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A PROPOSAL OF A PHYSICALLY BASED THICKNESS DEFINITION AND OF A NEW MEAN VELOCITIES DISTRIBUTION LAW IN A TURBULENT BOUNDARY LAYER ON THE GROUND OF LDA MEASUREMENTS

Paola GUALTIERI, Guelfo PULCI DORIA

Dipartimento di Ingegneria Idraulica ed Ambientale Girolamo Ippolito Via Claudio 21-80125 Napoli- Italy voice 0039-81-7683460 fax 0039-81-5938936 e-mail pulci@unina.it

ABSTRACT

In this paper, on the ground of previous LDA measurements and of the consequent three bands model of a boundary layer, a new physically based definition of its thickness, and a new mean velocities distribution law within it, .which can eliminate some difficulties connected with some classic distribution laws, are proposed. Both results well fit classic experimental data.

INTRODUCTION

Turbulent boundary layer is a fluid mechanics topic very much studied. The Authors studied experimentally for a long while a turbulent boundary layer generated along the bottom surface of a channel, with no longitudinal pressure gradient, and made measurements (particularly careful as every experimental point refers to more than 300000 data) of instantaneous velocities using an LDA. Details of channel and of measuring system are presented in (Greco, Pulci Doria 1983) (Gualtieri 1995). The Authors investigated about mean velocities, velocity fluctuations and power-spectra of turbulence, and recently about distributions of longitudinal integral length scales, as well as skewness and kurtosis of the instantaneous velocities (Pulci Doria 1991) (Gualtieri 1993) (Gualtieri, Pulci Doria 1997).

These last distributions have been interpreted in such that the thickness δ of the boundary layer is not constant but is really varying inside a certain band that, referring to Coles boundary layer definition (1956), is comprised between $0,85\delta$ and $1,95\delta$. Because of this phenomenon, the flow can be shared in three successive bands, whose widths are proportional to δ. In particular a boundary layer band ($y<0,85\delta$), an intermittence band ($0,85\delta<y<1,95\delta$), an external layer band ($y>1,95\delta$)(Gualtieri, Pulci Doria 1996).

Furthermore, in the first band, distribution laws relative to boundary layer are present, in the third band distribution laws relative to external layer are present, and in the intermittence band the distribution laws are weighed means between the boundary layer laws and the external layer laws, because of the intermittence phenomenon. In fact all happens as if, owing to intermittence, the boundary layer and the external layer laws would continue in the intermittence band, either from the bottom or from the top, in a percent of time.

CLASSIC LITERATURE ON BOUNDARY LAYER MEAN VELOCITIES DISTRIBUTIONS

Coles, on the ground of Clauser's ideas (1956), proposed a mean velocities distribution law that, as it is well known, is affected by an important failure: its derivative at the conventional (by Coles) end of the boundary layer is different from zero

Dean (1976), on the ground of Granville's ideas (1976), proposed a new mean velocities distribution law, in which he could eliminate the failure of Coles law. His law however, and this fact has not yet been underlined, is not continuous in its second derivative always at the end of the boundary layer (fig.1). In this and subsequent figures, for simplicity, the classic velocity defect divided by u* will be called Ud, and the non dimensional co-ordinate y/δ will be sometimes called Y.

More recently Pulci Doria and Taglialatela (1991), in an international context (Hancock, Bradshaw 1989) (Hoffmann, Mohammadi 1991) (Bandyopadhyay 1992) , proposed a new mean velocities distribution law, for taking into account the possible presence of turbulence in the external flow. This law respects the continuity of its derivative, but is affected too by lack of continuity in its second derivative at the end of the boundary layer, that, because of specific reasons, is 1,25 times thicker than Coles and Dean ones. (fig.2)

In his paper, Coles compared his mean velocities distribution law for boundary layer with no longitudinal pressure gradient with experimental points of Wieghardt: this comparison is reported in (fig.3). In his plot Coles represents the difference between the mean velocities values and the logarithmic law (that is the wake law multiplied by $2\Pi/K$) and calls this function "g". In fig.3 the Authors transfer also the Dean and Pulci Doria and Taglialatela mean velocities distribution laws, always as function "g". It is possible to observe:

1) The Coles law well fits the experimental points

till $y/\delta = 1$. Afterwards, it is slightly lower than the experimental points.

2) The Dean law is slightly higher than the experimental points till $y/\delta = 1$, and afterwards shows the same behaviour than Coles law.

3) The Pulci Doria and Taglialatela law fits well experimental points till $y/\delta = 1$, but afterwards becomes too high.

A NEW MEAN VELOCITIES DISTRIBUTION LAW

On the basis of previous observations about non continuity in Coles, Dean and Pulci Doria and Taglialatela mean velocities distribution laws and disagreements between these laws and the experimental data, and having defined a three bands model, which lets a good representation of Skewness, Kurtosis and Length Scales distributions, the Authors will propose a new mean velocities distribution law, relative to the simplest case of boundary layer with no longitudinal pressure gradient and with no turbulence in the external flow.
This law has to satisfy the following requisites:

a) It has to be based on the three band model.

b) It has to be continuous everywhere with its first and second derivatives at least.

c) It has to fit experimental points better than Coles, Dean, and Pulci Doria and Taglialatela laws.

As it is well-known, in mean velocities distribution problems, the true local quantity to which it is necessary to pay attention is not the value of mean velocity itself, but its derivative with respect to y/δ (that is the mean velocities vertical gradient distribution, henceforth "gradient").

Consequently, and according particularly to the requisite a), it is necessary:

1) To define the "gradient" in a boundary layer flow defined "hypothetical" because it would continue beyond the boundary layer band till an infinite distance from the wall, without meeting any external flow.

2) To multiply these "gradient" values by the weight function required by the three bands model which is worth 1 till $y_1/\delta=0{,}85$; varies linearly from $y_1/\delta=0{,}85$ to $y_2/\delta=1{,}95$; and is worth 0 above $y_2/\delta=1{,}95$ (so that the boundary layer law will continue in a percent also in the intermittence layer!)

3) To integrate this "gradient" in order to obtain the mean velocities distribution.

It is obvious that, the "gradient", has to be continuous with its first derivative at least (requisite b); and that the final law after integration will have to well fit the experimental Wieghardt points (requisite c).

In order to define the "gradient" in the "hypothetical" boundary layer flow, according with the previous requisites a) and b), the Authors followed these subsequent rules:

α) The law they had to obtain had to be equal to that of Coles till $y/\delta=0{,}925$

β) The "gradient" had to reach asymptotically a zero value as y/δ goes to infinity.

γ) The "gradient" had to be always continuous with its first derivative.

In order to follow these rules the Authors started from the Coles law (till $y/\delta=0{,}925$) (rule α), employed a hyperbolic law like $A/(y/\delta)^n$ from $y/\delta=1{,}075$ (rule β) and lastly chose the A and n values and the shape $[s(y/\delta)]$ of the function between $y/\delta=0{,}925$ and $y/\delta=1{,}075$ in order to follow the rule γ).

The function obtained through these previous points could stand for the new "gradient" valid in the "hypothetical" boundary layer flow.

As by the previous rules the new "gradient" had been defined (point 1), now the Authors could complete point 2) and 3) so obtaining the mean velocities distribution respectful of a) and b) requisites.

Lastly, in order to respect also requisite c), they compared the mean velocities distribution with Wieghardt points, and repeated the whole procedure many times changing A, n, and the shape $s(y/\delta)$ till also requisite c) had been achieved. So doing they obtained the true final distribution law (fig.4), that is strongly continuous till the second derivative (fig.5).

A direct comparison is presented in (fig.6) among the Authors new law, the Coles law and the experimental Wieghardt points. It is evident how this new law well fits the classic experimental points of Wieghardt, especially in the zone where $y/\delta>1$, where all the previous laws show important disagreements with them.

A NEW DEFINITION OF THE BOUNDARY LAYER THICKNESS

On the ground of the three bands model, and also of the new proposed law, it is possible to redefine the boundary layer thickness in a more rigorous way.

As it had been shown that the instantaneous thickness of the boundary layer varies, with respect to the Coles δ definition, from 0,85δ to 1,95δ, it becomes suitable to define a new boundary layer thickness δn as the mean between these two values, namely equal to 1,4 times the Coles δ definition. With respect to this new definition, the intermittence band lies symmetrically under and above the $y/\delta n=1$ point, from $y/\delta n = 0{,}6$, till $y/\delta n = 1{,}4$.

This new definition is a physically based one. Moreover, observing the shape of the new proposed law, it is possible to notice that, at the distance of 1,4 times the Coles δ definition (that is at δn distance), the mean velocity has attained almost the 100% value of U_∞ value (it is needed only a value equal to $0{,}0011u_*$ to attain just the 100% of U_∞).

A NEW "TOTAL WAKE LAW"

In both figs.3 and 6 the difference between the mean velocities values and those characteristic of the logarithmic law is represented. This difference represents the wake law multiplied by 2Π/K. It is possible to call this difference "total wake law".

Now, taking into account the new boundary layer thickness definition δn and the new mean velocities distribution, it is possible to define also a new "total wake law", which must be considered valid till y/δn=1,4. Only at that value of y/δn the intermittence band is completely exhausted and after this value the curve follows at all the simple law $U=U_\infty$, namely:

total wake law = U_∞ - (1/K)*ln (y/δ) – C

This new "total wake law" is reported in the following table.

y/δ	t.w.l.	y/δ	t.w.l.	y/δ	t.w.l.
.01	.000	.71	2.732	1.07	1.859
.04	.006	.75	2.678	1.09	1.818
.07	.040	.77	2.629	1.11	1.777
.11	.116	.78	2.612	1.13	1.738
.14	.231	.79	2.595	1.14	1.698
.18	.374	.80	2.550	1.16	1.659
.21	.545	.82	2.503	1.18	1.621
.25	.736	.84	2.455	1.20	1.584
.29	.942	.86	2.406	1.21	1.547
.32	1.152	.88	2.357	1.23	1.510
.36	1.367	.89	2.309	1.25	1.474
.39	1.584	.91	2.261	1.27	1.439
.43	1.797	.93	2.214	1.29	1.404
.46	2.005	.95	2.167	1.30	1.369
.50	2.200	.96	2.121	1.32	1.335
.54	2.377	.98	2.076	1.34	1.302
.57	2.530	1.00	2.031	1.36	1.269
.61	2.648	1.02	1.987	1.38	1.236
.64	2.723	1.04	1.944	1.39	1.204
.68	2.749	1.05	1.901	1.40	1.188

CONCLUSIONS

Taking into account the shape of the second derivative of mean velocities distribution and a previously defined three bands model in boundary layer with no pressure gradient, it has been possible to define a new mean velocities distribution law, more respectful of the physical behaviour and of the experimental data. Moreover both distribution law and three bands model suggest a new physically based definition of boundary layer thickness.

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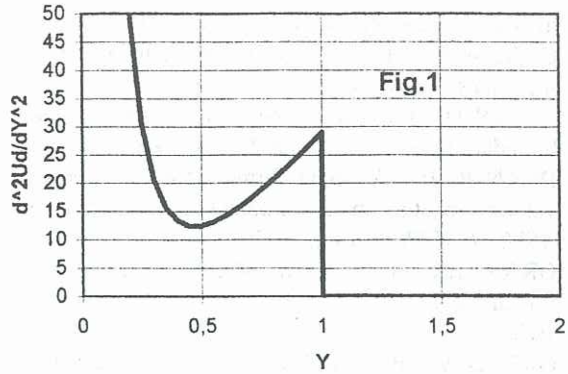

Fig.1

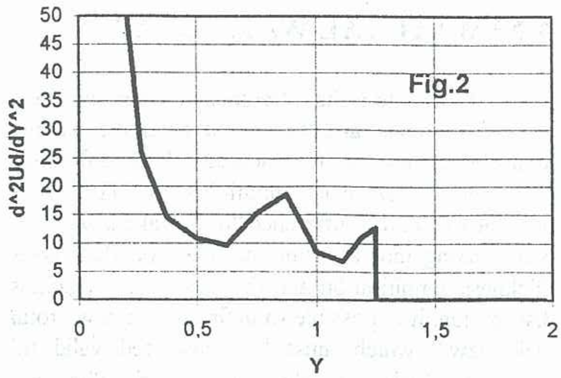

Fig.2

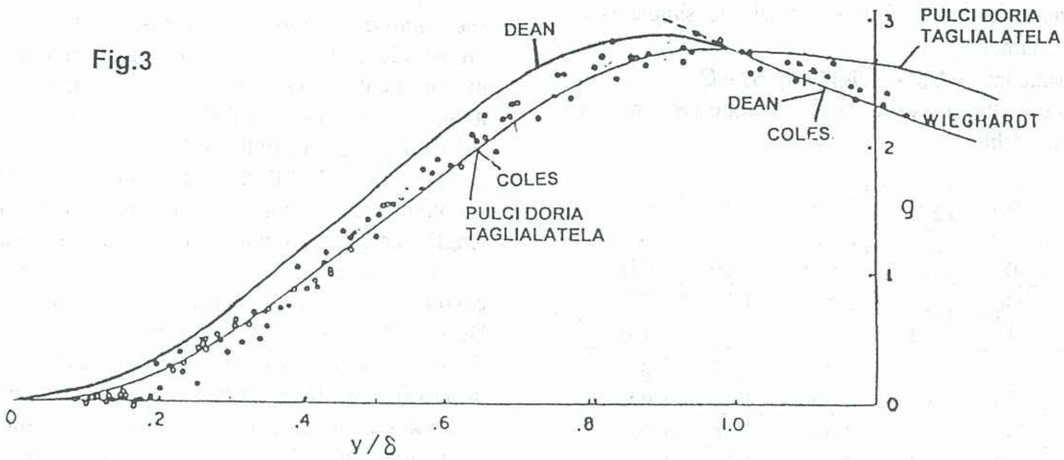

Fig.3

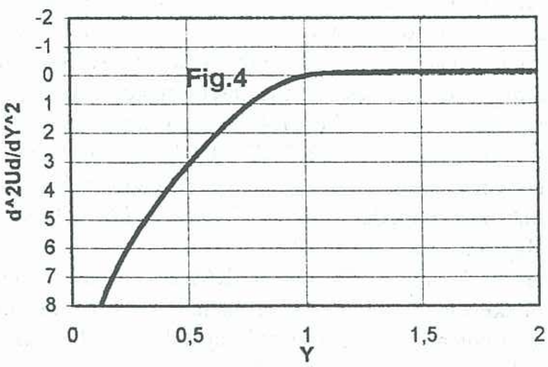

Fig.4

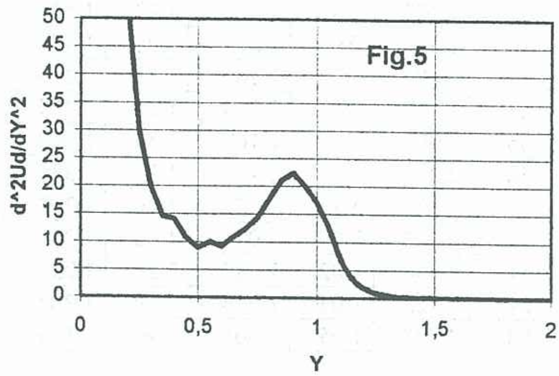

Fig.5

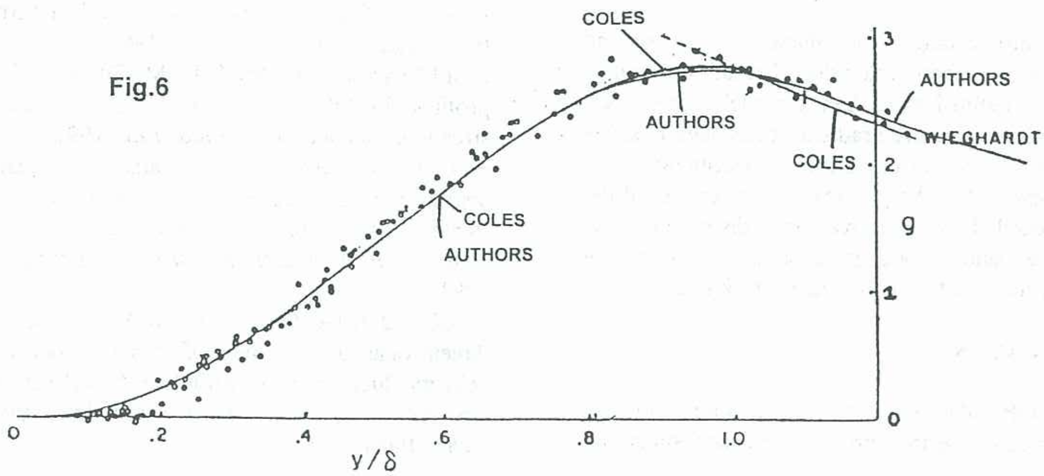

Fig.6