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TIME AND FREQUENCY DOMAIN ANALYSES OF LONG PERIOD WAVES IN PORT PHILLIP BAY

J.B. Hinwood[1], E.J. McLean[2], and H. B. Lewis[1]

[1] Department of Mechanical Engineering, Monash University, Clayton, Victoria, AUSTRALIA
[2] Department of Land & Water Conservation, Wollongong, NSW, AUSTRALIA

ABSTRACT

Time and frequency domain analyses have been developed and applied to determine the nature and origin if long period (>15 min) waves measured in Port Phillip Bay. Forcing mechanisms are the wind and the tides, while resonances also play a role.

INTRODUCTION

Water level records for stations in Port Phillip Bay (PPB) contain energy at periods ranging from a few minutes to tidal periods. Although the velocities directly generated at these periods are much smaller than the peak tidal velocities, near slack water they are often dominant and at all times add an oscillation which can strongly influence sediment suspension and transport, and thus both geomorphology and nutrient exchange. Time and frequency domain analyses will be demonstrated and used to identify and explain some features of long waves in Port Phillip Bay.

Frequency domain analyses of water levels for tidal harmonic analysis is routinely used, while several analytical studies and idealised experiments have presented information in terms of transfer function amplitudes, eg. Mei (1983), Lee (1971) and Miles (1974). Phase and coherence information have not been reported. Transfer function analysis is not routinely applied to field data sets.

Water level records taken in PPB for tidal analysis, or to support hydrographic data collection frequently contain oscillations of period 20 to 180 minutes with amplitudes of a few millimetres to a few centimetres. For example, the records obtained by the Marine Models Laboratory of the Department of Public Works (later the Port of Melbourne Authority) for major environmental studies in the 1970s frequently included events of one to three days duration with such oscillations. The dominant period was not consistent from event to event but was typically about 60-90 minutes.

The 60 – 90 minute period is actually less than estimates for the fundamental seiche period of PPB, but short sequences of data, analogue recording and other deficiencies prevented reliable phase and period estimates being made. Attempts by one of the authors to simulate seiche behaviour using the numerical model developed by Williams (1978) were unsuccessful, possibly because of the difficulty of determining the modal shape which was required as an initial condition in order to obtain a clean seiche signal.

More recently, using the present data set, Hinwood and McLean (1997) have shown that oscillations in northern PPB force an oscillation in the Yarra River. The Yarra River was shown to have a fundamental resonance of period 77 minutes.

While the tidal origin of the long wave has some supporters, and other believe that the waves originate in Bass Strait, most observers have noted that long waves are most likely to arise during periods of strong wind. Hinwood and McLean reported that long wave events tended to occur with strong wind events but did not present an objective correlation. They did report that for their data set direct correlation of wind speed with water level was not significant.

Thus there is no coherent explanation for the observed periods, no data on observed amplitudes, and no agreed explanation or dynamic model of the origin of these waves. The next section of this paper describes the data set and demonstrates characteristics of the long waves through frequency band analysis. In the subsequent sections, frequency and time domain analyses will be used to examine the roles of tidal excitation, wind forcing and seiching.

NATURE OF THE LONG WAVES

Data qualification

The principal data used in this paper were obtained from the Port of Melbourne Authority and comprise water level records for the month of October 1995 days at six stations. One of the stations is outside PPB in Bass Strait, four are in PPB and one is in the Lower Yarra River estuary, as shown in Figure 1. Data were recorded as six minute averages at six minute intervals.

Wind speed and direction data were obtained from the Environmental Protection Authority from Fawkner Beacon near the northern end of PPB. Atmospheric pressure data were obtained from Moorabbin Airport near the northern end of PPB. the meteorological data were instantaneous values recorded at one minute intervals. In most of the analyses, six one-minute values were averaged to produce records comparable with the water level records.

Prior to use, the data were screened for drop outs, spikes and other defects. Up to 1 per cent of the wind speed data were contaminated by spikes but defects in water level and atmospheric pressure were few and were usually single data values. Defective data were replaced by interpolated values.

Frequency band analysis

To reveal the main features of long waves the water level data were plotted as time series for selected frequency bands. The 7044 data values for each station were taken and the means removed. A window comprising a cosine taper over two per cent of the data values at each end of the records was applied. The data were Fourier transformed, separated in the frequency domain and inverse transformed to create the time series records.

Figure 2 shows selected frequency bands for Breakwater Pier over the 31 days. The lowest frequency band comprises diurnal and lower frequency constituents. It shows rises and falls in the daily mean sea level of up to ± 0.2m. These variations include tidal and meteorological forcing and short waves propagating past the mouth of Port Phillip Bay. Band 2 shows the semi-diurnal variations which are predominantly tidal. It should be remembered that the frequency domain analysis used to prepare the frequency band time series is not a tidal harmonic analysis. Consequently, interactions between closely spaced tidal harmonics which may give rise to diurnal inequality or longer period phenomena will not appear in Band 2 but in Band 1. A diurnal swing resulting from the diurnal tidal inequality is shown in Band 2 to the right of the record.

The four higher frequency bands show long waves. The amplitudes in each band vary widely over time from a few millimetres to a few centimetres. Band 3 comprises all constituents with periods between 205 and 600 minutes, and hence would include a number of tidal harmonics (eg M3-M7). The signal in this band appears to be dominated by one frequency constituent at Lorne and does not resemble the records in the three higher frequency bands. The signal in Band 3 becomes more characteristic of a broad band signal as one moves into PPB. At Breakwater Pier and Victoria Dock, Band 3 has maximum amplitude during the events which dominate the 3 higher frequency bands.

There is marked similarity between the envelopes of the three highest frequency bands and between corresponding bands from different stations. In particular, there are about 10 energetic events of duration about one day superimposed on a low level base signal.

TIDAL INFLUENCES

Tides propagate into Bass Strait and thence into PPB. Tides at this locality are semi-diurnal with spring-neap variation and diurnal inequality. Using the full record of 31 days, the spectrum at all stations showed that the semi-diurnal tide was dominant, with the next largest peak at diurnal frequencies. At higher frequencies, all the spectra fell fairly smoothly. Comparisons of the spectra at frequencies above semi-diurnal showed that the non-tidal energy at Lorne was up to 5 times greater than that at Queenscliff which in turn, was up to 5 times greater than at Hovell Pile and Breakwater Pier.

Examination of the data in Figure 2 provide a strong indication that the energetic long wave events shown in bands 4-6 are not linked to the phase or amplitude of the tide shown in Band 2.

The cross correlations and transfer functions between pairs of stations have been evaluated. Figure 3 shows these functions evaluated for the first 2,000 data points, a period of low wind, with Lorne selected as the reference station. Maxima at frequency interval 0.0013min^{-1} (1/12 hr^{-1}) are evident throughout the whole record.

Sample cross correlation plots are given in Figure 3; data for other stations paired with Lorne were similar. For frequencies up to 0.004 min $^{-1}$ and 0.006-0.007 min $^{-1}$, coherence is generally close to 1, but at other frequencies only the harmonics of M1 have high coherence.

Examination of the different constituents shows very large phase lag between Queenscliff and Lorne, which is approximately 90 degrees for a number of the tidal constituents (M3, M4, M6, M10, M12) but has a wide range of values for the other constituents; the lag does not represent a constant time interval for the different constituents. A further lag occurs between Queenscliff and West Channel Beacon/ Hovell Pile and then a smaller lag between those stations and Breakwater Pier/Victoria Dock.

Sample plots of the transfer function magnitudes and transfer function phase are shown in Figure 3. Strong peaks are evident at 0.002 (M3) and higher harmonics particularly M9 and M12. The magnitudes of the latter are up to 8 times larger in PPB than at Lorne. The occurrence of peaks at the tidal harmonic frequencies and the high coherence (≈ 1) at these frequencies, shows that each harmonic maintains a consistent phase, supporting the tidal origin of these peaks.

Discussion of tidal forcing

The background signal under low wind conditions in bands 3 and 4 occurs at frequencies which are harmonics of the lunar tide. However, the fact that the broad-band energetic long wave events shown in the frequency band analysis in Bands 4-6 bear no relationship to the tide is evidence that other forcing mechanisms than the tide are dominant in producing these events.

WIND FORCING

The frequency band analysis shows that there is a weak correlation between the three higher frequency long wave bands and the diurnal and long term water level variation (Band 1). At least the largest wave events occur close to the time of the high mean water level within the Bay although the correlation coefficient is not significant.

Figure 4 shows the speed of the northerly component of the wind with three long wave bands from the Breakwater Pier record. It is clear that rapid changes in the northerly wind speed correlate almost perfectly with the occurrence of the energetic long wave events in bands 4, 5, and 6; they are also evident in the record of Band 3. Trials of other wind parameters did not produce strong correlations. This figure provides firm evidence that the energetic events are excited by the wind.

The spectra referred to in the previous section showed that during these events, the energy in the long waves was relatively broad band, and the coherence plots showed that only at the tidal harmonic frequencies was there a strong correlation between the records at Lorne and at stations within PPB. Thus, the waves generated by the wind in Bass Strait and recorded at Lorne are not the same waves as those recorded within PPB. Hence, during these energetic events, there is direct excitation of the long waves within PPB, which is independent of the direct excitation occurring at Lorne.

SEICHING IN PORT PHILLIP BAY

Helmholtz mode

The nature of seiching in a partially enclosed Bay has been discussed and reviewed Mei (1983). The lowest mode of seiching is known as the Helmholtz or pumping mode and is driven by the "stiffness" provided by the gravitational potential energy associated with the elevation of water in the Harbour and the "inertia" provided by the kinetic energy of the flow through a restricted entrance. In this mode, the water surface elevation is nearly uniform over the Harbour Basin and is 90° out of phase with the external forcing. For a harbour of plan area S, entrance area A, and a uniform inlet channel of length L, the frequency of the Helmholtz mode, f_H, is:

$$f_H = \frac{1}{2\pi}\left(\frac{gA}{SL}\right)^{1/2}$$

Referring to the transfer function amplitude and phase shown in Figure 3, consider the transfer function over the frequency range 0 – 0.00118 min $^{-1}$, which includes the diurnal and semi-diurnal frequencies. At zero frequency, all stations show an amplitude ratio of 1, and zero phase shift relative to Lorne. Over the diurnal and semi-diurnal range, the amplitudes all fall to about half that of Lorne and there is a phase lag of about 30° at Queenscliff and 90° at all four PPB stations which are north of the Sands. All stations have a small response peak at 0.002 min $^{-1}$ (M3), but at higher frequencies up to 0.003 min.$^{-1}$, amplitudes continue to fall and the phase lags between Lorne and northern PPB stations remain at about 90° (Hovell Pile, West Channel Beacon) or increase (Breakwater Pier, Victoria Dock). A damped linear oscillator has a phase lag of 90° close to its resonant frequency. This would put the resonant frequency in the range 0.0011 –0.0017 min $^{-1}$, ie a period of 12.5±2.5 hr.

The results of the transfer function analysis suggest that the northern part of PPB may be considered as the harbour basin and the channels through the Sands as the inlet channel. Adopting an aggregate area and average length for these channels yields a period of the Helmholtz mode of 12 ± 3 hours, consistent with the above. The behaviour of the water level at Queenscliff suggests that a two-basin model would be better.

Closed basin modes

The closed basin modes have been fully described by Lamb (1928), while Mei has included the effect of the radiation of energy from the harbour mouth for some cases - this lowers the natural period only slightly provided that the entrance area is small. Analytical solutions are available for rectangular, circular and elliptical basins of constant or parabolically varying depth, and for a few other cases. In most harbours and bays the dominant seiche mode is the fundamental sloshing mode.

The transfer function amplitudes provide strong evidence for the existence of resonant periods of 124 and 169 minutes. At each of these periods, the amplitude of the component at Lorne is amplified in PPB by up to six times, while the coherence is close to 1. Excluding the Yarra River station (Victoria Dock), at other periods, the amplitudes are generally about the same as at Lorne.

The most likely oscillation is the fundamental sloshing mode from north to south. predictions of the period are given in table 2 based on the analytical solution for an elliptical basin with parabolic depth profiles (Mei). The basin may be that part of PPB north of the Sands or the whole of the Bay. A first order correction for the enhanced kinetic energy over the Sands which adds about 8 minutes to the predicted period for the whole of the bay has been included.

Mode	North of Sands		Whole Bay	
Period (min)	117	124	138	169
	Expected phase	Measured phase	Expected phase	Measured phase
Station:				
Queenscliff	--	--	0	-50
Hovell Pile	0	0	0	0
W Channel Beacon	0	29	0	-35
Breakwater Pier	180	163	180	105

Table 1: Measured and predicted periods and phases of N-S fundamental basin modes in PPB. Phases relative to Hovell Pile.

For the fundamental sloshing mode in the northern part of the bay, it would be expected that all stations on the same side of the nodal diameter would have the same phase, 180° different to those on the other side. Table 1 gives the expected and measured phases for the two oscillations for this mode. The stations are grouped at each end of these basins and hence cannot reliably distinguish between a propagating wave and a seiche. The transfer function phases provide qualified support for the occurrence of a seiche at the period of 124 minutes but not for 169 minutes.

The phases of the 169 minute oscillation are not consistent with a seiche within either the northern part or the whole north-south basin of PPB. However, in view of the transfer function amplitude and coherence data, a 169 min. resonance is present. The mode shape of this resonance is presently not known, but it may be a coupled two-basin oscillation or may result from an oscillation in the South Channel, which lies along the southern side of the Sands.

DISCUSSION AND CONCLUSIONS

The frequency band analysis produced time series records in Bands 4-6 (12 – 205 min.) showing short intense periods of wave energy superimposed on a low level background. Spectral analysis showed that the signal in Band 4, particularly the background, was dominated by the overtides, with high coherence between stations. Bands 5 and 6 and the high energy events in Band 4 appear to be broadband phenomena. The coherence between the broad band phenomena within PPB was moderate but generally not significant, while between Lorne and PPB the coherence was low. This supports an origin within PPB for these wave events. Most of these strong events coincided with strong changes in the north-south wind over PPB. Following such a change, wave energy increased very rapidly.

Two periods stood out in the transfer function amplitude plots, 124 minute (M9) and 169 minute (M12). In each of these amplification of the oscillation at Lorne reached about six times in PPB. This amplification shows that the Bay has resonances at these periods. Comparison of calculated and measured period and phase data for the 124 minute oscillations confirm the occurrence of a sloshing-mode seiche in the northern part of PPB. Phase and period data show that the 169 minute oscillation is not the fundamental north-south sloshing mode seiche, which would have a period of 137 minutes over the whole Bay. The data are inadequate to evaluate more complicated modes involving the South Channel or Corio Arm which could account for the longer observed period.

The presence and origins of multiple forced and free modes of oscillation could not have been made without the combined use of time and frequency domain analyses. The techniques are simple to apply but interpretations require care, particularly when record lengths are short and statistical significance cannot be rigorously established.

REFERENCES

HINWOOD, JB and MCLEAN, EJ (1997) "Forcing of long period waves in the Yarra estuary", Proc. 13th Australasian Conf. Coastal and Ocen Eng., Christchurch, NZ, 7-11 Sept. 1997, 2:1071-76.

LAMB, H (1928) "Hydrodynamics", Dover, New York.

LEE, J-J (1971) "Wave-induced oscillations in harbours of arbitrary form", *J. Fluid Mech.*, 45:375-394.

MEI, CC (1983) "The Applied Dynamics of Ocean Surface Waves", Wiley-Interscience, New York, 740pp.

MILES, JW (1974) "Harbor seiching", *Ann.Rev. Fluid Mech.*, 6:17-35.

WILLIAMS, BJ (1978) "Mathematical Modelling for the Management of Waste Water Systems", PhD thesis, University of Melbourne, Melbourne, AUSTRALIA.

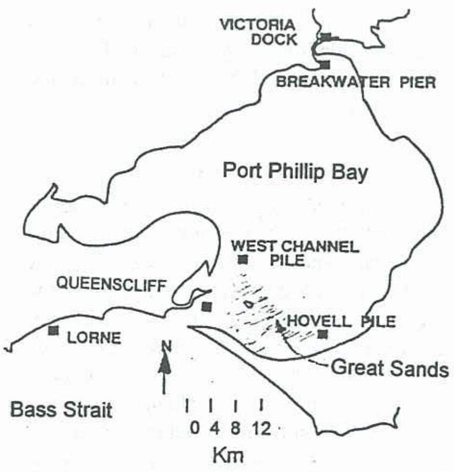

Figure 1: Location of recording stations.

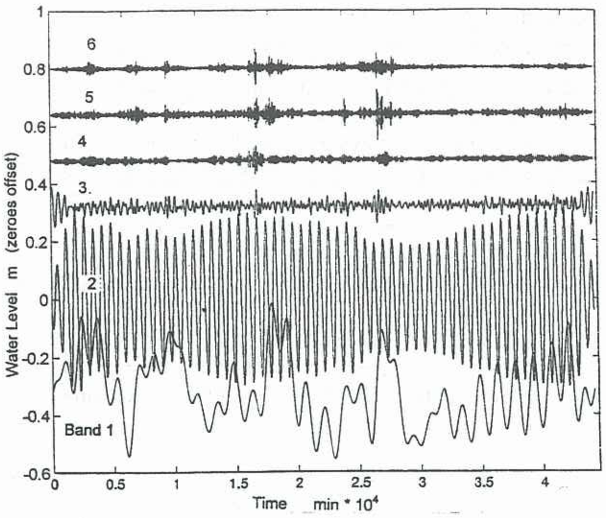

Figure 2: Frequency band analysis of water level

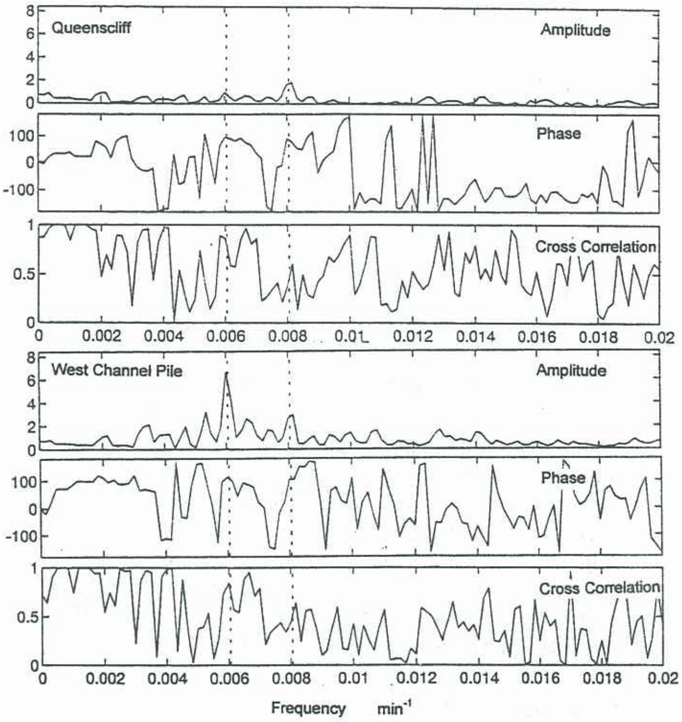

Figure 3: Transfer function relative to Lorne.

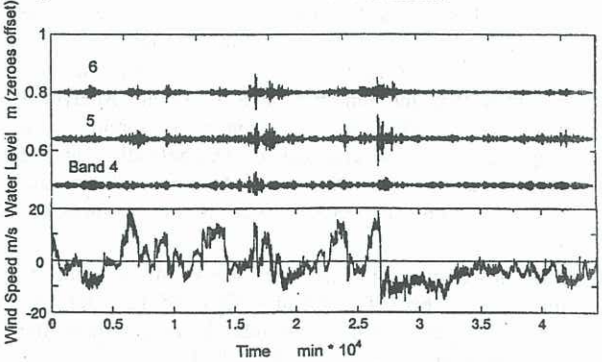

Figure 4: Wind speed and long waves (Bands 3 - 6 at Breakwater Pier)