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DETECTION OF VORTICAL STRUCTURE AND SOUND SOURCE IN A RECTANGULAR JET BY PRESSURE MEASUREMENT

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ABSTRACT

The three-dimensional vortical structure in a rectangular jet was detected by pressure measurement on the basis of the idea that vorticity-concentrated regions are well correspond to low pressure regions. The sound source term of Ribner's equation, the second time derivative of the pressure, was also calculated with the pressure data and discussed in relation to the vortical structure. The results reveal that the pressure measurement is very useful and efficient to detect the three-dimensional vortical structure and the sound source term, and that the intense sound source term is closely related to the vortical structure and its motion.

INTRODUCTION

The dynamics of coherent structures has been noted as a key feature of turbulent shear flows, and many research works have been done on the structures. Since the coherent structures are characterized by vortical structures, it is crucial in the experimental works to educe the vortical structures by measuring vorticity usually from velocity data. However, the educed results have serious errors owing to the structure 'jitter' and to the differential process required in the vorticity measurements.

It is well known that the vortical structures are closely related to the instantaneous pressure as shown by the Poisson's equation for pressure:

$$-\frac{1}{\rho}\nabla^2 \tilde{p} = \frac{\tilde{\varepsilon}}{2\nu} - \frac{1}{2}\tilde{\omega}_i^{\,2}, \qquad (1)$$

where ρ is the density of fluid, $\tilde{p}$ is the instantaneous pressure, $\tilde{\varepsilon}$ is the instantaneous dissipation, ν is the kinematic viscosity and $\tilde{\omega}_i$ is the instantaneous vorticity. This equation suggests that the vorticity contributes to $\nabla^2\tilde{p}$ being positive (Bradshaw and Koh, 1981), and that the low-pressure region including a minimum pressure point is closely related to the vorticity-concentrated region. If the mean pressure is uniform over the flow field, the pressure fluctuations show direct footprints of the vortical structures. Indeed the results of direct numerical simulation (Kasagi et al., 1995) reveal that the low-pressure regions in the fluctuating-pressure field correspond to the vortical structures. Thus the detailed pressure measurements in the turbulent shear flows are expected to give us useful information on the vortical structures. Although a significance of the pressure fluctuations has been pointed out, there have been few works on the direct pressure measurements which are usually attended with serious errors owing to the disturbance caused by a pressure probe inserted in the flow. Toyoda et al. (1994) developed a probe to measure the fluctuating static pressure in the turbulent flows, and confirmed that the direct pressure measurements with the probe were very effective to detect the large-scale vortical structures in jets.

Lighthill (1952) showed that turbulence generates sound, and Powell (1964) - Howe (1975) developed a vortex sound theory based on Lighthill's equation. In order to reduce aerodynamic noise, it is important to know the noise generating mechanism related to vortical structures.

Ribner (1962) proposed a theory of aerodynamic sound, called 'dilatation theory', developing Lighthill's equation. The Ribner's equation is deduced on the assumption of $M \rightarrow 0$ and constant entropy as follows:

$$\frac{\partial^2 p^{(1)}}{\partial t^2} - c_0^{\,2}\nabla^2 p^{(1)} = -\frac{\partial^2 p^{(0)}}{\partial t^2}, \qquad (2)$$

where c_0 is the sound speed, $p^{(0)}$ is the local pressure in flow and $p^{(1)}$ is the sound pressure. The sound source term in the right-hand side is the second time derivative of $p^{(0)}$.

As mentioned above, pressure is closely related to vortical structures and sound source in flows, and the pressure measurements are useful to detect both of them.

There are interesting several proposals on the generating mechanism of aerodynamic sound : merging of vortices (Laufer et al., 1973; Ffowcs Williams and Kempton, 1978), breakdown of vortex rings via cut-and-connect process (Takaki and Hussain, 1985), interactions of vortices (Kambe, 1986), stretching of vortex filament (Ishii et al., 1997) and so on.

The vortex motions in the rectangular jet are three-dimensional and have various features (Toyoda and Hussain, 1989) : merging, stretching and cut-and-connect interaction.

Thus the rectangular jet is a good example to discuss the sound generating mechanism related to the vortex motions. The rectangular jet is also noted as a technique to reduce the jet noise or to limit some direction of sound emission as reported in the case of elliptic jet at low Mach number (Bridges and Hussain, 1987).

In the present study, the three-dimensional vortical structure in the rectangular jet of aspect ratio 4 was detected by the pressure measurements, and the second time derivative of the pressure, a sound source term of Ribner's equation, was calculated with the pressure data and was discussed in relation to the vortical structure.

EXPERIMENTAL APPARATUS AND PROCEDURES

The experiments were carried out for an jet issuing from a sharp-edged rectangular orifice. The equivalent diameter *De* of the orifice is 50 mm, the velocity at the jet exit *Ue* is 4 m/s and the Reynolds number *Re* (=*UeDe*/ν) is 1.3×10^4. In order to regulate the formation of vortices, the jet was excited by a loud speaker inside the wind tunnel. The excitation frequency (=141 Hz) was a quarter of the natural frequency of the shear layer generated from the orifice edge, and the excitation intensity u'/Ue at the jet exit (u' : the rms of fluctuating velocity, *Ue* : the mean velocity) was 0.03. Under the excitation the stable interaction of vortices was evolved.

The pressure probe (Toyoda et al., 1994) to measure fluctuating static pressure is shown in Fig.1. The static pressure tube with four small holes is connected at the end to a condenser microphone. The structure and the dimension of the probe were determined so as to minimize errors in the measurement of fluctuating static pressure.

In order to obtain instantaneous spatial pressure distribution by the pressure probe, the pressure measurements were carried out by using phase-average technique. The arrangement of the probes for the measurement of the phase-average pressure is shown in Fig. 2. The reference velocity and the fluctuating pressure were measured respectively by a single normal hot-wire probe fixed near the jet center at $x/De \fallingdotseq 1.0$ and by the pressure probe moved over the flow field. The phase-averaging of fluctuating pressure was carried out at 36 phase angles of the reference signal from $\theta=0$ to $\theta=2\pi$ with an increment of $\pi/18$. The sampling number for phase-averaging is about 210 for each phase. The non-dimensionalized phase-average pressure $\langle p^{(0)} \rangle$ and sound source term $\langle -\partial^2 p^{(0)}/\partial t^2 \rangle$ of Ribner's equation are defined by

$$\langle p^{(0)} \rangle = 2\,[\,p^{(0)}\,]_p / \rho Ue^2, \qquad (3)$$

$$\langle -\partial^2 p^{(0)}/\partial t^2 \rangle = 2\,[\,-\partial^2 p^{(0)}/\partial t^2\,]_p\,(De/Ue)^2 / \rho Ue^2, \qquad (4)$$

where $[\ ]_p$ designates a phase-average value.

The traversing of the pressure probe, the data acquisition and the data processing were automatically controlled by personal computers.

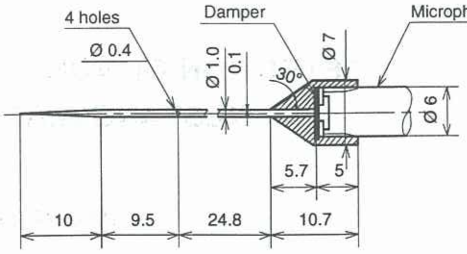


Fig. 1 Pressure probe

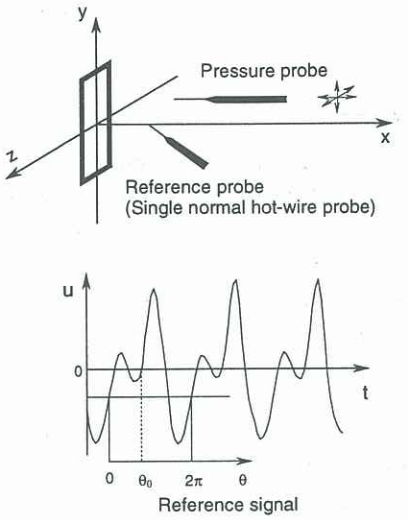
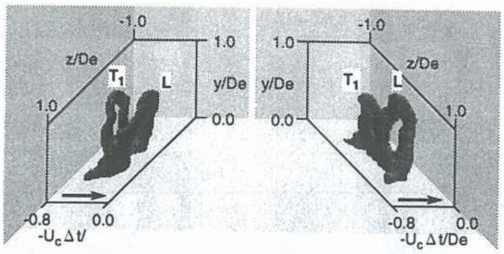


Fig. 2 Arrangement of the probes for the phase-average measurement

RESULTS AND DISCUSSIONS

The pressure measurement was carried out in the major plane (xy plane), the minor plane (xz plane) and the yz planes (at *x*/*De*=1.0, 1.3, 1.5, 1.7, 2.1, 2.3), and the sequential contours of the phase-average pressure in the yz planes enable us to make the three-dimensional views of the pressure field by using Taylor hypothesis. The results are shown in Figs. 3 and 4, where the convection velocity *Uc* of the vortical structure for applying Taylor hypothesis is assumed to be equal to $U_0/2$ (U_0 : jet-center velocity) . The evolving large-scale vortices are paired and interacted as shown in Fig. 3. The interacted structure of the leading vortex (L) and the trailing vortex (T) at *x*/*De*=1.0 agrees well with that predicted by the visualization experiment (Toyoda and Hussain, 1989). The three-dimensional pressure field at *x*/*De*=1.5 in Fig. 4 (a) indicates stretching of the leading vortex in the minor-axis direction and engulfing of the trailing vortex into the upstream leading vortex. The pressure field with the threshold level of lower pressure is shown in Fig. 4 (b), which indicates the skeleton of the vortical structure. The figure suggests the partial merging of the leading and the trailing vortices. Such a complicated structure cannot be detected by other



$<p^{(0)}> \leqq -0.45$

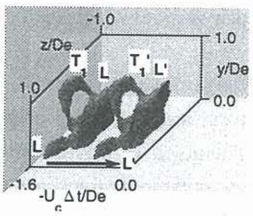
Fig. 3 Three-dimensional pressure field at $x/De=1.0$

measurement techniques.

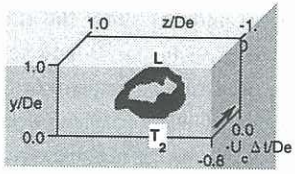
The pressure contours in the three planes and the three-dimensional views enable us to predict the vortical structure in the rectangular jet. The predicted model is shown in Fig. 5. The leading vortex (L) bends downstream owing to the self-induced velocity and engulfs the downstream trailing vortex [T_1' in Fig. 5 (b)]. Farther downstream the leading vortex stretches in the minor-axis direction [Fig. 5 (c)] and splits into small vortex rings [Fig. 5 (d)]. The outer part of the trailing vortex (T_1) stretches upstream [Fig. 5 (b)], and the inner part (T_2) rushes into the leading vortex near the jet center [Fig. 5 (b) and (c)]. The vortex ring (L) spreads outward in the minor-axis direction, and the compound vortical structure consisted of the parts of leading and trailing vortices is formed [Fig. 5(c)]. The compound vortex bifurcates in the major-axis direction via the cut-and-connect process (Kida et al., 1989) near the jet center [Fig. 5(e)].

Considering the velocity field measured by a single normal hot-wire probe (Hiramoto and Toyoda, 1996), the authors propose the deformation model of the compound vortex ring formed by the interaction of leading and trailing vortices as shown in Fig. 6. Figure 6 shows the bifurcation-and-reconnection of the compound vortex ring. The paired vortices contact at the jet center at $x/De \fallingdotseq 1.5$ [Fig. 6(b)], and the interaction generates the bridges (A) and the threads (B) shown in Fig. 6(c), which are caused by vortex annihilation and cross-linking (Hussain and Husain, 1989) in the jet center region. At the next stage, two vortex rings are formed as shown in Fig. 6(d). The bifurcated vortex rings incline inside, move toward the jet center, collide each other, and form a vortex ring via the cut-and-connect process again as shown in Figs. 6 (e) and (f).

Of interest is how the vortex motions mentioned above are related to the sound source term. The second time derivative of pressure, the sound source term of Ribner's equation, was calculated with the pressure data. Figures 7 (a) and (b) show the distributions of phase-average pressure $<p^{(0)}>$ and sound source term $<-\partial^2 p^{(0)}/\partial t^2>$ in the major plane at three phases with an increment of $\pi/18$. In Fig. 7(a), low-pressure regions, shown by dark zones, are induced by vortex motions. L and T_1 in Fig. 7(a) indicate the sections of the leading and the trailing vortices in Fig. 5, and a low-pressure region T_2 is



(a) $<p^{(0)}> \leqq -0.23$



(b) $<p^{(0)}> \leqq -0.6$

Fig. 4 Three-dimensional pressure field at $x/De=1.5$

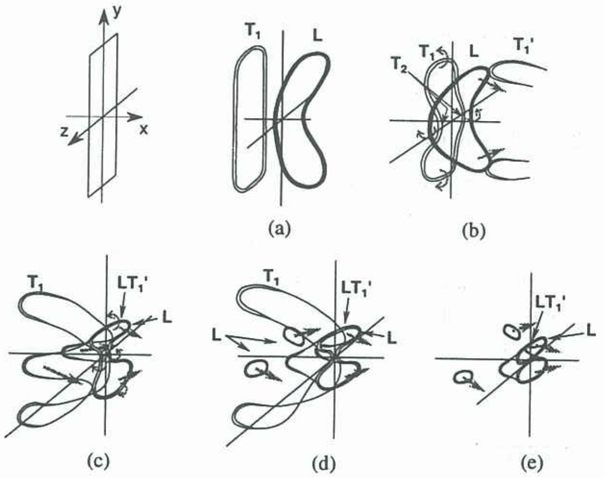


Fig. 5 Three-dimensional vortical structure

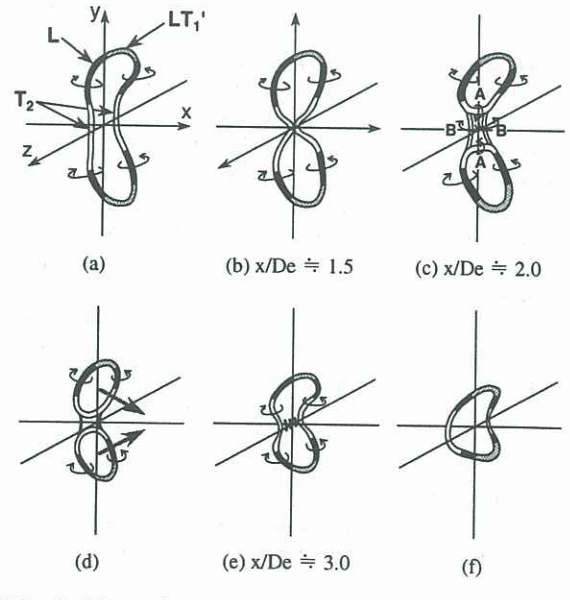


Fig. 6 Bifurcation-and-reconnection of the compound vortex ring

induced by the pair vortices which are the parts of the trailing vortex ring.

In Fig. 7(b), the positive and negative regions of Ribner's sound source term are shown by bright and dark zones respectively. The negative regions of sound source term correspond to low-pressure regions, and the positive regions are located on both sides of negative regions in the direction of vortex convection.

Figures 8 (a) and (b) show the three-dimensional distributions of pressure $<p^{(0)}>$ and sound source term $<-\partial^2 p^{(0)}/\partial t^2>$. Regarding symmetry of flow with xy- and xz- planes, the three-dimensional views were constructed from the measured region which is positive region of y and z axes. In Fig. 8(a), the low pressure region shows deformation and interaction of vortices, as shown in Fig. 5(b). In Fig. 8(b), it is obvious that the negative region of sound source term is closely related to the low pressure region, vorticity concentrated region, in Fig. 8(a). The positive region of sound source term is intense at the jet center where the trailing vortex stretches and rushes into the leading vortex ring.

CONCLUSIONS

The present study is concluded as follows.

(1) The pressure measurement by using the phase-average technique is very useful and efficient to detect the three-dimensional vortical structure and the sound source term in the rectangular jet.

(2) The sound source term of Ribner's equation is closely related to the vortical structure and its motions.

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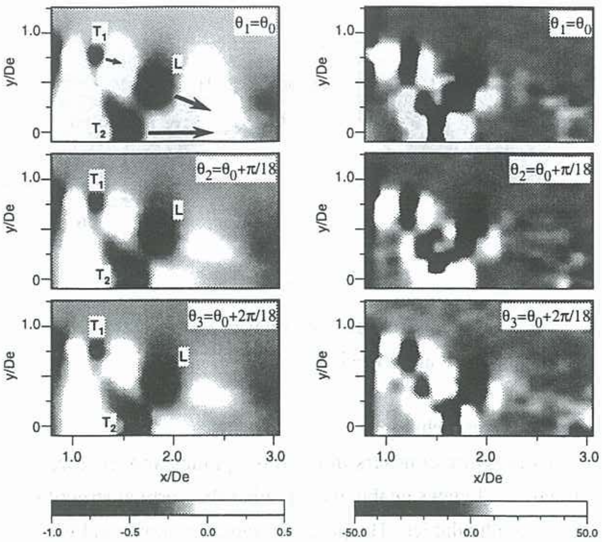
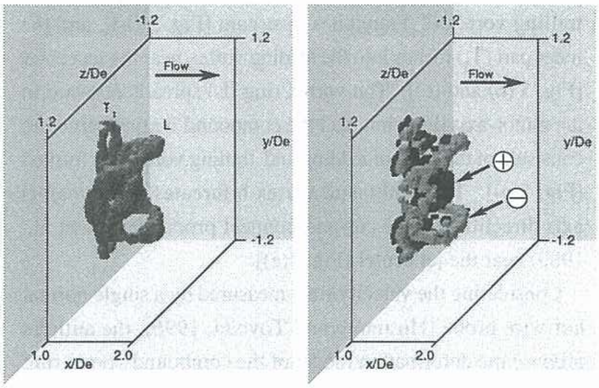


$<p^{(0)}>$ $<-\partial^2 p^{(0)}/\partial t^2>$

(a) Pressure (b) Sound source term

Fig. 7 Phase-average values in the major plane



(a) Pressure
($<p^{(0)}> \leqq -0.6$)

(b) Sound source term
($\oplus$:$<-\partial^2 p^{(0)}/\partial t^2> \geqq 70.0$,
$\ominus$:$<-\partial^2 p^{(0)}/\partial t^2> \leqq -55.0$)

Fig. 8 Three-dimensional phase-average values

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