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NUMERICAL STUDY OF FLOW IN A ROTATING SHOCK TUBE

G.D. HORROCKS, J.A. REIZES and S.G. MALLINSON

Faculty of Engineering
University of Technology Sydney, NSW, AUSTRALIA

ABSTRACT

Contact surface instabilities which occur in a rotating shock tube are analyzed by numerical simulation. Two types of instability have been identified: Richtmyer-Meshkov and Rayleigh-Taylor. The Richtmyer-Meshkov instability was found to be an inviscid phenomenon which can be eliminated by the addition of a small amount of viscosity to the flow. The Rayleigh-Taylor instability was found to be sensitive to the levels of both viscosity and thermal conductivity.

INTRODUCTION

Shock waves and steep pressure gradients occur in high speed machinery and pipe flows. A frequently used test case for shock wave flow is the shock tube (Liepmann and Roshko, 1957). This is a long, thin channel, closed at both ends with a diaphragm separating two gases at different states. When the diaphragm is suddenly removed, a shock wave propagates through the gas at the lower pressure and a rarefaction wave through the higher pressure gas.

The one-dimensional inviscid shock tube problem is well understood (Liepmann and Roshko 1957). For the two- and three-dimensional problem, both the inviscid and viscous shock tube problems have also been extensively studied numerically and experimentally (see, for example, Glass and Patterson (1955) and Sharma and Wilson (1995)). However, little work has been done on an apparently simple abstraction of the standard shock tube problem, that of the rotating shock tube (see Figure 1), with only a few studies to be found in the open literature.

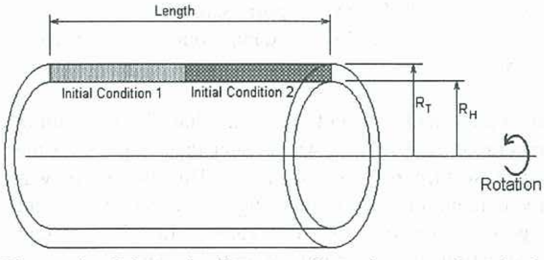

Figure 1 : Schematic diagram of rotating annular shock tube. R_T = radius of outer wall, R_H = radius of inner wall.

Larosiliere and Mawid (1995) performed a numerical study of inviscid flow flow in an annular cylindrical shock tube rotating about its central axis (see Figure 1). The investigation focused on how the shock "leans" and forms an "S" shape due to the three dimensional effects caused by the rotation. They also found that the temperature ratio has a dramatic effect on the stability of the contact surface. It was suggested that if, initially, the high pressure side was hotter than the low pressure side, then the contact surface would be unstable after interacting with either the shock or rarefaction waves.

A numerical study of the flow in a rotating annular shock tube is described here. In particular, the contact surface instability noted in previous studies has been explored in some detail.

NUMERICAL METHOD

Numerical simulations were performed using the commercial finite-volume Navier-Stokes solver CFX4.2. A simple rectangular grid in cylindrical coordinates is used to discretize the volume, with constant grid spacing in the axial direction and radial directions. The results presented by Larosiliere and Mawid (1995) suggest that the flow is axisymmetric, and so periodic boundary conditions are used in the circumferential direction. The spatial gradients are calculated using the higher-order 'Superbee' upwind scheme, with a flux limiter. It is second-order accurate. Temporal integration is accomplished using implicit backward differencing, which is first-order accurate in time.

The permissible time-step, Δt, is dependent on the individual problem, with the Courant-Friedrichs-Lewy (CFL) criterion being the fundamental limiting factor for the flow, that is, $U\Delta t/\Delta x < 1$, where U is the maximum velocity and Δx is the minimum grid element length.

The physical geometry of the domain is a circular annulus, as shown in Figure 1. For all models performed in this paper, the shock tube length is L = 1.0 m, the diaphragm is placed at $L/2$, the aspect ratio L / R_T is 1.5, and the radius ratio R_H / R_T is 0.934.

The numerical model uses primitive variables, hence no non-dimensionalisation scheme is required. However, for ease of comparison, the time used in this paper will be non-dimensionalised as $t^* = t\, a_\infty / L$, where t^* is the non-dimensional time, t is the dimensional time and a_∞ is the reference acoustic velocity, defined as that which exists at the axis of rotation on the low pressure side. The rotational speed of the annulus, Ω,is represented by the outer wall Mach number, $M_\Omega = \Omega R_T / a_\infty$.

The initial condition is defined as constant total enthalpy in a rotating frame of reference, also known as constant rothalpy (Larosiliere and Mawid, 1995). Isothermal initial conditions, which are modeled for some of the simulations involving thermal conductivity, are found to have no significant effect on the results.

VALIDATION

To assess the accuracy of the simulation, the code was used to solve a one-dimensional inviscid shock tube problem, with no rotation or heat transfer. The results from the code are compared with the analytical solutions in Figure 2. The initial conditions used are P_1=100kPa, T_1=288K on the low pressure side and P_2=200kPa, T_2=288K on the high pressure side. The figure represents the situation at $t^* = 0.25$. The agreement between computation and theory is good with minor differences near the shock, contact surface and rarefaction wave. It is found that the shock and contact surface are smeared over four mesh points regardless of mesh density. Figure 3 shows the effect of increasing the number of mesh points. The shock front steepens, in closer agreement with theory, but this causes an overshoot in the pressure distribution downstream of the shock. This could be eliminated by adding some artificial viscosity to increase the dissipation. For reasons that will become apparent, it was decided not to do this.

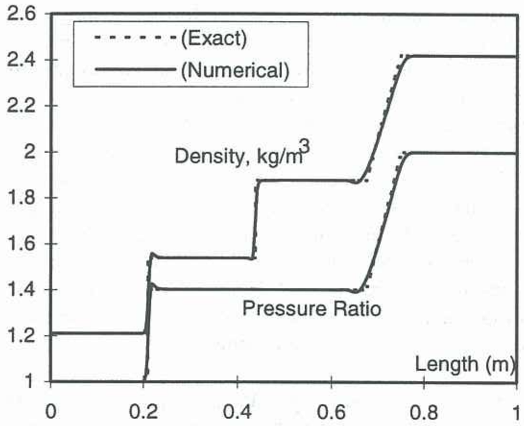

Figure 2 : Pressure and density distributions for one-dimensional shock tube simulation using a 400 node mesh at $t^* = 0.25$.

RESULTS AND DISCUSSION

Numerical simulations have been performed for the rotating shock tube configuration considered by Larosiliere and Mawid (1995). Figure 4 shows the density contours for p_2/p_1 = 2.0, T_2/T_1 = 1.0 and M_Ω = 1.0, which may be compared directly with their Figure 4. The time steps shown are for t^* = 0.25, 0.5, 0.75 and 1.0. The mesh was the same as that used by Larosiliere and Mawid, 450x30 in the longitudinal and radial directions, respectively. The viscosity (10^{-8} Pa s) and thermal conductivity (10^{-8} W m^{-1} K^{-1}) were effectively zero. The results from the current code are essentially identical to the previous work, except for the final time step. The shock wave travels from right to left along the shock tube, while the rarefaction wave travels from left to right. The contact surface also travels from right to left, albeit at a much slower rate than the shock. The contact surface leans because of the small initial density variation across the tube which leads to the axial velocity near the inner wall being less than that at the outer wall. After the shock has been reflected from the end wall, it travels back along the tube and passes through the contact surface. Larosiliere and Mawid observed the contact surface to remain smooth and to retain an "S" shape, whereas here, the contact surface has a small instability superimposed on this "S" shape, located slightly below the centre of the channel. A magnified view of the contact surface is shown in figure 5. The perturbation on the "S" shape is believed be a Richtmyer-Meshkov instability (Richtmeyer, 1960; Meshkov, 1969), caused by the reflected shock traveling through the distorted contact surface.

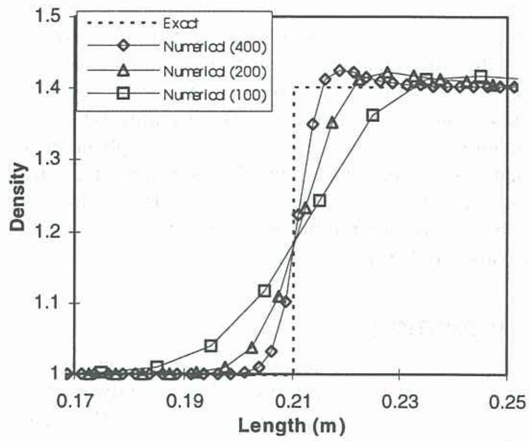

Figure 3 : Effect of increasing mesh density for one-dimensional shock tube problem, t^* =0.25.

When the present simulation was run again, using a viscosity equal to 1.788×10^{-5} Pa s (corresponding to that for air at 288K) and restarting from t^* = 0.75, the contact surface was found to be the same leaning "S" shape, but without the small roughness near the centre of the contact surface, that is, the same result as the previous work. A magnified view of the contact surface from this viscous simulation is shown in figure 6. Larosiliere and Mawid specifically add a "small amount of fourth-difference artificial dissipation...to suppress non-linear instabilities". It appears that, in their calculations, this dissipation damps out the instability observed in Figures 4 and 5. Yet, for an inviscid and zero heat transfer solution, these instabilities are inherent in the flow. The addition of a small amount of viscosity, whether artificially (Larosiliere and Mawid), or purposefully (present study), is therefore sufficient to damp out this particular instability.

Larosiliere and Mawid noticed that if the initial temperature ratio, T_2/T_1, was greater than unity, then the contact surface became unstable. This instability was only evident in one of their figures and they did not suggest a reason for its cause. In the present investigation, it was found for initial temperature ratios greater than unity that, in some instances, the contact surface become unstable before any waves had passed through it. This instability also occurred even if the viscosity of the fluid or the thermal conductivity was raised to levels far in excess of those experienced in air at the same conditions.

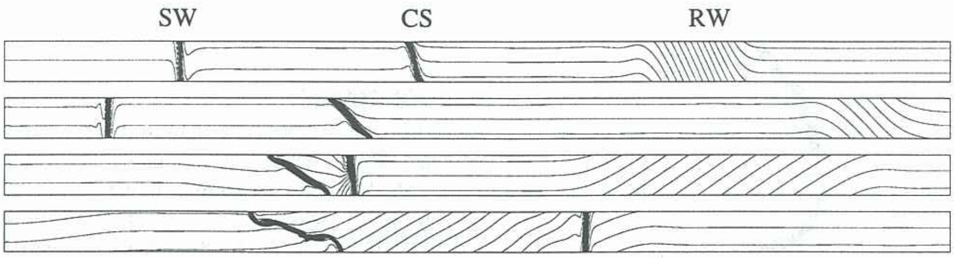

Figure 4 : Density contours for rotating shock tube with $p_2/p_1 = 2.0$, $T_2/T_1 = 1.0$ and $M_\Omega = 1.0$. Time increasing down the page with t* = 0.25, 0.5, 0.75 and 1.0. SW = shock wave, CS = contact surface, RW = rarefaction wave.

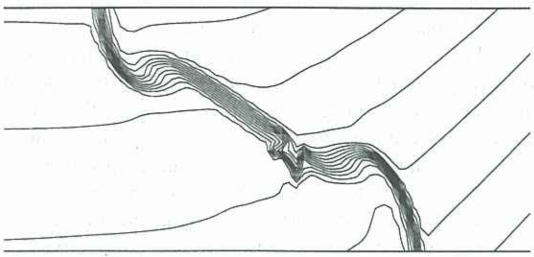

Figure 5 : Magnified view of the contact surface at $t^* = 1.0$ from Figure 4.

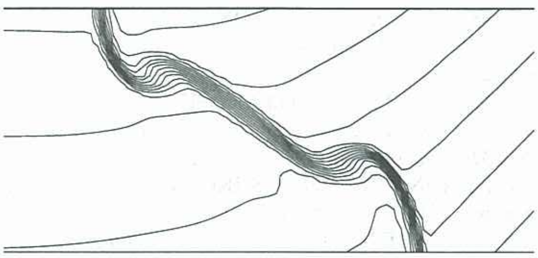

Figure 6 : Magnified view of the contact surface at $t^* = 1.0$, where the simulation was restarted from t^*=0.75 in Figure 4, now using laminar viscosity = $1.788\text{x}10^{-5}$ Pa.s.

An example of this is shown in Figure 7, where the initial conditions are $p_2 / p_1 = 2.0$, $T_2 / T_1 = 4.0$ and $M_\Omega = 1.0$. The time steps shown are $t^* = 0.0638$, 0.1276, 0.1913 and 0.2551. The contact surface leans in the opposite direction to that seen in Figures 4-6. The reason for this is unclear at this stage. Neither the shock nor the rarefaction wave have interacted with the contact surface during this time. However, as can be seen on the magnification of the density contours at the final time step (Figure 8), the contact surface has started to become unstable. The mechanism of this instability is very different to the instability visible in Figures 4 and 5, as it is not triggered by interaction with any waves. It is

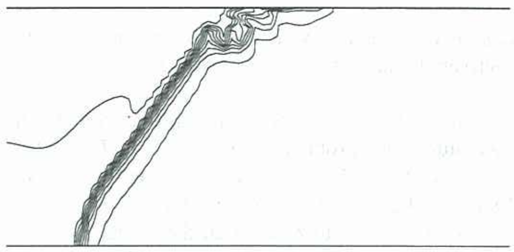

Figure 8 : Magnified view of the contact surface at $t^* = 0.2551$ for $p_2/p_1 = 2.0$, $T_2/T_1 = 4.0$ and $M_\Omega = 1.0$.

believed that this is a type of Rayleigh-Taylor instability, that is analogous to a dense gas initially above a lighter gas in a gravitational field. Here, the centripetal force acts in a similar manner to gravity.

From the magnification of the contact surface (Figure 8), it can be seen that the instability starts at the outside rim of the tube. The bulk fluid flow is from right to left, but the start of the instability is a small jet, which flows against the general fluid stream from left to right. The instability continues to grow from that shown in Figure 8, until the instability consumes the entire contact surface, at approximately $t^* = 0.9$ (Figure 9). The instability

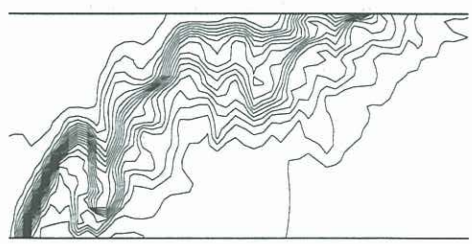

Figure 9 : Magnified view of the contact surface at $t^* = 0.8930$ for $p_2/p_1 = 2.0$, $T_2/T_1 = 4.0$ and $M_\Omega = 1.0$.

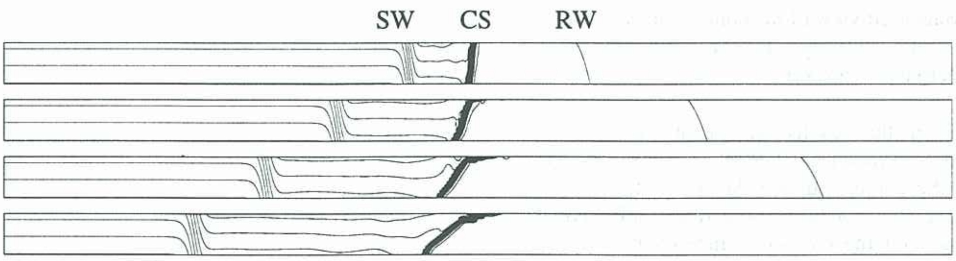

Figure 7 : Density contours for rotating shock tube with $p_2/p_1 = 2.0$, $T_2/T_1 = 4.0$ and $M_\Omega = 1.0$. Time increasing down the page with t* = 0.0638, 0.1276, 0.1913, 0.2551. SW = shock wave, CS = contact surface, RW = rarefaction wave.

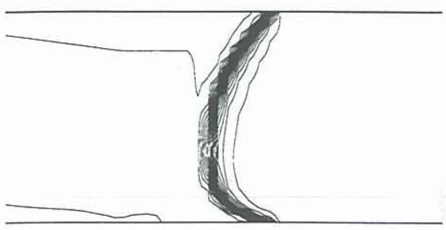

Figure 10 : Magnified view of the contact surface at $t^* = 0.2551$ for $p_2 / p_1 = 2.0$, $T_2 / T_1 = 4.0$ and $M_\Omega = 1.0$, viscosity = 1.788×10^{-1} Pa s.

stops growing, and now causes bulk mixing of the flow, effectively broadening the contact surface.

The effect of viscosity was investigated for the high temperature ratio problem ($p_2/p_1 = 2.0$, $T_2/T_1 = 4.0$ and $M_\Omega = 1.0$), with the viscosity ranging between 1.788×10^{-5} Pa s and 1.788×10^{-1} Pa s. The thermal conductivity was set to zero for all these simulations. As the viscosity was raised, boundary layer effects increased, distorting the contact surface into a "c" shape. The instability was present until the viscosity was 1.788×10^{-1} Pa.s (10^4 times that of air at 288 K), where the highly viscous fluid stopped the instability from forming (see Figure 10).

The effect of thermal conductivity was investigated for the high temperature ratio problem ($p_2/p_1 = 2.0$, $T_2/T_1 = 4.0$ and $M_\Omega = 1.0$), with the thermal conductivity ranging between 2.531×10^{-2} W m^{-1}K^{-1} and 2.531×10^{3} Wm^{-1}K^{-1}. The viscosity was set to zero for all these simulations. As the thermal conductivity was raised, the contact surface started to broaden as the effects of thermal conductivity started to become significant across the contact surface. For a thermal conductivity equal to 2.531×10^{2} W m^{-1}K^{-1} (10^4 times that of air at 288 K), the contact surface had broadened sufficiently to prevent the instability from forming (see Figure 11).

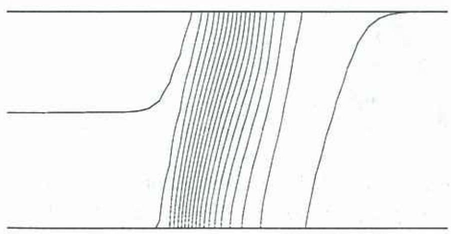

Figure 11 : Magnified view of the contact surface at $t^* = 0.2551$ for $p_2 / p_1 = 2.0$, $T_2 / T_1 = 4.0$ and $M_\Omega = 1.0$, thermal conductivity =2.531×10^{2} W m^{-1}K^{-1}.

It is clear from the results shown above that the instability is not a Prandtl number effect - viscosity acts on distorting the contact surface due to friction at the wall, preventing the instability occurring; and thermal conductivity acts on the contact surface by broadening the temperature gradient, stopping the instability. They are separate and independent actions.

CONCLUSION

The contact surface instabilities noticed by Larosiliere and Mawid (1995) in a rotating shock tube has been studied in greater detail. It has been found that the instability takes two forms: Richtmyer-Meshkov and Rayleigh-Taylor. The Richtmyer-Meshkov instability is caused by the interaction of the shock wave with the contact surface and could be eliminated by the addition of a small amount of viscosity. For this reason, it is suggested that the artificial viscosity employed by Larosiliere and Mawid did not permit them to see this form of instability. The Rayleigh-Taylor instability is caused by the centrifugal force acting in a similar manner to gravity in natural convection. Varying the viscosity and thermal conductivity altered the shape of the contact surface, causing curvature of the contact surface near the wall and contact surface broadening, respectively. In contrast to the Richtmeyer-Meshkov instability, the Rayleigh-Taylor instability was found to occur even when the viscosity (and also the thermal conductivity) of the simulated gas was raised to values far in excess of air.

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