13th Australasian Fluid Mechanics Conference
Monash University, Melbourne, Australia
13-18 December 1998

TIME-DEPENDENT SIMULATION OF TURBULENT FLOWS IN AXISYMMETRIC SUDDEN EXPANSIONS

Baoyu GUO, Tim A.G. LANGRISH and David F. FLETCHER

Department of Chemical Engineering
University of Sydney, NSW 2006, AUSTRALIA

ABSTRACT

This work uses a numerical method (CFD) to simulate the time-dependent flow without external swirl in three-dimensional axisymmetric sudden expansions. Both the swinging and the precessing motion of the jet caused by the instability are predicted for turbulent mean flow, resulting in intense flow oscillation. A frequency parameter (Strouhal number) of the precession agrees with the reported experimental data. The underlying mechanism for the self-induced precession is discussed.

INTRODUCTION

Incompressible flow in sudden expansions is a classical example of many aspects of fluid flow phenomena (e.g., boundary layer development, shear layers, recirculation zones). The primary feature of flow for this geometry is a jet confined by the surrounding walls. Extensive investigations have been conducted for two-dimensional flows. When the Reynolds number is above about ten, the non-linear advection term in the Navier-Stokes equations becomes dominant compared with the linear diffusion term. Due to the non-linear behaviour of the physics, the flow becomes unstable and yields asymmetric flow patterns even in a symmetrical geometry. In this case, there are two types of flow instabilities, the pitchfork bifurcation and the Hopf bifurcation (reviewed by Mizushima *et al*, 1996). The mean flow field asymmetry and hence solution multiplicity remain a feature of flows in the turbulent regime. In the case of the two-dimensional channel with a suddenly expanded part, it is essential to the symmetry breaking bifurcation that the two regions of the recirculation vortices are separated, and a resultant pressure difference arises which causes the jet to remain asymmetric. However, when a cross flow exists between the two recirculation zones, periodic motions of jets have been seen experimentally and numerically (Gebert *et al*, 1997).

Flows in a circular cylinder with a suddenly expanded part are encountered in many engineering applications, such as spray dryers and burners. The recirculation vortex behind the sudden expansion of the circular cylinder is toroidal, with the pressure being continuous in the entire toroidal region, so it was traditionally believed that the symmetry breaking pitchfork bifurcation cannot be expected in this case. However, a swirl or precession has been observed in spray dryers (Langrish, 1998) and utilised in burners (Hill *et al*, 1992). It has been shown that this precession also occurs in a simple fully confined jet flow by Hill *et al* (1995), who visualised the precession in axisymmetric confined jets and measured the frequencies. This flow pattern is preferred in burners and mixing vessels as it encourages the large-scale mixing process, but should be avoided in spray dryers because it produces undesirable back-mixing and particle deposition on the wall, thus ruining the quality of products.

The investigations of the two dimensional situation have highlighted the mechanism of the asymmetry and the oscillation of the flow. The restrictions in a two dimensional sudden expansion do not necessarily exist in its three dimensional, axisymmetric counterpart, because the fluid can flow transversely around the jet. Therefore, unlike for two dimensional cases, steady asymmetric flow is not expected in an axisymmetric sudden expansion, whereas oscillations are possible. However, very limited information is available on the instability characterised by a precessing jet.

This work uses the commercial CFD package CFX4 to simulate the time-dependent flow numerically in axisymmetric sudden expansions. Both swinging and precession motions are discovered to exist simultaneously for the turbulent mean flow, which cause intense flow oscillation. The model and numerical methods used in the calculation are given. The frequencies of the precessing jet are compared with the reported experimental data. The ability of the CFD approach to predict such flow phenomena is thus validated and helps to gain some insight regarding the mechanism of the instability. To our knowledge, no other numerical simulations of this instability have been reported to date.

GEOMETRICAL MODEL AND PHYSICAL PROPERTIES

The geometry shown in Fig.1 consists of two pipes of diameter d and D. The x-axis of the Cartesian coordinates is on the centre-axis and the origin is at the centre of the expansion face. The simulation regime has been taken as $l=10d$ and $L\geq16D$. Neumann boundary conditions are imposed for both the inlet and the outlet, so that the velocity components and turbulence quantities are assumed to be fully developed.

A fully three-dimensional structured grid has been used and constructed to have small orthogonality deviations. At the centre of the larger cylinder, it consists of five blocks, which when viewed in the axial direction show four blocks making an O-grid round an H-grid (a so-called butterfly mesh). In the annular region, a refining structure has been used to avoid over concentration of cells at the centre. Unlike cylindrical grids, the solution with this model is able to vary across the centre axis of the geometry. The cell density along the axis is non-uniform, and increases as the expansion plane is approached, with the neighbouring cells varying in size by no more than 7%. The final grids have as many as 95,000 cells in total. Further increases in the extent of the simulation regime and the grid density have no identifiable effects on frequencies of the oscillations, so the simulation results are grid independent.

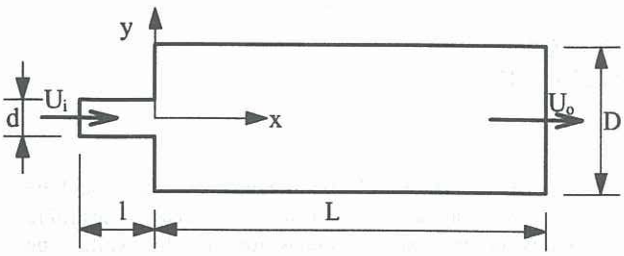


Figure 1 : Schematic diagram of the geometry

The fluid is chosen as incompressible, Newtonian, and its physical properties have been set as 1.295 kg/m^3 for the density (ρ) and 1.717×10^{-5} kg/ms for laminar viscosity (μ), respectively. These values are close to those of air, but adjusted in some cases to give the desired Reynolds numbers.

SOLVER PARAMETER

The Reynolds number (Re) is of the order of 10^5 based on the inlet diameter d and the average inflow velocity. The turbulent, time-averaged Navier-Stokes equations have been solved, and the standard k-ε turbulence model has been used.

The CFD package CFX4 (CFX, 1997) has been employed, which provides a finite-volume based flow solver. The Neumann condition may be implemented in the code simply by setting mass flow boundary for inflow and pressure (or mass flow) boundary for outflow. The higher order differencing scheme QUICK has been used for velocities and Van Leer has been used for k and ε. SIMPLEC or PISO is employed for pressure correction. The equation solvers are BLOCK STONE for velocities and AMG for pressure.

A steady calculation has been used as an initial estimate for the transient calculations. The sum of the absolute values of the net mass fluxes into or out of every cell in the flow has been tested as a convergence criterion for mass conservation. The final mass source residual has been less than 0.1% of the total mass flow rate.

Adaptive time stepping has been used to ensure that every step is converged to the criterion. Second-order, fully implicit Backward Euler differencing in time has been used. A typical time step is 0.02s. The inclusion of the second order extrapolation technique improves the initial guess to the solution at each time step. This reduces the number of iterations required for convergence.

RESULTS AND DISCUSSION

Velocities and pressure are recorded at several points during the transient calculation. After a period of chaotic transition, coherent oscillations of the variables at the monitoring points have been observed as shown in Fig.2. The velocity components v in the y direction and w in the z direction oscillate intensely. However, the variations of v and w velocities look alike except that there is a phase difference between them. Moreover, there is an oscillation of higher frequency superimposed on a fundamental oscillation. This leads us to identify the frequencies of the two apparent oscillations. Because of the axisymmetric nature of the geometry, it is sensible to analyse the velocity component in the cross-stream plane or transverse velocity (with the amplitude of $(v^2+w^2)^{0.5}$ and the direction expressed by a phase angle $\pm\arctan(w/v)$). In polar coordinates, the amplitude indicates the motion in the radial direction, while the phase angle represents the motion in the azimuthal direction. It can be seen from Fig.3 that the amplitude appears more regular and oscillates periodically about a fixed value (~0.25m/s). So, the above phenomena indicate that there are two different motions (swinging and swirl or rotation) of the jet which occur simultaneously at that point.

On Fig.4, the phase of the transverse velocity runs from $-\pi$ to $+\pi$ (or $+\pi$ to $-\pi$, depending on the precessing direction) in a cycle (the corresponding time interval is defined as precession period), so that the existence of the precession is evident. A periodic function of higher frequency is superimposed on the straight lines from $-\pi$ to $+\pi$ (or $+\pi$ to $-\pi$). This periodic function corresponds to the oscillation of the amplitude on Fig.3. Therefore the orientation of the swinging motion also rotates with the precessing motion. This means that a higher order oscillation occurs relative to a moving coordinate that rotates at the fundamental frequency of the precession.

It has been noticed that the self-induced precessing instability can be reproduced with different grids, e.g., the length of the domain, the number of blocks, relative size of the blocks, cell density distribution. Moreover, the initial guesses, from steady calculations, may be different from run to run because they are not well converged and the flow patterns are asymmetric due to numerical disturbances. However, after a period of time in the transient computation, a stable, relatively regular and oscillating solution can always be achieved for the geometry and the velocity considered. This is also true when the transient simulation starts directly from the default zero flowfield. The subsequent jet precesses either in the clockwise or the counter-clock direction, and the direction does not affect the frequency of the precession.

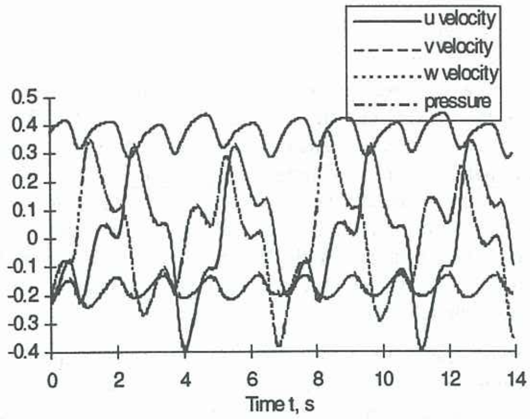


Figure 2 : Flow variables at a monitoring point (U_i=10 m/s, Re = 100,000)

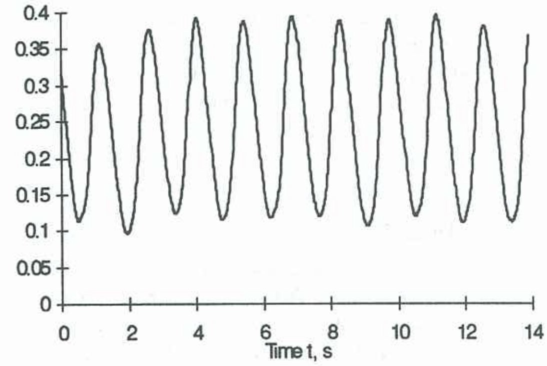


Figure 3 : Amplitude of transverse velocity (U_i=10 m/s, Re=100,000)

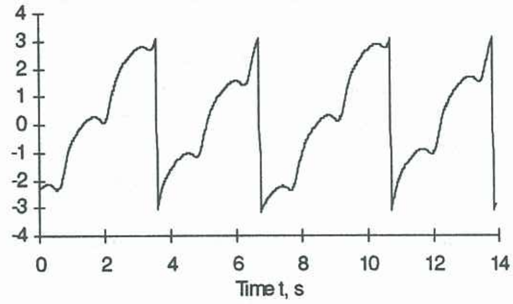


Figure 4 : Direction of transverse velocity (U_i=10 m/s, Re=100,000)

Different Monitoring Points

At the present monitoring point (4 diameters downstream from the expansion on the centreline), the precession oscillation appears to be stronger than the swinging oscillation. However, this is not always the case at other positions. The intensity of the swinging oscillation just downstream of the expansion is significant relative to the precession and becomes gradually weaker further downstream, due to the spreading of the jet. Then the precession appears dominant. In spite of this, the flow variables show the same periodicity at different points. Although the periods for both the swinging and precession are independent of position, there exists a phase lag at a downstream point. This demonstrates the spiral nature of the jet.

Examination of the flowfield shows that the jet displaces from the centreline and reattaches to the wall about 2 diameter (D) from the expansion. There is an azimuthal flow just behind the step, and a large recirculation zone takes place within 4~5 diameters (~10 step heights) from the sudden expansion. The flow variation becomes insignificant beyond 10 diameters although a slight swirl flow can be still observed.

Inlet Velocity Profile

The inflow profiles have some influence on the results if the inlet pipe is less than 10 diameters (d) in length. When a uniform velocity profile is implemented on the inlet at the expansion plane, the periods for both motions increased by 5~10% in this case. Moreover, more calculation time is required before the aforementioned phenomena establish.

Comparison with Experimental Data

Hill *et al* (1995) measured the precession frequency f_p for various configurations over a wide range of Reynolds number (5,000~46,000). The Strouhal number was defined as

$$St = \frac{f_p \sqrt{\rho} D^2}{\sqrt{M}} \qquad (1)$$

where M is a point source of axial momentum located on the centreline of a long axisymmetric duct of diameter D. Its value here is equal to the momentum flux at the inlet.

For a sudden expansion, the Strouhal number based on the averaged outflow velocity, U_o, is straightforward and differs from eq (1) only by a constant factor:

$$St = \frac{f_p d}{U_o} = \frac{f_p E^2 d}{U_i} \qquad (2)$$

where E is the expansion ratio (E=D/d) and U_i is the average inlet velocity.

Table 1: Frequencies from simulations

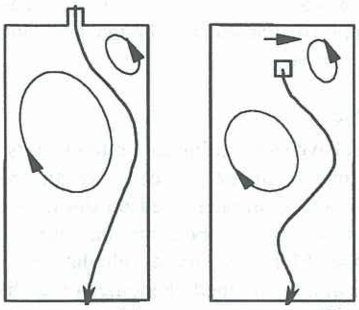
d, m	D, m	U_i, m/s	Re	f_s, 1/s	f_p, 1/s
0.1	0.5	10.0	100,000	0.70	0.28
0.1	0.5	7.72	58,226	0.52	0.22

The swinging frequency (f_s) and precession frequency (f_p) in current simulations are listed in Table 1. Based on eq.(2), the value for the Strouhal number is 7.04~7.08×10^{-2} in the current simulations and 6.36~8.82×10^{-2} from the data of Hill *et al*. The fluid used by Hill *et al* was water whilst the present calculation uses air. Moreover, the geometric model used by us is much larger in dimension although the expansion ratio is similar. The present expansion ratio (E=5) falls within the range (E=3.75~14.0) tested by Hill *et al*. The k-ε model is most reliable at Reynolds number above 20,000, so we have used higher Reynolds numbers. Nevertheless, the present results satisfactorily agree with the results of Hill *et al*.

No information has been reported in the literature on the swinging oscillation. It can be seen that the frequency for swinging increases as the precession frequency increases. More data are needed to determine this relation.

Mechanism

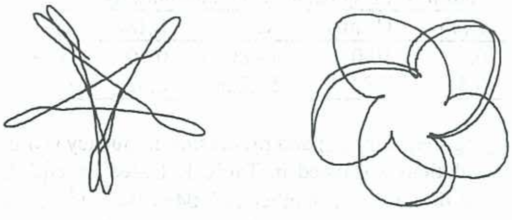
As the momentum transfer between the jet and wall through the recirculating vortex is complex, we may explain the formation of stable jet oscillation in terms of the jet movement as a solid body.



(a) (b)

Figure 5 : Schematic 2-dimensional flow pattern.
(a) steady asymmetric pattern.
(b) cross flow & jet oscillation.

For a simple 2-dimensional sudden expansion as shown in Fig.5(a), the initial displacement of the jet from the centreline causes a negative pressure gradient to form across the jet due to the asymmetric pattern. The jet moves from the centreline towards the wall in response to this pressure gradient, and the transverse component of the jet momentum simultaneously increases. If the pressure increases more than the momentum as the displacement increases, then a steady asymmetric flow pattern will form permanently. However, if a cross flow exists across the jet, as shown in Fig.5(b), the pressure difference can be damped. Eventually a point can be reached where the jet momentum exceeds the pressure gradient and the jet movement reverses. This cross-flow is necessary for the sustained swinging oscillation (Gebert *et al,* 1997). Consequently, the swinging oscillation is produced by a dynamic balance between the pressure difference across the jet and the transverse momentum of the jet.



(a) (b)

Figure 6 : Schematic pattern of jet motion for an axisymmetric sudden expansion
(a) at 0.2D; (b) at 4D

For the three-dimensional case, a cross flow may exist in the annular space between the central jet and surrounding wall, so the swinging oscillation is not difficult to understand. However, when the swinging jet deflects from the centreline, the jet moves towards the wall so that it hits the wall at an oblique angle, and an azimuthal component of momentum arises. The swinging jet appears as if the wall reflects it. A coupling between the two patterns of motions arises and a sustained precession results. This is shown by the results of the current simulation. Firstly, the final solutions, when stable, always feature the co-existence of two periodic motions: swinging and precession. Moreover, Fig.6 shows the velocity vector direction varying with time at two centreline points, which qualitatively indicates the pattern of the jet motion. It clearly shows the deflection and reflection of the swinging motion, which has a radial as well as an azimuthal component.

Therefore, the interaction between the swinging and precessing motions of the jet is necessary for retaining the precession. In this sense the precession is extracting energy from the swinging oscillation.

CONCLUSION

By solving the time-averaged Navier-Stokes equations for turbulent flow, CFD can be used to predict the instability characterised by the precessing jet in axisymmetric sudden expansions. The frequencies simulated with an expansion ratio of 5 agree with the experimental data of Hill *et al* (1995). Moreover a swinging motion of the jet is found to accompany the precession and may be the cause of the precessing jet.

ACKNOWLEDGEMENTS

This work is supported by an Australian Research Council Large Grant. The support of Professor B.S.Haynes for this project is greatly appreciated. Regarding this phenomenon, discussion with Dr K.R.Morrison, University of Canterbury, New Zealand, is gratefully acknowledged.

REFERENCES

CFX, Solver Manual, CFX International, AEA Technology, Harwell, Didcot, Oxon., UK, 1997.

GEBERT, B.M., DAVIDSON, M.R. and RUDMAN, M.J., "Computed oscillations of a confined submerged liquid jet", *International Conference on CFD in Mineral & Metal Processing and Power Generation*, CSIRO, 411-417, 1997.

HILL, S.J., NATHAN, G.J. and LUXTON, R.E., "Precession in axisymmetric confine jets", *Twelfth Australasian Fluid Mechanics Conference*, Australia, 135-138, 1995.

HILL, S.J., NATHAN, G.J. and LUXTON, R.E., "Processing and axial flows following a sudden expansion in an axisymmetric nozzle", *Eleventh Australasian Fluid Mechanics Conference, Australia*, 1113-1116, 1992

HONEYANDS, T.A. and MOLLOY, N.A., "Oscillations of submerged jets confined in a narrow deep rectangular cavity", *Twelfth Australasian Fluid Mechanics Conference*, Australia, 493-496, 1995.

LANGRISH, T.A.G., Personal Communication, 1998.

MIZUSHIMA, J., OKAMOTO, H. and YAMAGUCHI, H., "Stability of flow in a channel with a suddenly expanded part", *Phys. Fluids* 8(11), 2933-2942, 1996.