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THREE-DIMENSIONALITY OF A PLANE MIXING LAYER

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ABSTRACT

An experimental study has been conducted to investigate the three-dimensional structure of a plane, single-stream mixing layer through X-probe hot-wire measurements. By comparing results obtained from a number of spanwise velocity profiles, it is shown that the mixing layer thickness varies by around 3% over the central 60% of the flow. The spanwise variation in the maximum Reynolds stresses is comparable to the accuracy with which they are measured. The "turbulence" level in the potential core increases roughly as the square of the downstream distance from an exit level of 0.35%. The increase is caused by the irrotational motion induced by the expanding mixing layers but some unexpected features were observed in the normal stresses in the potential core.

INTRODUCTION

This paper reports on the documentation of the initial region of a plane turbulent jet where a potential core separates two, single-stream mixing layers. With the streamwise distance x measured from the wind tunnel exit, the region of interest is $x/D \leq 5$, where D is the exit height. Ultimately we intend to study the impingement of this jet on flat plate placed in the region $x/D \leq 5$. Since the deflection of the streamlines during impingement will inevitably lead to three-dimensionality through the angular momentum instability, it is necessary to document the three-dimensionality of what will be the upstream flow. Furthermore, Tu and Wood (1996) showed that the impinging flow near the stagnation point was apparently influenced by the free-stream turbulence in the potential core, so this was also documented.

In this paper, the three-dimensionality is quantified by measuring the mean velocity and Reynolds stresses over the range $0.01 \leq x/D \leq 5$ at z/D = -2, -1, 0, 1, and 2, where the spanwise coordinate z has its origin at the tunnel centreline. In some cases, the spanwise averages of the measurements are compared to those measured along the tunnel centreline (z/D = 0). There have been remarkably few measurements of the spanwise uniformity of nominally plane mixing layers. Wood (1982) and Bell and Mehta (1992) found that variations in U decreased with x presumably because the rapid growth of mixing layers rapidly stretches any mean secondary vorticity to eventually negligible amounts. Wood (1982), the only other measurements we know of in a single stream mixing layer, concluded that the major non-uniformity was a bodily wrinkling of the layer. In other words, the vertical location of the mixing layer varied more than mean velocity within it.

EXPERIMENTAL APPARATUS

The experiments were conducted downstream of the exit of a conventional blower tunnel at a free-stream velocity, U_0, of 30 m/sec. D was 52 mm and the width of the tunnel was 350 mm, giving an exit aspect ratio of 6. To improve the two-dimensionality of the flow, endplates were installed at the tunnel edges, z/D = -3 and 3. The exit "turbulence" level was 0.35%.

Measurements were made using a DANTEC 55P51 X-probe with 5 μm diameter tungsten sensing elements about 1.5 μm in length and about 1 mm apart. The sensing length was separated from the prongs by stubs formed by copper coating to a thickness of about 20 μm. The probe was held by a traverse system driven by stepper motors, which moved it vertically, and rolled the probe about its axis to obtain all six Reynolds stresses using the technique of Cutler and Bradshaw (1991). The anemometer output was fed to a fully automated data acquisition system controlled by a PC and interfaced to the stepper motor drivers.

The probes were calibrated statically in the potential core (x/D = 0) using the method of Clausen and Wood (1988). Calibration was done before and after the measurements. If the difference in the calibration constants were large, the measurements were discarded, otherwise, the difference was distributed linearly among all the data points.

RESULTS AND DISCUSSION

Figure 1 shows the streamwise development of the average mixing layer thickness δ and that along the centreline. δ was determined by fitting the measurements of the mean streamwise velocity, U, to an error function

$$U = [1 + \mathrm{erf}(\xi)\,]\,/\,2 \qquad (1)$$

where ξ is the normalised y-coordinate

$$\xi = \left(y - y_{0.5}\right)/\,\delta \qquad (2)$$

$y_{0.5}$ is the value of y where $U = 0.5U_0$. The spanwise distribution of $y_{0.5}$ is shown in Figure 3. δ is related to the conventional thickness, $b = y_{0.9} - y_{0.1}$, by $\delta \approx b/1.8$.

A self-preserving mixing layer grows linearly from an effective origin x_0 (the hypothetical value of x at which δ is zero). This is indicated in Figure 1 where, $\bar{x}_0$, the average x_0, is used to scale δ. Self-preservation occurs for $x/D \geq 2$. x_0 is negative, which is typical for a mixing layer growing from a laminar exit boundary layer. Figure 1 shows spanwise variations of δ of less than ±3%.

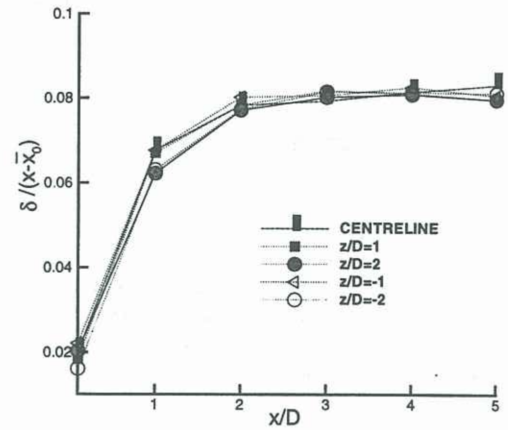

Figure 1: Development of mixing layer thickness.

Figure 2 shows the spanwise variations in mixing layer growth rate, $d\delta/dx$, which was based on a linear least-squares fit to δ for $x/D > 2$, and the variations in the effective origin x_0. Figure 3 shows the development of $y_{0.5}$. Along the centreline, dδ/dx is 0.090, and the average rate is 0.081. Both are larger than the value of 0.072 obtained by Mehta et al. (1991). The variation in x_0 is 55 per cent, but is anti-correlated with the variation in $d\delta/dx$, so the net effect is only a small variation in δ. Furthermore, the spanwise variation of the primary shear stress $\overline{uv}$, shown in Figure 4, is not correlated with the that of $d\delta/dx$ whereas the momentum integral equation requires a direct relation for self-preservation. The level of maximum $\overline{uv}/U_0^2$, which should be constant in the self-preserving region, is about 0.010. This result is consistent with previous measurements, eg Wood and Bradshaw (1982) and Mehta et al. (1991) and with the momentum integral equation which can be used to determine $\overline{uv}$ from the profiles of U. It is generally accepted that X-probes measure the Reynolds stresses to an accuracy of around 10% so the spanwise variations shown in Figure 4 are not significantly larger than the measurement error. Notice also the very low values of $\overline{uv}$ at $x/D = 0.01$, which indicate that the non-turbulent nature of the boundary layer at the contraction exit. At least for $x/D < 3$ the spanwise variations in $y_{0.5}$ do not change with x, suggesting that the mixing layer is wrinkled or warped in the spanwise direction. The amount of wrinkling is roughly constant with x, and so is large compared to δ at small x, but it then decreases in significance.

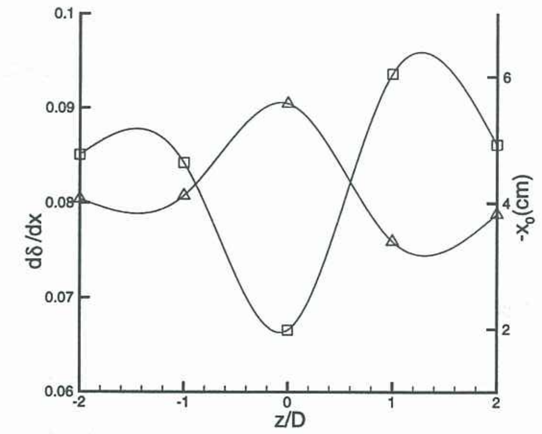

Figure 2: Spanwise variation of the growth rate and effective origin. dδ/dx; Δ : x_0 ; □ .

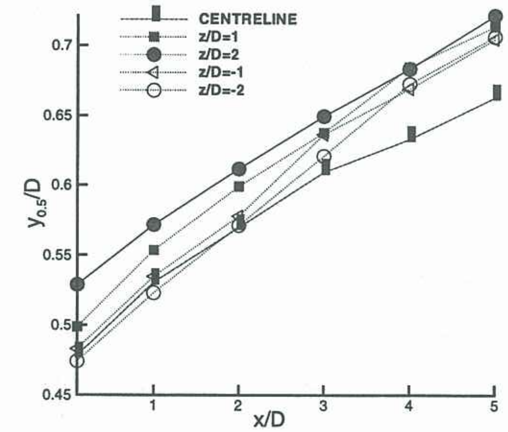

Figure 3: Development of $y_{0.5}$.

	Present Results	Wood and Bradshaw (1982)
$\overline{u^2}/U_0^2$	0.028	0.025
$\overline{v^2}/U_0^2$	0.016	0.014
$\overline{w^2}/U_0^2$	0.020	0.015
$\overline{uv}/U_0^2$	0.010	0.009

Table 1: Comparison of the maximum Reynolds stress in the self-preserving region

The downstream development of the three normal stresses and the primary shear stress is summarised in figure 5: $\overline{u^2}$ is the streamwise normal stress; $\overline{v^2}$ is in the normal direction and $\overline{w^2}$ is in the spanwise direction. Each of the stresses reaches a peak value before relaxing to self-preservation, which is typical of mixing layers forming from a laminar exit boundary layer, Bradshaw (1966). For $x/D > 2$, the Reynolds stresses were constant to well within the expected accuracy of their measurement. Table 1 compares the current data with the single stream mixing layer results of Wood and Bradshaw (1982). Figure 5 also shows that all stresses in the centreline are below the average, by an amount that is within the accuracy of measuring the Reynolds stresses.

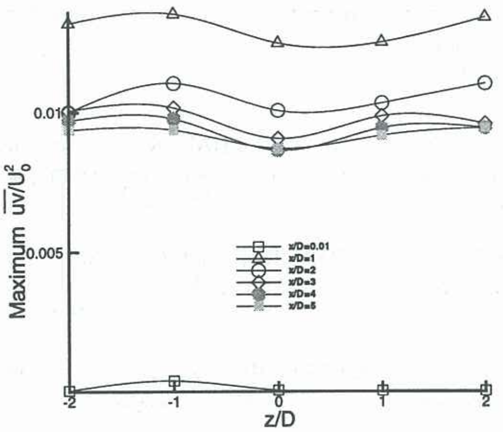

Figure 4: Spanwise variation of maximum shear stress.

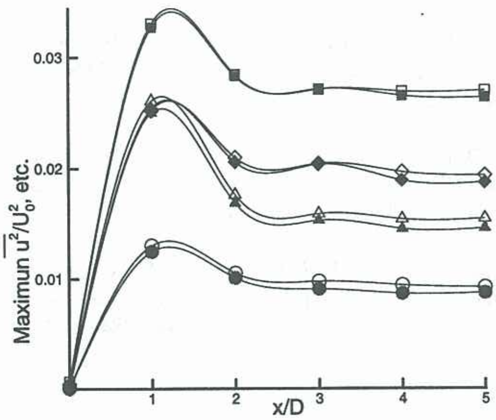

Figure 5: Development of the maximum Reynolds stresses. $\overline{u^2}/U_0^2$; □ : $\overline{v^2}/U_0^2$; Δ : $\overline{w^2}/U_0^2$; ◊ : $\overline{uv}/U_0^2$; O. Filled symbols show centreline values, unfilled symbols show spanwise averages.

As an indication of the fluctuation levels in the potential core, Figure 6 shows the development of the normal stresses along the tunnel centreline. The initial free-stream "turbulence" (which we show below contains a contribution from the unsteadiness) is quickly swamped by the rapidly increasing contribution from the irrotational motion imposed by the expanding mixing layers. Because mixing layers grow linearly, they are the most rapidly growing shear layers, and a single stream mixing layer is the most rapidly growing of all. The linear growth should cause the induced normal stresses should increase as x^4, Wood and Ferziger (1984), which is generally consistent with the results in Figure 6. On the other hand, the well known constraint on the irrotational flow

$$\overline{v^2} = \overline{u^2} + \overline{w^2} \qquad (3)$$

which was found to be accurate by Sunyach and Mathieu (1969) over all x in the potential core, is here violated in the region around $x/D = 3$. Repeated independent measurements showed that $\overline{v^2} < \overline{u^2}$ consistently in this region. Equn (3) is strictly applicable only for flows with streamwise homogeneity, in which case the mixing layers would not increase in thickness. However, it is not clear how the departures from it arise from the relationships between the normal stresses that follow from irrotationality - and which reduce to Equn (3) under homogeneity, see for example Wood and Ferziger (1984).

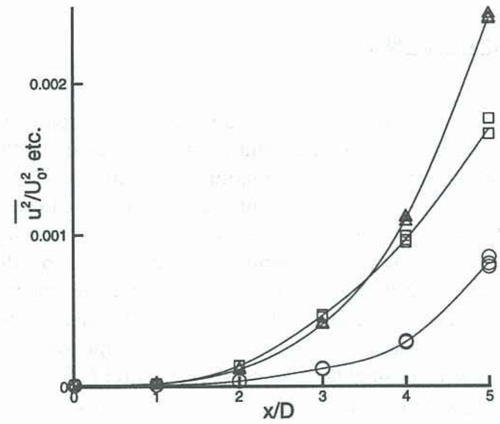

Figure 6: Development of the centreline normal Reynolds stresses. $\overline{u^2}/U_0^2$; □ : $\overline{v^2}/U_0^2$; Δ: $\overline{w^2}/U_0^2$; O.

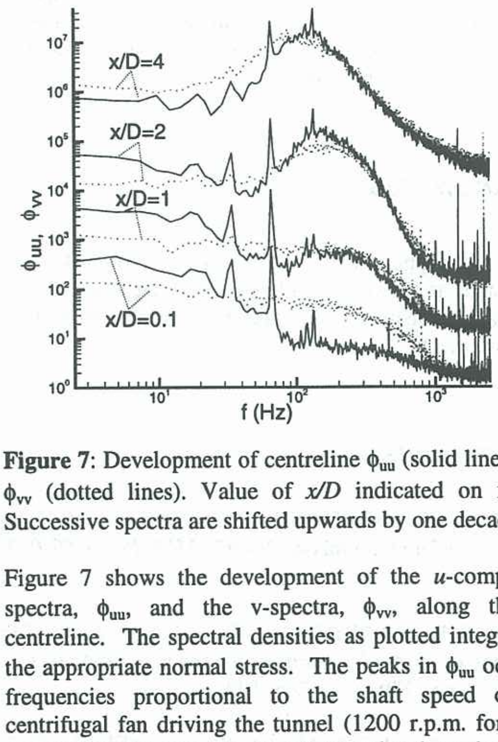

Figure 7: Development of centreline ϕ_{uu} (solid lines) and ϕ_{vv} (dotted lines). Value of *x/D* indicated on figure. Successive spectra are shifted upwards by one decade.

Figure 7 shows the development of the *u*-component spectra, ϕ_{uu}, and the v-spectra, ϕ_{vv}, along the jet centreline. The spectral densities as plotted integrate to the appropriate normal stress. The peaks in ϕ_{uu} occur at frequencies proportional to the shaft speed of the centrifugal fan driving the tunnel (1200 r.p.m. for these measurements) and are, therefore, related to the blade passing frequency and its harmonics. The peaks do not occur in ϕ_{vv} or ϕ_{ww} (the latter results are not shown) and therefore are associated with unsteadiness - the timedependent x-direction motion of the whole potential core. Since $\overline{v^2} \approx \overline{u^2}$ at the tunnel exit, the magnitude of the unsteadiness is likely to be small. As x increases, the spectra fill out due to the increasing contribution of the induced motion. Theoretically, this component peaks at a frequency inversely proportional to x, and the results show a definite decrease in the peak frequency as x increases. It is interesting that ϕ_{vv} is greater than ϕ_{uu} at low frequencies only at $x/D = 4$, which is the only station where $\overline{v^2} > \overline{u^2}$. Since the integral scale of the induced streamwise motion is zero, Wood and Ferziger (1984), the low frequency behaviour of the spectra suggests that $\overline{u^2}$ is augmented in the region $x/D < 4$ by other means.

CONCLUSIONS

An experiment was conducted to document the spanwise variation of mean velocity and Reynolds stresses in the developing single stream mixing layer in the initial region of a plane turbulent jet. The streamwise development of these quantities was typical of a mixing layer developing from a laminar boundary layer on the contraction wall. The spanwise variations in mixing layer growth rate and effective origin, while significant, are anti-correlated, so they do not cause a large variation in mixing layer thickness. Across the central 60% of the flow, this thickness varied by less than ±3%. Self-similarity was achieved for $x/D \geq 2$ and no significant variations in the maximum Reynolds stresses were found. The mixing layer was found to be wrinkled or warped in the spanwise direction.

The fluctuation levels in the potential core were quickly dominated by the induced motion caused by the linearly growing shear layers. Only a small amount of unsteadiness was present.

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