13th Australasian Fluid Mechanics Conference
Monash University, Melbourne, Australia
13-18 December 1998

THE DEGENERATION OF BASIN-SCALE INTERNAL WAVES IN LAKES

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ABSTRACT

Mechanisms for the transfer of energy by the degeneration of basin-scale internal waves in lakes are identified. By ordering the timescales over which each of these mechanisms act, regimes are defined in which particular processes are expected to dominate. Comparison of the predictions of this timescale analysis with the results from laboratory experiments confirms its applicabilty and supports the conclusion that in most lakes nonlinear steepening dominates the degeneration of basin-scale internal waves created by moderate to strong wind events. Since solitary waves are more likely to break at the sloping boundaries, leading to localised turbulent mixing and enhanced dissipation, the transfer of energy from an initial basin-scale seiche to shorter solitary waves has important implications for water quality.

INTRODUCTION

In lakes the major source of energy for the internal wavefield is the action of the wind over the surface of the lake, generating basin-scale internal standing waves known as seiches (Mortimer, 1952). Field observations show that these basin-scale waves decay at a rate far greater than can be accounted for simply by internal dissipation (Imberger, 1994). Furthermore, the internal wave spectra in lakes, although continuous, varies in time in response to wind forcing. Saggio & Imberger (1998) showed that after the passage of a storm the internal wave spectrum in Lake Biwa contained generally higher energy levels with pronounced peaks, corresponding with an increase of energy of not only the basin-scale waves but also of a group of higher frequency waves that they identified as nonlinear waves. The observed decay times and the time varying spectra suggest that energy is transferred, apparently rapidly, within the internal wave spectrum from the basin-scale waves to waves of higher wavenumber and frequency.

This paper investigates mechanisms that contribute to the degeneration of the internal basin-scale seiche. In particular, the relative importance of viscous damping, nonlinear steepening, internal bore formation and shear instabilities are considered and, by ordering the timescales over which each of these mechanisms acts, regimes are identified in which particular mechanisms would be expected to dominate the degeneration of basin-scale internal waves in lakes. The applicability of these regimes is demonstrated by comparing the predictions with the results of laboratory experiments.

TIMESCALES AND REGIMES

Each of the four degeneration mechanisms considered here operates over a timescale that is determined by the amplitude of the initial seiche, the dimensions of the basin and the stratification. In deriving these timescales we use a quasi two-layer rectangular model with a rigid lid and neglect the effects of rotation. The length, depth and width of the basin are given by L, H and W, respectively, and the equilibrium layer depths by h_1 and h_2. The fluid in each layer has a constant density and we define an effective gravity $g' = g\Delta\rho/\rho_0$. The linear internal long-wave speed is then given by $c_0 = (g'h_1h_2/H)^{1/2}$ and the period of the standing basin-scale wave by $T_i = 2L/c_0$. The initial condition consists of a tilted interface where η_0 is the initial interface displacement at the ends of the basin and the velocity field is assumed to initially evolve in accordance with linear theory. The two-layer assumption is valid so long as the thickness of the thermocline δ_ρ is thin compared with the thickness of the surface layer h_1 ($\delta_\rho \ll h_1$).

Viscous damping timescale: T_d

In the two layer rectangular model viscous losses occur in boundary layers at the solid boundaries and in the shear layer at the density interface. It can be shown (e.g. Spigel & Imberger, 1980) that the interface displacement decays as $\eta = \eta_0 e^{-\alpha_d t/T_i}$, suggesting a dissipative timescale

$$T_d = \frac{T_i}{\alpha_d} \tag{1}$$

T_d is the e-folding time for the decay of the wave amplitude. In the laboratory, where the boundary layers are laminar, there is a rigid lid and interfacial losses are significant, we have

$$\alpha_d = \frac{\pi \delta_b A_b}{2V} + \frac{\nu H T_i}{2\delta_\rho h_1 h_2} \quad (2)$$

where $\delta_b = (\nu T_i/\pi)^{1/2}$ is the thickness of the boundary layer, ν the kinematic viscosity, A_b is the area of the boundary layer and V is the total volume. In lakes, where the boundary layers are turbulent, there is a free upper surface and the interfacial losses are relatively small, Spigel & Imberger (1980) simplified (2) to

$$\alpha_d = \frac{\delta_b A_b}{V} \quad (3)$$

where the boundary layer thickness is now given by $\delta_b = (U_{\max} T_i^{1/2} e)/(471\nu^{1/2})$, $U_{\max}$ the maximum velocity in the lower layer and e is the grain roughness.

Nonlinear steepening timescale: T_s

The initial steepening of a long wave due to nonlinear effects can be described by the equation (e.g. Long, 1972)

$$\eta_t + c_0 \eta_x + \beta \eta \eta_x = 0 \quad (4)$$

where $\beta = \frac{3}{2} c_0 (h_1 - h_2)/(h_1 h_2)$. Balancing the unsteady and nonlinear terms leads to a steepening timescale for a basin-scale internal wave of length L and initial amplitude η_0

$$T_s \sim \frac{L}{\beta \eta_0} \quad (5)$$

As the wave steepens its horizontal lengthscale decreases until dispersive effects, neglected in the derivation of (4), eventually balance the nonlinear steepening and solitary waves evolve.

Timescale for formation of internal bores: T_b

In a two-layer model we can define upper and lower layer Froude numbers as $F_1^2 = U_1^2/g'h_1$ and $F_2^2 = U_2^2/g'h_2$, in which case the flow in the thin upper layer becomes supercritical when $F_1^2 + F_2^2 = 1$ (Wood & Simpson, 1984). When the upper layer is supercritical the seiche cannot propagate against the current and an internal bore develops. The time at which the flow first becomes critical at the centre of the basin (where the velocities are a maximum) is given by

$$T_b = \frac{T_i}{4} h_1 \eta_0 \left(\frac{H h_2^2}{h_1^3 + h_2^3} \right)^{1/2} \quad (6)$$

If $T_b > T_i/4$ the flow never becomes supercritical and a bore of this type will not form.

Kelvin-Helmholtz timescale: T_{KH}

The basin-scale seiche results in considerable interfacial shear that is periodic and has a maximum value at the node. Defining a local Richardson number in terms of the thickness of the thermocline, $Ri = g'\delta_\rho/(\Delta U)^2$, and using a stability criterion of $Ri > 0.25$, the minimum shear necessary for Kelvin-Helmholtz billows is thus $\Delta U = 2(g'\delta_\rho)^{1/2}$. Since the wave-induced shear initially increases (for $0 < t < T_i/4$), the flow is always stable before the time

$$T_{KH} = \frac{L}{\eta_0} \left(\frac{\delta_\rho}{g'} \right)^{1/2} \quad (7)$$

If $T_{KH} > T_i/4$ the flow will remain stable. The derivation of T_{KH} assumes that the flow is steady and takes no account of the finite time required for the formation of billows. It can be shown that this is valid for $\eta_0/L \ll 0.1$, which will be true for most lakes.

The shear instability timescale T_{KH} refers to shear instabilities at the node of a standing basin-scale wave. Kelvin-Helmholtz billows can also be generated by the shear induced by smaller scale propagating internal waves such as solitary waves and surges.

Regimes

Depending on the relative ordering of these four timescales, a number of *regimes* can be defined in which a particular mechanism is expected to dominate. By equating timescales, the regime boundaries can be defined in terms of the amplitude of the initial basin-scale seiche η_0 and the depth of the interface h_1. Since T_d and T_{KH} both depend on other characteristics of the lake and stratification (in particular on L, g' and δ_ρ), the regime boundaries need to be calculated individually for each lake. As an example, figure 1 has been calculated for the laboratory experiments described below using a representative value of δ_ρ=1 cm.

Within a regime more than one mechanism can contribute to the degeneration of the initial basin-scale wave, but as our experimental observations confirm it is likely that the first mechanism to occur will be the most important.

Regime 1 (Damped seiche): $T_d < T_s$

If the damping timescale is shorter than the timescale for nonlinear steepening, the amplitude of the initial basin-scale wave will be damped before it can steepen and evolve into solitary waves. In this case, linear theory is expected to capture the dynamics of the flow. The largest amplitude seiche that will be damped before it evolves into solitary waves is determined by setting $T_d = T_s$ using (1) and (5)

$$\eta_0 = \frac{\alpha_d}{3} \left(\frac{h_1 h_2}{h_2 - h_1} \right) \quad (8)$$

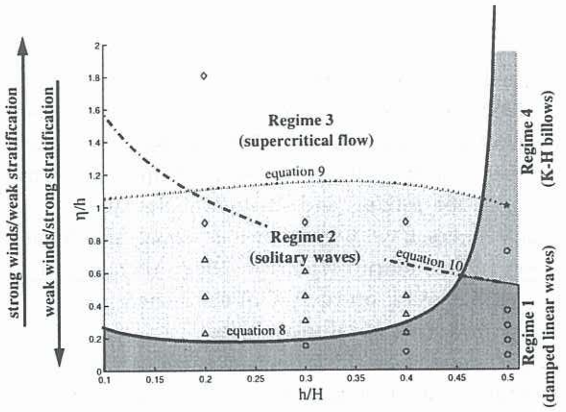


Figure 1: The regime diagram for the laboratory experiments. The regime boundaries are determined by the curves: $T_d = T_s$ (—), $T_b = T_i/4$ (···) and $T_{KH} = T_i/4$ (-·-·-). The laboratory observations are also plotted (⋆ K-H billows and bore, ◇ broken undular bore, △ solitary waves, □ steepening , o damped linear waves).

Regime 2 (Solitary waves): $T_s < T_d$

In this regime the initial basin-scale wave will steepen due to nonlinear effects and will evolve into a train of solitary waves. This mechanism results in the transfer of energy within the internal wavefield from the basin-scale seiche to waves of higher wavenumber and frequency. Figure 1 shows that Regime 2 covers most of the parameter space for lakes and reservoirs under moderate to strong wind events.

Regime 3 (Internal bore): $T_b < T_i/4$

This defines the regime in which, at some time, the velocity in the surface layer exceeds the linear long wave speed and a bore forms. The energy in the initial wave is lost to mixing and enhanced dissipation in the case of turbulent bores and to shorter solitary waves in the case of undular bores. Setting T_b to its maximum value of $T_i/4$ yields the amplitude of the smallest thermocline tilt that would generate a supercritical flow

$$\eta_0 = h_1 \left(\frac{h_1^3 + h_2^3}{h_2^2 H} \right) \tag{9}$$

This displacement corresponds to upwelling of the thermocline which is an extreme and complex event, suggesting that internal bores of this type are unlikely to occur in lakes.

Regime 4 (K-H billows): $T_{KH} < T_i/4, T_{KH} < T_s$

In this regime the wave induced shear across the thermocline causes the local Richardson number to fall below 0.25 and Kelvin-Helmholtz billows form at the node. By setting T_{KH} to its maximum value of $T_i/4$ the minimum thermocline tilt that would lead to billowing is given by

$$\eta_0 = 2 \left(\frac{\delta_\rho h_1 h_2}{H} \right)^{1/2} \tag{10}$$

However, since $T_s < T_{KH}$ (except in the special case when $h_1 \sim h_2$) any seiche is likely to steepen and evolve into solitary waves before billows develop. If the solitary waves extract sufficient energy from the basin-scale wave, its amplitude in turn will be reduced so that the shear is no longer sufficient to induce billowing at the node (although the solitary waves may induce local billowing). We therefore include in our definition of Regime 4 the requirement that $T_{KH} < T_s$. These conditions will only usually be satisfied when the thermocline is near the mid-depth in a lake.

LABORATORY EXPERIMENTS

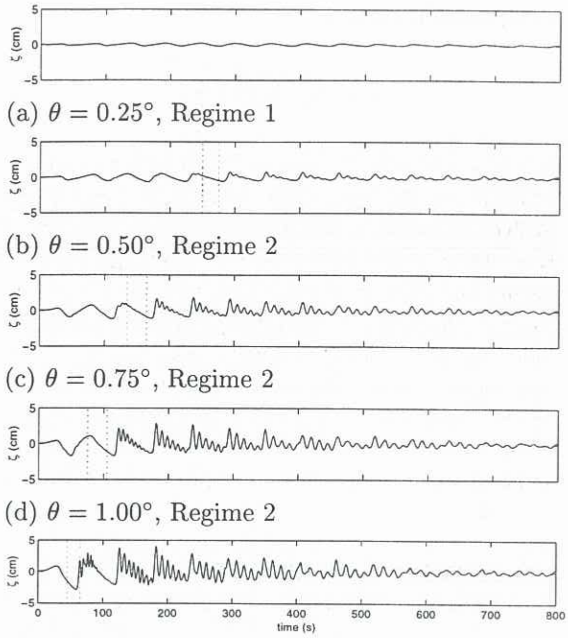
To determine the applicability of these regimes a series of laboratory experiments was carried out in a tilting tank, 6 m long 29 cm wide and 30 cm deep. After filling with a two-layer stratification, using fresh and salt water, the tank was slowly rotated until it was tilted at a small angle to the horizontal, θ. To commence the experiment the tank was quickly returned to a horizontal position so that the interface was then inclined at the original angle of tilt of the tank. The resulting flow was recorded on video and still photographs and by ultra-sonic internal wavegauges.

The experimental variables considered in this study were: the angle of tilt, θ (0.125°–2.77° $\pm$ 0.03°) and the interface depth, h (5.8–14.5$\pm$0.2 cm). The overall density difference between the upper and lower layers was kept constant at $\Delta\rho \approx 20 \pm 2$ kg/m^3.

In the laboratory experiments, to accomodate the ultrasonic wavegauges, the interface was always below the mid-depth position.

Experimental results

When the initially tilted tank was returned to the horizontal the fluid immediately responded to the baroclinic pressure gradient. The evolving wavefield is best observed by examining the time series of interfacial displacements recorded by the ultrasonic wavegauges. Figure 2 shows a typical timeseries from the wavegauge at the centre of the tank for a number of experiments with varying initial angles of tilt. In those experiments in which the the nonlinearity was very weak (that is, for very small angles of tilt or when the interface was near the mid-depth) the interface oscillated as a basin-scale wave until the motion was damped by viscous effects. However, as the nonlinearity of the initial wave was increased (by increasing the angle of tilt or by reducing the depth of the thinner layer), the initial basin-scale wave was observed to steepen and evolve into a train of solitary waves.



(a) $\theta = 0.25°$, Regime 1

(b) $\theta = 0.50°$, Regime 2

(c) $\theta = 0.75°$, Regime 2

(d) $\theta = 1.00°$, Regime 2

(e) $\theta = 1.50°$, Regime 2

Figure 2: As the angle of tilt increases, the initial wave is observed to steepen more quickly and the number and amplitude of the emerging solitary waves increases.(h/H=0.3)

In some experiments with large tilts or small depth ratios an internal bore or surge was observed. The timescale analysis confirms that these surges were generated by nonlinear steepening of the initial basin-scale wave and were not the result of supercritical flow in the thinner layer. Shear instabilities induced by the basin-scale wave were only observed in one experiment in which the interface was at mid-depth and was inclined through the maximum angle.

Figure 1 plots the observations from the laboratory experiments on the regime diagram determined by the theoretical timescales previously developed.

DISCUSSION AND CONCLUSIONS

The results of the laboratory experiments described above confirm the general applicability of the timescale analysis and proposed regime classification. Furthermore, they support the hypothesis that the nonlinear steepening of the initial basin-scale wave and subsequent formation of solitary waves is the dominant mechanism for the degeneration of basin-scale internal waves in lakes. The predictions of this analysis were also compared to a number of field observations[1] which showed that for moderate and strong wind events most lakes fall within Regime 2 (solitary waves).

An important feature of the generation of solitary waves by the nonlinear steepening of an initial seiche is that energy is transferred within the internal wavefield from the basin-scale wave to waves of higher wavenumber and frequency. This transfer of energy to these shorter solitary waves has important consequences for mixing and dissipation in lakes since solitary waves have been shown to shoal and break at sloping boundaries (Helfrich, 1992, Michallet and Ivey, 1998), losing up to 70% of their energy on first interaction with typical lake slopes.

The timescale for the formation of solitary waves is important in determining where and in what direction the solitary waves first appear and, hence, where the waves will shoal and dissipate their energy. Breaking solitary waves are likely to be a major energy source for sustaining the turbulent benthic boundary layer in lakes and for driving the vertical transport of nutrients so that this process has important consequences for the aquatic ecology of lakes.

The significance of nonlinear steepening and solitary waves in lakes has implications for the way in which we model the hydrodynamics of these systems. Most numerical models employed to study the hydrodynamics of lakes make the hydrostatic approximation. While these models have been used to satisfactorily simulate most flows, by neglecting vertical accelerations they are unable to capture the evolution and propagation of solitary waves and therefore the transfer of energy from the basin-scale seiche to these shorter waves and the subsequent rapid dissipation and mixing caused by their shoaling.

[1]The results of this comparison are not included here but are available from the authors on request

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