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RESPONSE OF CYLINDRICAL STRUCTURES TO VORTEX SHEDDING IN THE NATURAL WIND

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INTRODUCTION

The alternate shedding of vortices from circular cylindrical structures is a well-known phenomenon in fluid mechanics that has been observed for more than a hundred years. Under certain conditions it can cause severe cross-wind oscillations on structures in the natural wind, such as chimneys, lighting poles and observation towers, usually at or near a critical velocity at which the vortex shedding frequency coincides with a natural frequency of the structure. However, the accurate prediction of the magnitude of the deflections, and the equivalent static forces acting on the structure, is difficult due to the many variables involved. The principal variables are : mean velocity profile and turbulence characteristics in the approach flow, Reynolds Number, taper, natural frequency, mode shape, damping, mass distribution and surface roughness.

This paper critically reviews four commonly-used methods for calculation of cross-wind response of structures in the natural wind due to vortex shedding. The methods can be divided into two classes : a) sinusoidal or harmonic excitation models, and b) random excitation models. The random excitation models are generally more complex to apply. In some cases, the cross-wind excitation and response due to lateral turbulence exceeds that due to vortex shedding, and extensive calculations for vortex shedding would be unjustified in that case.

SINUSOIDAL EXCITATION MODELS

The assumption that the vortex shedding phenomenon generates near- sinusoidal cross wind forces on circular cylinders can be linked to the work of Scruton and co-workers in the nineteen fifties and sixties[1], although in his formulation he treated the excitation forces solely as a form of negative aerodynamic damping. This model is a good one for situations in which large oscillations occur, and the shedding has 'locked-on' to the cross-wind motion of the structure. The assumption of sinusoidal excitation leads to responses which are also sinusoidal.

With this model, the ratio of vibration amplitude at the tip of a uniform cantilevered cylinder, to the cylinder diameter, can be evaluated as :

$$a/d = k\, C_L/(4\pi\, Sc\, St^2) \qquad (1)$$

where k is dependent weakly on the mode shape of vibration (for a power law with exponent of n, k is equal to (2n+1)/(n+1)

C_L is the amplitude of the sinusoidal lift coefficient
Sc is the mass-damping parameter or 'Scruton Number'
St is the Strouhal Number for vortex shedding

The Scruton Number is defined as :

$$Sc = 4\pi\, m\, \varsigma/(\rho\, d^2) \quad \text{or } 2m\, \delta/(\rho\, d^2) \qquad (2)$$

where m is the mass per unit length along the structure (or effective mass for a structure vibrating in a non-uniform mode shape)

ς is the critical damping ratio
δ is the logarithmic decrement (= $2\pi\,\varsigma$, for low damping)
ρ is the density of air
d is the diameter of the cylinder

Ruscheweyh [2] has modified the basic sinusoidal model by the use of a 'correlation length'. The term 'correlation length' is one that is normally applied to random processes or excitation, and a better term would be 'excitation length'. The vortex shedding forces are applied over a height range less than the total height of the structure in this model. This model has been adopted for the draft wind loading code of the European Community [3], for which the following equation is given :

$$a/d = K_w \,.\, K.\ c_{lat} \,.\, (1/St^2).\ (1/Sc) \qquad (3)$$

where, K_w is an 'effective correlation length factor', which does not exceed 0.6, and decreases with increasing aspect ratio,

K is a mode shape factor, equal to 0.13 for a cantilever.

c_{lat} (=C_L) is the lateral force coefficient, depending on Reynolds Number

Ruscheweyh's model apparently agrees well with measurements of vortex-induced vibrations on a number of steel chimneys [2]. However, similar agreement can be obtained by use of a simpler equation based on the basic sinusoidal model and Equation (1). In this case,

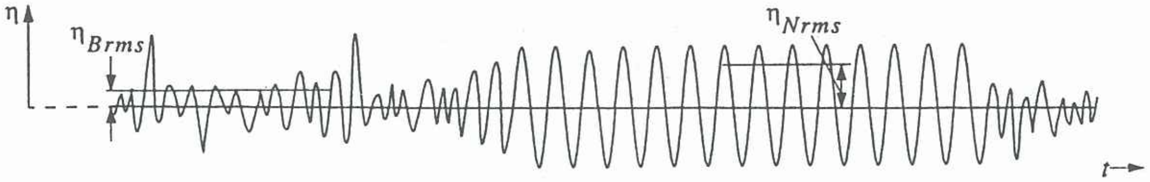

Figure 1 : Switching between wide-band random and sinusoidal response (from [6]).

C_L, St can be taken as 'constants' and combined with k, to give:

$$a/d \approx K / Sc \qquad (4)$$

The value of K is about 0.5.

RANDOM EXCITATION MODELS

Random excitation models of vortex shedding excitation were developed over many years by Vickery and co-workers [4,5]. With some approximations, the peak deflection at the tip as a ratio of diameter can be written in the following form for a uniform cantilever :

$$a/d = g\frac{\left[n_1 S_{CL}(n_1)\right]^{1/2}\left(\rho d^2/m\right)}{16\pi^{3/2}\zeta^{1/2}St^2}f(\mu) \qquad (5)$$

where $S_{CL}(n)$ is the spectral density of the generalised crosswind force coefficient

$f(\mu)$ is a function of mode shape

g is a peak factor which depends on the resonant frequency, but is usually taken as 3.5 to 4

Equation (5) has some similarities with Equation (1), but it should be noted that in the case of random vibration, the response is inversely proportional to the *square root* of the damping, whereas in the case of Sinusoidal excitation, the peak response is inversely proportional to the damping (see Equations (3), (4)). The peak factor (ratio between peak and r.m.s. response) is also much greater than the value of $\sqrt{2}$ in the sinusoidal model. The spectral density includes the effect of correlation length on the fluctuating forces.

In Vickery's procedure [5], the spectral density of the local lift force is represented by a Gaussian function, and lock-in is dealt with by an amplitude-dependent aerodynamic damping, within the random excitation model. The method has been calibrated to the response of large concrete chimneys.

HYBRID MODEL

Item 96030 of the Engineering Sciences Data Unit (E.S.D.U.) covers the response of structures of circular and polygonal cross-section to vortex shedding. A computer program and spreadsheet is provided to implement the methods. ESDU 96030 [6] covers uniform, tapered and stepped cylindrical or polygonal structures, and also yawed flow situations.

The method used in ESDU 96030 appears to be a hybrid of the two previously described approaches. For low amplitudes of vibration, a random excitation model similar to that of Vickery, has been adopted. At high amplitudes, i.e. in lock-in situations, a sinusoidal excitation model has been adopted, with a cross-wind force coefficient that is non-linearly dependent on the vibration amplitude. Figure 1 illustrates the type of response postulated by ESDU – the response is postulated to switch intermittently between a random wide-band response and a constant amplitude sinusoidal type, as 'lock-in' occurs.

The effect of cross-wind turbulence excitation is also included in this method. This contribution becomes more significant with increasing wind speed, and thus is more important for larger cylinders (e.g. large diameter reinforced concrete chimneys with high critical wind speeds)

COMPARISON OF COMPUTED RESPONSES

A comparison of the computed response to vortex shedding of a) a 100 metre steel chimney, b) a 250 metre reinforced concrete chimney, and c) a 25-metre thin-walled, steel lighting pole is described in this section.

The relevant details of the three structures are given in Table 1 following.

Property	Structure 1	Structure 2	Structure 3
Height (m)	100	250	25
Diameter (m)	4.9	20	0.55-0.20 (tapered)
Surface roughness (mm)	0.1	1	0.15
Natural frequency (Hz)	0.5	0.3	0.5
Mode shape exponent	2	1.6	2
Mass/unit height (Kg/m) (top third)	1700	50,000	30
Critical damping ratio	0.005	0.01	0.005

Table 1 : Structural Properties

These represent a wide range of structural types for which the cross-wind response needs to be assessed. In all three cases, the structures were assumed to be located in open country terrain, with relevant velocity profile and turbulence properties based on the values given in the Australian Standard, AS1170.2-1989 [7]. In this comparison, only the first mode of vibration was considered.

The maximum r.m.s. ratio of tip deflection/mean diameter, for the three structures have been calculated by the following methods and tabulated in Table 2 :

a) Method given in the European pre-standard [3].

b) The simple formula of Equation (4), taking K equal to 0.5.

c) Vickery's random excitation approach [5], (Structures 1,2 only)

d) ESDU 96030 and 96031 [6,8], using the spreadsheet supplied with [8]

Method	Structure 1	Structure 2	Structure 3
(a)	0.080	0.032	0.016
(b)	0.095	0.027	0.020
(c)	0.214	0.0045	n.a.
(d)	0.308	0.0054	0.014

Table 2 : Calculated values of maximum r.m.s. tip deflection/diameter (at or near critical velocity)

Figures 2 and 3 show the variation of r.m.s. deflection / diameter ratio with windspeed, as calculated by Methods (c) and (d) respectively.

DISCUSSION AND CONCLUSIONS

The four methods compared in Table 2 clearly give significant variations in estimated response to vortex shedding, for all three structures. In the case of Structure (1), all methods predict large amplitudes characteristic of 'lock-in', although Methods (c) and (d) predict higher amplitudes. Methods (a) and (b), based on sinusoidal excitation, overestimate the response of Structure 2 (a large reinforced concrete chimney), which is subject to wide-band excitation with low amplitudes. Methods (c) and (d) are also compared for Structure 2 in Figures 2(b) and 3(b). Although the computed maximum responses at the critical wind speed (about 35 m/s) are similar, the ESDU approach (Method (d)) shows higher response at higher wind speeds, due to the inclusion of response due to lateral turbulence.

Vickery's model [5] is currently applicable to high Reynolds Numbers only, and has not been applied to Structure 3, which is clearly in the sub-critical regime.

The other methods predict a low response amplitude for Structure 3 which has a very low critical velocity in the first mode, although this type of low-mass pole or mast has a history of occasional large vortex shedding responses, sometimes in higher modes, and often producing fatigue problems, e.g.[9]. One of the main problems in predicting their behaviour is in predicting the structural damping ratio, which is often very amplitude dependent.

A reasonable amount of research has gone into the problem of vortex-shedding induced cross-wind vibration, but there are still significant differences in the predictions by the various accepted methods, and only one of these can claim to be a universal one for structures of all sizes [6]. However, the use of the simple relationship of Equation (4), (Method (b)) as an easy-to-use rule-of–thumb, seems appropriate for design codes or standards, to provide for quick checks by structural engineers.

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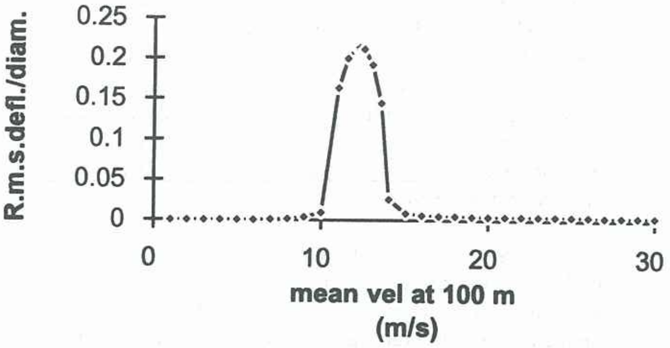
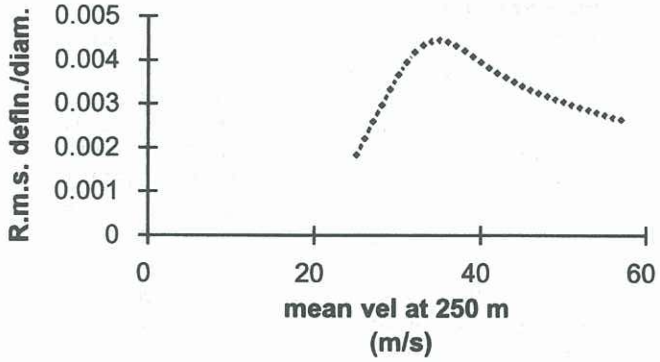
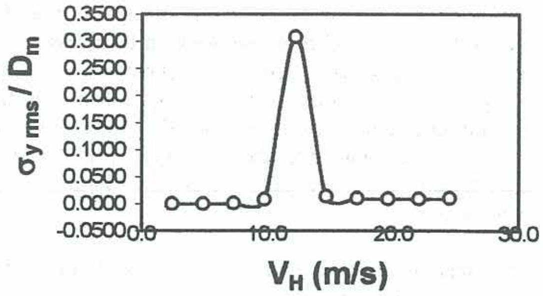


(a) Structure 1 (100-metre steel chimney)

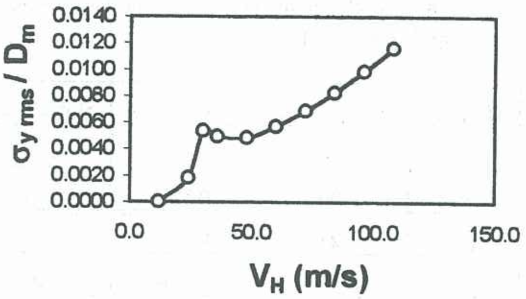


(b) Structure 2 (250-metre r.c. chimney)

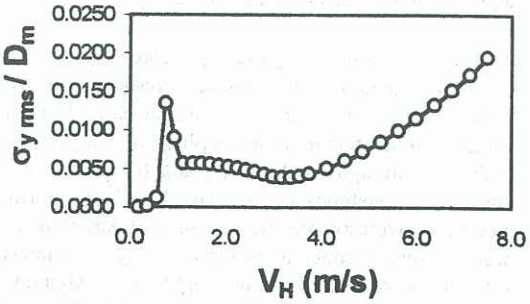
Figure 2. R.m.s. deflection / diameter versus windspeed
(as predicted by Method (c) [Vickery])



(a) Structure 1 (100-metre steel chimney)



(b) Structure 2 (250-metre r.c. chimney)



(c) Structure 3 (25-metre steel lighting pole)

Figure 3. R.m.s. deflection / diameter versus windspeed
(as predicted by Method (d) [ESDU])