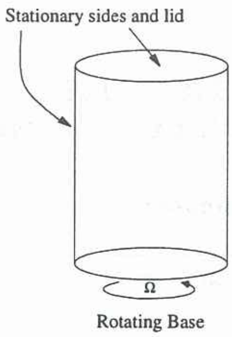

Figure 1: Torsionally driven cylinder geometry.

ports an examination of the perturbation vorticity in a torsionally driven cylinder as the Reynolds number is increased from conditions below those required for breakdown to those just above. A parallel investigation of the stability of this flow undertaken by Brydon and Thompson (1998) (submitted to this conference) has shown the close coincidence between the appearance of negative eigenvalues of wavelength equivalent to the vortex breakdown dimension and the onset of vortex breakdown.

The geometry consists of a closed cylindrical container, with a rotating lower lid to produce the required swirling flow along the axis - see figure 1.

The breakdown in a torsionally driven cylinder is presumed here to have the same mechanism of formation as that in the open pipe and delta wing flows, as the bubbles produced in all of these flows have the same general characteristics (eg. a stagnation point at the upstream end of the breakdown bubble, and a region of recirculation behind the stagnation point). Hence wave trapping should also be observed in the torsionally driven cylinder if it is to be considered an explanation for the occurrence of vortex breakdown generally.

Results for this study were produced using the CFD programs included in the CFX version 4.2 commercial software package. The basis of the code for steady state simulations is a conservative finite difference method. For this study an upwinding scheme was used, resulting in 3rd order accuracy in the advection term and 2nd order accuracy for the diffusion. A test of the grid independence of these results has been conducted - the 100x40 grid was found to give a result within 5% of the result obtained from grids up to 350x140, where the test of convergence was based on the minimum and maximum streamfunction values.

For the transient runs a fully implicit backward difference time stepping procedure was used.

Initial studies have confirmed the accuracy of CFX in steady state flow prediction for the geometry and 100x40 uncompressed grid considered here, when compared with the experimental results of Graham *et al* (1998), and an experimental study is planned in order to confirm the time dependent results.

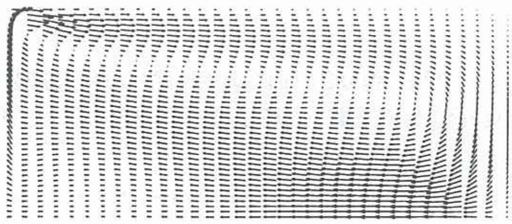

Figure 2: Vector field at Re = 1400.

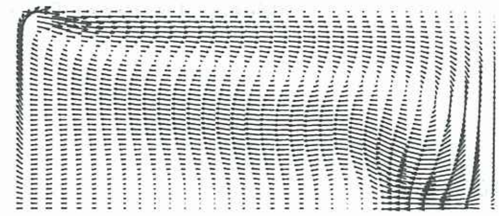

Figure 3: Vector field at Re = 1870.

For the Reynolds numbers at which the numerical work is conducted only the axisymmetric bubble type of breakdown is observed, so the solution domain has been restricted to a single 2 dimensional plane parallel to the cylinder axis, in order to reduce the computational workload.

The intention of this study is to examine the perturbation vorticity along the centreline of the cylinder, where the perturbation vorticity η_{pert} is defined as:

$$\eta_{pert} = \eta - \eta_i$$

where η is the vorticity at each point along the centreline for each timestep, and η_i is the vorticity of the initial condition at the same axial location.

In the same fashion as Darmofal and Murman's presentation, for easier visualisation η_{pert} for successive timesteps is increased by a small amount and moved to the right so that the vorticity for each timestep is displaced from that for the previous step.

First an initial condition is generated for each run. The mass source tolerance (a continuity parameter) is used as the convergence criterion in generating these solutions. From this initial condition a time dependent evolution of the flow is commenced. The Reynolds number is stepped up via an increase in the lid's angular velocity and the perturbation vorticity along a line one grid point away from the axis monitored as the flow adjusts to the increase in swirl.

STEADY STATE RESULTS

Two initial conditions are used: 1400 (well below breakdown) and 1870 (some retardation of the flow along the axis). Vector plots of these initial conditions can be seen in figures 2 and 3 - the rotating lid is to

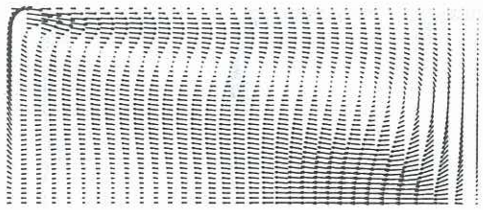

Figure 4: Vector field at Re = 1460.

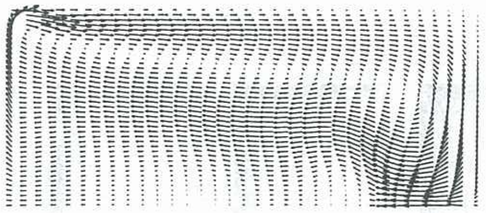

Figure 5: Vector field at Re = 1930.

the left, and the bottom of the figure represents the centreline of the cylinder.

In these studies the Reynolds number is increased by 60, ie. from 1870 to 1930, and from 1400 to 1460. The steady flow states at Re = 1460 and Re = 1930 can be seen in figures 4 and 5.

In the 1400 to 1460 transition there is no breakdown or even divergence of the streamlines - this has been used as a baseline transition to compare the higher Reynolds number results with. The 1870 to 1930 transition is from a no-recirculation bubble flow (but some divergence of the streamlines) to recirculation bubble flow. Here is identified a significant difference between the open pipe and torsionally driven cylinder types of breakdown: Darmofal and Murman's increase in swirl is less than 1%, and the transition is from parallel streamlines to a fully developed bubble. In the torsionally driven cylinder the transition to breakdown is more like an evolution of the flow, and it occurs over a much larger Re range ($\sim 10\%$). To increase the Reynolds number by such a large amount however introduces large transients, and so the small Re change only is included here. The transition here between the no breakdown and breakdown cases is therefore not as distinct as in Darmofal and Murman's pipe study.

TIME DEPENDENT RESULTS

Figures 6 and 7 reveal the behaviour of the perturbation vorticity for the 2 transitions: Re = 1400 to 1460, and 1870 to 1930, using a timestep of 0.5 seconds. In figures 8 and 9 the perturbation vorticity in the initial stages of the flow evolution is plotted - here the timestep is 0.01 seconds. The wave motion along the centreline observed by Darmofal and Murman is not seen here, and it is apparent from this work that a study of the propagation of the vorticity

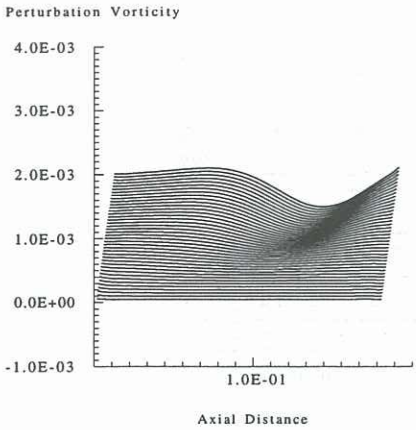

Figure 6: Perturbation vorticity, 1400 to 1460 transition, timestep 0.5.

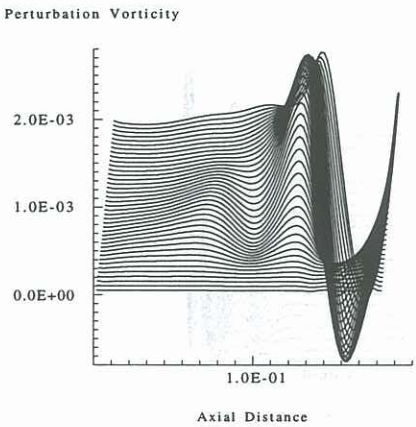

Figure 7: Perturbation vorticity, 1870 to 1930 transition, timestep 0.5.

change through the whole cylinder domain is required in order to determine its origin.

The series of contour plots in figure 10 reveals the perturbation vorticity over the entire domain for the Re = 1870 to 1930 transition for every 10th timestep.

The rotating lid is at the bottom of each figure. These plots show the perturbation vorticity from just after the initiation of the Reynolds number change (timestep 10) to the development of both breakdown bubbles at Re = 1930. A breakdown bubble begins to develop approximately one third of the axial length from the rotating lid at timestep 230. The second bubble appears at approximately two thirds of the axial length at timestep 290. The change in vorticity is seen to descend from the point where the rotating lid meets the cylinder wall. From there the vorticity is transported diagonally across the cylinder to the point where the breakdown bubbles evolve.

CONCLUSION

From the results presented here it can be seen that the change in azimuthal vorticity propagates through

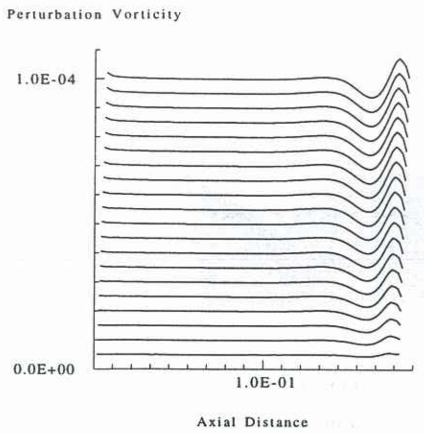

Figure 8: Perturbation vorticity, 1400 to 1460 transition, timestep 0.01.

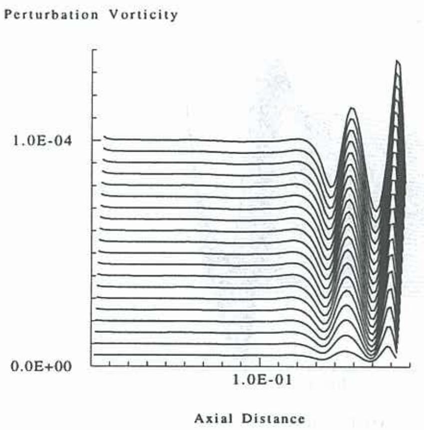

Figure 9: Perturbation vorticity, 1870 to 1930 transition, timestep 0.01.

the cylinder domain in a more complicated manner compared with its propagation along the axis in the open pipe. Further study of the relationship between the azimuthal vorticity change and the onset of breakdown in the torsionally driven cylinder is required in order to relate the wave trapping model for vortex breakdown to this geometry.

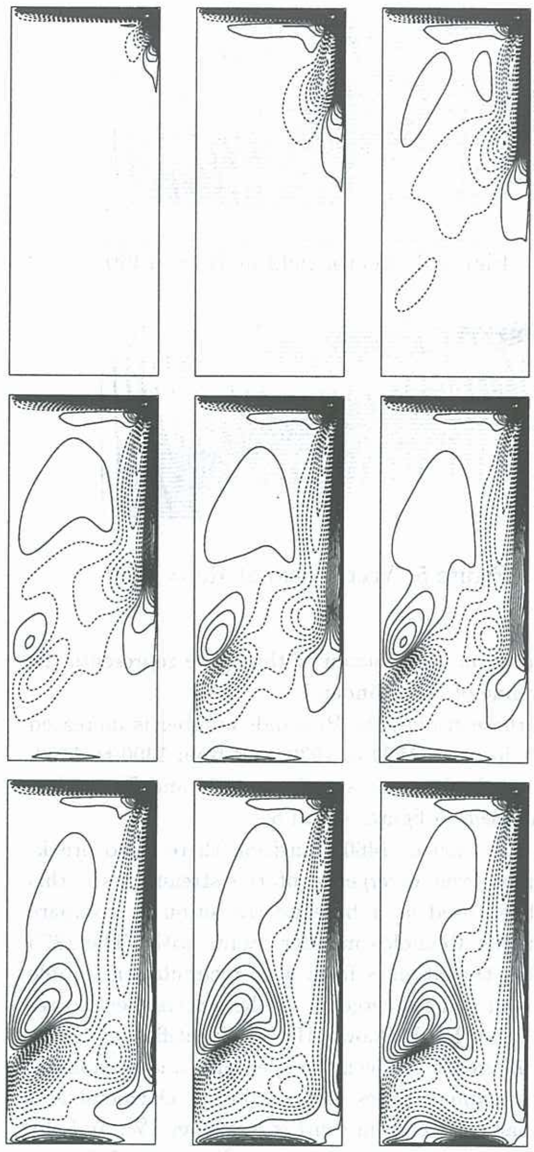

Figure 10: Perturbation vorticity contours, timesteps 10,50,100,150,200,230,290,350,400

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