13th Australasian Fluid Mechanics Conference
Monash University, Melbourne, Australia
13-18 December 1998

HYDRODYNAMIC SECTIONAL ADDED MASS AND DAMPING FOR A SHIP MOVING WITH FORWARD SPEED

Damien S. HOLLOWAY and Michael R. DAVIS

School of Civil and Mechanical Engineering
University of Tasmania, Hobart, Tasmania, AUSTRALIA

ABSTRACT

Two dimensional sectional added mass and damping calculations for a ship moving with forward speed using traditional frequency domain strip theory and a new time domain fixed reference frame strip theory are compared.

NOTATION

a zero speed sectional added mass
a_{FH} forward speed sectional added mass in heave
a_{FP} forward speed sectional added mass in pitch
a_F forward speed sectional added mass for combined heave and pitch
A_{ij} vessel added mass coefficient
b, B damping (subscripts as for a)
F_3 heave force $= \int \Delta F\, dx$
F_5 pitch moment $= -\int x\Delta F\, dx$
ΔF sectional force per unit length of hull $= \frac{dF_3}{dx}$
h_x complex local heave amplitude at x
n normal direction on hull-fluid boundary
n_z vertical component of unit normal
U forward speed of ship
x longitudinal hull coordinate (relative to LCG)
$z = a + \frac{b}{i\omega}$, subscripts as for a (similarly Z)
ρ fluid (water) density
θ_{53} phase lag of pitch relative to heave
ω frequency of oscillation
ξ_3, ξ_5 complex global heave/pitch amplitude

INTRODUCTION

An oscillating ship hull, in the absence of waves, experiences hydrodynamic forces traditionally categorised as added mass (arising from inertia of the surrounding water) and damping (from energy dissipation in the radiated waves), in addition to a hydrostatic force. In order to analyse these forces "strip theories" are often used, in which it is assumed on the basis of hull slenderness and some restrictions on the wave encounter frequency that the flow surrounding the hull is essentially two-dimensional. The cross sectional forces are then expressed in terms of these sectional added mass and damping coefficients. For a detailed description of the most used form of strip theory see Salvesen, Tuck and Faltinsen (1970).

Several variants of strip theory exist, including a fixed reference frame theory (Holloway, 1998). The two dimensional hydrodynamic problem associated with the radiated and diffracted waves is solved in strips fixed in space rather than moving forward with the boat. The advantage is that there are no longer any forward speed terms in the free surface boundary condition, and this boundary condition is now purely two dimensional. The only neglected dependence on x is $\frac{\partial^2\phi}{\partial x^2}$ in Laplace's equation, which will be small for a slender hull, hence this type of theory is frequently referred to as a $2\frac{1}{2}$D theory. The theory is valid for large vessel Froude numbers, unlike traditional (2D) strip theories.

This paper compares estimates of added mass and damping from the two mentioned strip theories, primarily for the purpose of validating the new theory.

BACKGROUND THEORY

Added Mass and Damping Coefficients

Sectional Forces According to Conventional (Two-Dimensional) Strip Theory.

Let ϕ be the radiated wave potential for a ship oscillating with small amplitude and moving with forward speed U in otherwise undisturbed water. Given the linearised form of pressure evaluated in a reference frame moving with the forward speed of the ship as $-\rho\left(\frac{\partial\phi}{\partial t} - U\frac{\partial\phi}{\partial x}\right)$ one obtains $\Delta F = -\rho\left(p - U\frac{\partial P}{\partial x}\right)$ where $p = \int_{section} \frac{\partial\phi}{\partial t} n_z dl$ and $P = \int_{section} \phi n_z dl$.

The hull boundary condition for periodic motion is $\frac{\partial\phi}{\partial n} = \left(\frac{\partial}{\partial t} - U\frac{\partial}{\partial x}\right)\left(e^{i\omega t}\left(\xi_3 - x\xi_5\right)\right) n_z$, which is satisfied by $\phi = i\omega\left(\xi_3 + \xi_5\left(x - \frac{U}{i\omega}\right)\right) e^{i\omega t}\phi_0$ where $\frac{\partial\phi_0}{\partial n} = n_z$ on the hull surface. ϕ and ϕ_0 must also satisfy $\frac{\partial^2\phi}{\partial t^2} + g\frac{\partial\phi}{\partial z} = 0$ in a fixed reference frame on the free surface, but if $\frac{U}{\omega}$ is small compared with the wavelength it becomes sufficient to satisfy this boundary condition in the above moving reference frame. This gives $P = \frac{e^{i\omega t}}{\rho}\left(\xi_3 - \xi_5\left(x - \frac{U}{i\omega}\right)\right) i\omega z$ and $p =$

$i\omega P$, where we define $z(x,\omega) = a(x,\omega) + \frac{b(x,\omega)}{i\omega} = \int_{section} \phi_0 n_z dl$. These sectional added mass and damping forces are calculated using a two dimensional frequency domain free surface panel method similar to that described in Doctors (1987), showing excellent agreement (Holloway, 1998). Figure 1 shows an example of further validation against analytical results of Ursell (1949).

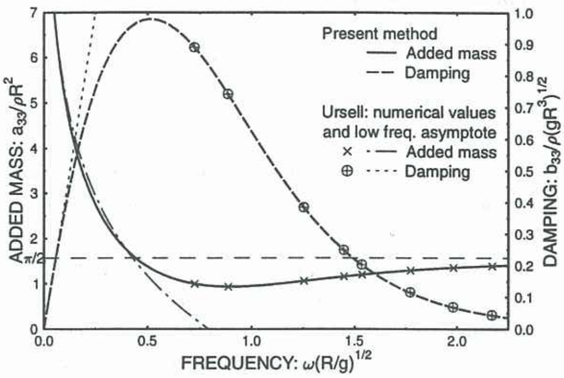

Figure 1: Added mass and damping for a semi-circular cylinder, radius R: comparison with analytical results of Ursell (1949).

If it is assumed for slender hulls that $\int_{section} \frac{\partial\phi}{\partial x} n_z dl = \frac{\partial P}{\partial x}$ (i.e. that $\int_{\frac{\partial}{\partial x} section} \phi n_z dl = \int_{section} \phi \frac{\partial n_z}{\partial x} dl = 0$) then it follows

$$\Delta F = e^{i\omega t}\left\{\left(\xi_3 - \xi_5\left(x - \frac{U}{i\omega}\right)\right)\omega^2 z + U\left[\left(\xi_3 - \xi_5\left(x - \frac{U}{i\omega}\right)\right) i\omega\frac{\partial z}{\partial x} - i\omega\xi_5 z\right]\right\}.$$

The complex sectional force amplitude for a unit heave and pitch motions are therefore

$$\frac{\frac{\partial\Delta F}{\partial\xi_3}}{e^{i\omega t}} = \omega^2 z + i\omega U\frac{\partial z}{\partial x}, \tag{1}$$

$$\frac{\frac{\partial\Delta F}{\partial\xi_5}}{e^{i\omega t}} = -\left(\omega^2 x + i\omega 2U\right) z + \left(U^2 - i\omega U x\right)\frac{\partial z}{\partial x}. \tag{2}$$

Forward Speed Sectional Added Mass and Damping Coefficients for Heave or Pitch.

The total vessel added mass and damping coefficients in terms of total hydrodynamic forces are according to their standard definition (Salvesen *et al.*, 1970) $Z_{ij} = \frac{\frac{\partial F_i}{\partial\xi_j}}{\omega^2 e^{i\omega t}}$, thus $\frac{\partial Z_{3j}}{\partial x} = \frac{\frac{\partial\Delta F}{\partial\xi_j}}{\omega^2 e^{i\omega t}}$ (and obviously $\frac{\partial Z_{5j}}{\partial x} = -x\frac{\partial Z_{3j}}{\partial x}$). We could define forward speed sectional added mass and damping coefficients for heave and pitch motion as $a_{FH} + \frac{b_{FH}}{i\omega} = \frac{\partial Z_{33}}{\partial x}$ and $a_{FP} + \frac{b_{FP}}{i\omega} = \frac{\partial Z_{35}}{\partial x}$. Note that, to avoid infinite values at the centre of pitch, a_{FP} and b_{FP} are the forces for a unit global pitch acceleration and velocity rather than the usual unit local heave acceleration and velocity. Substituting (1) and (2) into these expressions gives

$$a_{FH} = a + \frac{U}{\omega^2}\frac{\partial b}{\partial x} \tag{3}$$

$$a_{FP} = -ax - \frac{2Ub}{\omega^2} + \frac{U^2}{\omega^2}\frac{\partial a}{\partial x} - \frac{Ux}{\omega^2}\frac{\partial b}{\partial x} \tag{4}$$

$$b_{FH} = b - U\frac{\partial a}{\partial x} \tag{5}$$

$$b_{FP} = -bx + 2Ua + \frac{U^2}{\omega^2}\frac{\partial b}{\partial x} + Ux\frac{\partial a}{\partial x}. \tag{6}$$

One can easily verify that $\int \frac{\partial A_{ij}}{\partial x} dx$ and $\int \frac{\partial B_{ij}}{\partial x} dx$ recover the usual expressions for A_{ij} and B_{ij}, given for example in Salvesen *et al.* (1970).

Coefficients for Combined Heave and Pitch.

Suppose the boat oscillates with a particular combination of heave and pitch. The local heave at a point x forward of the origin will be $h_x = \xi_3 - x\xi_5$, noting that the quantities h_x, ξ_3 and ξ_5 are all complex. If we define $a_F(\xi_3,\xi_5) + \frac{b_F(\xi_3,\xi_5)}{i\omega} = \frac{\frac{d\Delta F}{dh_x}}{\omega^2 e^{i\omega t}}$, then from $\Delta F = \frac{d\Delta F}{dh_x} h_x = \frac{\partial\Delta F}{\partial\xi_3}\xi_3 + \frac{\partial\Delta F}{\partial\xi_5}\xi_5$ we obtain $z_F(\xi_3 - x\xi_5) = z_{FH}\xi_3 + z_{FP}\xi_5$. Letting $\xi_5 = \frac{|\xi_5|}{|\xi_3|}\xi_3 e^{-i\theta_{53}}$, in which θ_{53} represents the phase lag of pitch relative to heave, gives

$$z_F = \frac{z_{FH}|\xi_3| + z_{FP}|\xi_5| e^{-i\theta_{53}}}{|\xi_3| - x|\xi_5| e^{-i\theta_{53}}}. \tag{7}$$

The right hand side of (7) may be evaluated from zero speed sectional coefficients using equations (3)–(6) and values of $|\xi_3|$, $|\xi_5|$ and θ_{53} obtained from full strip theory response calculations.

Time Domain ($2\frac{1}{2}$-Dimensional) Strip Theory

This theory solves the two-dimensional potential flow problem for fixed strips of water, rather than the traditional strips attached to each cross section (details in Holloway, 1998). It differs from 2D theory in its inclusion of forward speed effects in the free surface boundary condition, and in its capacity to include steady flow terms (approximately accounted for in 2D theory through the so called m_j-terms (Newman, 1978)). It has in common with the 2D theory the full unsteady hull boundary condition. It is only the omission of longitudinal derivative term in Laplace's equation, justified on the basis of hull slenderness, that distinguishes it from a full 3D theory.

The left hand sides of (3)–(6) are evaluated from the time domain strip theory in terms of $\frac{\partial\Delta F}{\partial\xi_j} \simeq \frac{1}{\varepsilon}\left([\Delta F]_{\xi_j=\varepsilon} - [\Delta F]_{\xi_j=0}\right)$, using the above definitions of $\frac{\partial A_{ij}}{\partial x}$ and $\frac{\partial B_{ij}}{\partial x}$. Note that $[\Delta F]_{\xi_j=0}$ is not necessarily zero if steady flow terms are included in the strip theory. Similarly, the left hand side of (7) is obtained from $\frac{z_F}{\omega^2 e^{i\omega t}} = \frac{d\Delta F}{dh_x} = \frac{\Delta F(\xi_3,\xi_5) - \Delta F(0,0)}{(\xi_3 - x\xi_5)}$.

The time domain strip theory shows significantly better agreement with experimental results than conventional (2D) strip theory, particularly (as expected) at high speed (Holloway (1998)). The theory also makes use of a 2D time domain panel method, which has been validated extensively against numerical and analytical solutions of Dawson (1977), Doctors (1987), Faltinsen and Zhao (1991), Giesing and Smith (1967), Maskell and Ursell (1970), Roberts (1987), Ursell (1949) and Yeung (1982) for steady and unsteady translating and oscillating submerged and surface piercing bodies (details in Holloway (1998) and papers under preparation).

NUMERICAL RESULTS AND DISCUSSION

Sectional added mass and damping distributions were calculated for a SWATH (small waterplane area twin hull) at two different speeds and a conventional hull form at a single speed. The hulls are shown in figure 2. The frequency and relative heave and pitch amplitude and phase used were those corresponding to peak heave response as calculated by the $2\frac{1}{2}$D strip theory, and are indicated on figure 3.

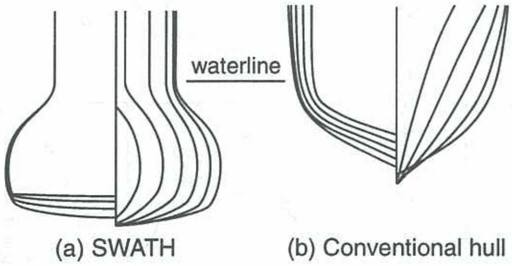

Figure 2: Body view of test hulls

A comparison of the 2D and $2\frac{1}{2}$D theories (figure 3) shows differences attributable to the differences in the theories. One would expect the hull boundary condition to strongly influence added mass and the free surface boundary condition to influence damping, and this accounts for the greater differences in the results in the latter case. In particular the SWATH model shows a smoother distribution of damping forces around the transition from its submerged bow to the surface piercing sections aft (around section 8). The sharp changes in the 2D theory are because, unlike in the $2\frac{1}{2}$D theory, the potential problem is solved for individual sections in total isolation, and forward speed effects are only included afterwards in combining coefficients.

Plotting damping against frequency, conventional sections show one peak while SWATH sections show two peaks with an intermediate frequency of zero damping (due to cancellation of the waves produced by the keel and by the shoulders). The frequency chosen for the SWATH tests by chance almost coincided with the frequency of zero damping for a significant part of the hull length, consequently the damping in the 2D theory results is dominated by the $-U\frac{\partial a}{\partial x}$ and $2Ua + Ux\frac{\partial a}{\partial x} + \frac{U^2}{\omega^2}\frac{\partial b}{\partial x}$ terms of equations (5) and (6). The strong dependence of damping on $\frac{\partial a}{\partial x}$ and $\frac{\partial b}{\partial x}$ contributes to the sharp changes near section 8.

Finally, one can observe some numerical "noise" in the $2\frac{1}{2}$D theory results. Several factors contribute, including the presence of chines for the SWATH and non-sinusoidal forces. These effects diminished as the number of panels per section and the number of sections were increased.

CONCLUSION

In comparing the two theories it must be emphasised that they are not mathematically equivalent except at zero speed. Significant differences are therefore inevitable, but assuming that the theories are not applied outside their range of validity one would also expect strong similarities between the results. These expectations are clearly borne out by the results.

REFERENCES

DAWSON, C.W., "A practical computer method for solving ship-wave problems",in *Proc. Int. Conf. Numer. Ship Hydrodynamics,* 30–38, 1977.

DOCTORS, L.J., "Application of the boundary-element method to bodies oscillating near a free surface", *Int. Symp. Comp. Fluid Dyn.,* 1987.

FALTINSEN, O.M. and ZHAO, R., "Flow predictions around high-speed ships in waves", in *Mathematical Approaches in Hydrodynamics* (ed. T. Miloh), 265–288, SIAM, 1991.

GIESING, J.P., and SMITH, A.M.O., "Potential flow about two-dimensional hydrofoils", *J. Fluid Mech.,* **28,** 113–129, 1967.

HOLLOWAY, D.S., "A high Froude number time domain strip theory applied to the seakeeping of semi-SWATHs", *PhD thesis,* Univ. of Tasmania, 1998.

MASKELL, S.J., and URSELL, F., "The transient motion of a floating body", *J.Fluid Mech.,* **44,** 303–313, 1970.

NEWMAN, J.N., "The theory of ship motions", *Advances in Appl. Mech.,* **18,** 221–283, 1978.

ROBERTS, A.J., "Transient free surface flows generated by a moving vertical plate", *Quart. J. Mech. and Appl. Math.,* **40,** 129–158, 1987.

URSELL, F., "On the heaving motion of a circular cylinder in the surface of a fluid", *Quart. J. Mech. and Appl. Math.,* **2,** 218–231, 1949.

SALVESEN, N., TUCK, E.O., and FALTINSEN, O., "Ship motions and sea loads", *Trans. SNAME,* **78,** 250–287, 1970.

YEUNG, R.W., "The transient heaving motion of floating cylinders", *J. Eng. Math.,* **16,** 97–119, 1982.

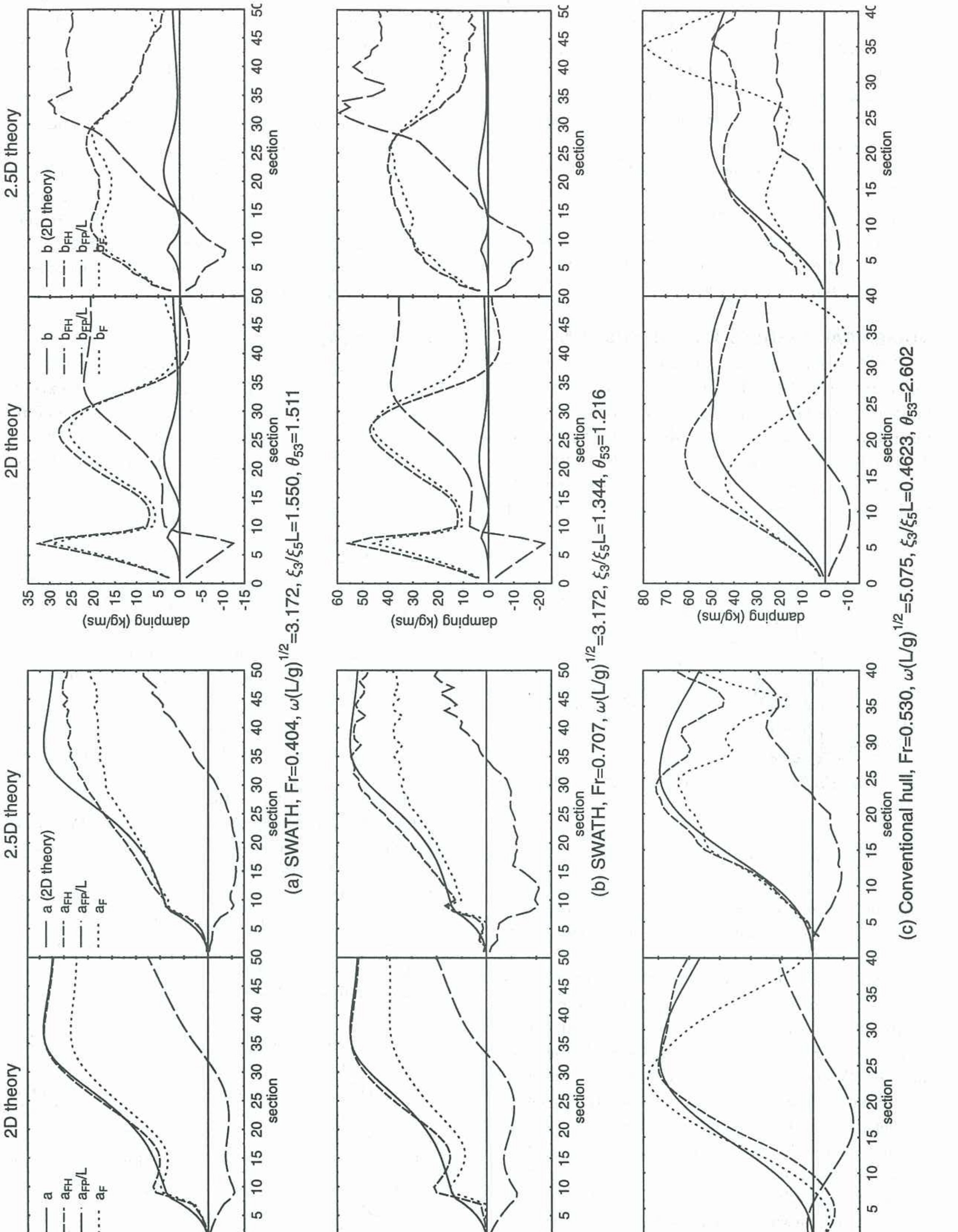

Figure 3: Sectional added mass and damping distribution at forward speed: comparison of 2D (conventional) and $2\frac{1}{2}$D strip theories.