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A Study of Waves Developed by Caldera Collapse

J.P.P. GRAY and J.J. MONAGHAN

Department of Mathematics and Statistics
Monash University, Clayton, Victoria, AUSTRALIA

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ABSTRACT

The following paper outlines a study of the production and propagation of water waves using SPH. Initially the SPH code is tested to see that it agrees with experiment. The method is then applied to model waves that are typically generated during the collapse of the caldera of a volcano.

INTRODUCTION

The purpose of this study is to investigate the propagation and production of solitary waves in water. The eventual aim being to simulate the types of waves produced during the collapse of caldera volcanoes surrounded by bodies of water. The water motions that occur during such a collapse are complicated, involving the overturning of waves. The numerical method used is that of Smoothed Particle Hydrodynamics (SPH) reviewed by Monaghan (1992). While other methods are able to simulate the travelling motions of waves it is only recently that overturning waves have been simulated beyond the stage where they hit the water. Variations of the finite difference Marker and Cell (MAC) method as given by Miyata (1986) and Mader (1986) are able to handle breaking waves but are complicated to code. The fully nonlinear potential flow models of Grilli *et al* (1997) is capable of handling the breaking and overturning of waves but cannot simulate flow once the overturning wave has rejoined the body of the fluid.

We begin by doing some simple tests on solitary waves to ensure that the SPH technique is sufficiently accurate. The methods mentioned above may be more accurate for these particular problems but they cannot easily handle the problem we wish to investigate.

The solitary wave was initiated using the classical small amplitude form for its height and velocity field. We compare the results to the simulations by Chan and Street (1970) and experiment.

The small amplitude form is only approximate so we also generated the waves in the simulation by moving the left hand end of the tank into the fluid. This gives the fluid an impulse which is similar to what occurs in the experiments.

Finally, a simple model of a caldera collapse is simulated to illustrate the kinds of waves that are generated in the water surrounding this type of volcanic eruption.

SPH

SPH is a particle method. It allows one to solve the Navier-Stokes equations without the use of a grid. This makes it ideal for free-surface problems as particles automatically define the free surface.

Associated with each particle are certain properties such as position, velocity and density. The method employs the use of a kernel (e.g. A Gaussian) which smooths out the properties of each particle over a distance of $2h$, where h is the smoothing length. The major property of the kernel is that it approximates the delta function in the limit $h \to 0$. Particles within a distance of $2h$ of each other interact. A given particle property is found by a summation of this property over the surrounding particles.

The momentum equation in SPH is given by,

$$\frac{d\mathbf{v_a}}{dt} = -\sum m_b \left(\frac{P_a}{\rho_a^2} + \frac{P_b}{\rho_b^2} + \Pi_{ab}\right) \nabla_a W_{ab} + F_a. \tag{1}$$

Where the viscosity term Π_{ab} is,

$$\Pi_{ab} = -\frac{\alpha h}{\bar{c}_{ab}} \frac{\mathbf{v_{ab}}.\mathbf{r_{ab}}}{|\mathbf{r_{ab}}|^2} \left(2\bar{c}_{ab} - 3h\frac{\mathbf{v_{ab}}.\mathbf{r_{ab}}}{|\mathbf{r_{ab}}|^2}\right) \tag{2}$$

The continuity equation is used to calculate density,

$$\frac{d\rho_a}{dt} = \sum_b m_b(\mathbf{v_a} - \mathbf{v_b}).\nabla_a W_{ab}. \tag{3}$$

The original version of SPH dealt only with compressible flows, but is applied here to model incompressible flow. Using the approach of Monaghan (1994) incompressible flow is modelled by incorporating a stiff equation of state for the fluid, making the fluid slightly compressible. The method is therefore similar to Chorin's (1967) artificial compressibility method. We use the equation of state (Batchelor (1973)).

$$P = B\left(\left(\frac{\rho}{\rho_0}\right)^\gamma - 1\right) \tag{4}$$

Where $\gamma = 7$ and $B = \frac{200gD}{\rho\gamma}$ was chosen to give a Mach number $M = 0.1$ and therefore density variations of 1%.

RESULTS

Wave runup against a wall

We initiated waves in two different ways. In our first series of computations we used an analytical form for a solitary wave as our initial condition. In the second series we initiated the wave by moving the left hand wall of the tank.

Solitary Wave - Analytical Form

Chan and Street (1970) modelled water waves using a variation of the MAC method. They compared their SUMMAC method with a series of experiments, (Camfield and Street (1968)), in which a solitary wave of amplitude H, propagated over a body of water of depth D. Eventually the wave runs into the wall at the end of the the tank and up the wall to a maximum height R. At this point the wave has zero vertical velocity and will then fall down the wall reforming the solitary wave.

We use the approximate analytical form for a small amplitude wave (Lighthill 1980), as our initial condition.

$$\eta = \frac{H}{cosh^2\left(\sqrt{\frac{3}{4}\frac{H}{D^3}}(x - Ut)\right)} \qquad (5)$$

The simulation illustrated in Figure 1 is for a wave with $D = 21cm$ and $H = 0.6D$. The figure shows the initial wave setup, the wave propagating towards the wall and the wave after it has run up the wall.

Figure 2 shows how the SPH computations compare to the simulations and experiments discussed above. The SPH results are in excellent agreement with experiment even though the analytical form (5) is only an approximation to the actual velocity field. However the wave in the simulation quickly adjusts to a steady form consistent with the equations of motion. It is to be expected (for small amplitude waves) that after the interaction with the wall the wave will reform with the same profile as before although travelling in the opposite direction. When $H/D = 0.2$ the agreement between the incident and reflected waves was good. This is also in agreement with the method of Chan and Street. As expected large amplitude waves produced a reflected wave with a different profile to the incident wave.

Solitary Wave - Generated by Moving Wall

The previous section showed that SPH could be used to model the propagation of waves, but it can be extended to simulate both the initiation and propagation of waves. We produced waves by accelerating the left hand wall of the tank to the velocity $V = 0.1gt_s$, for a time $t_s = t_*\sqrt{D/g}$. The wall was then moved

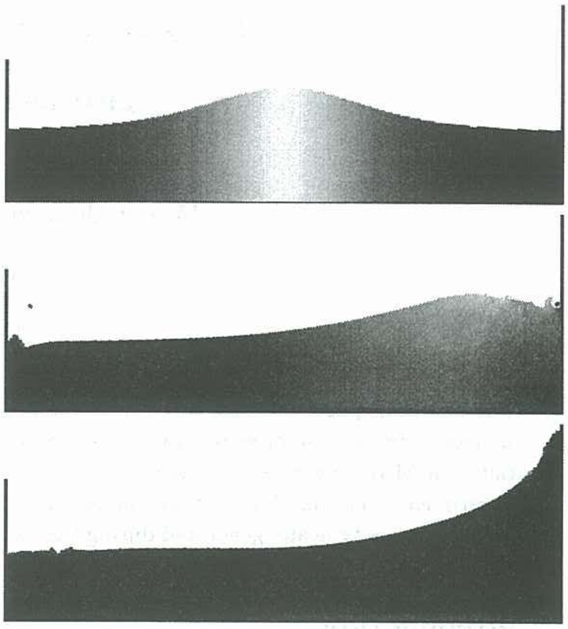

Figure 1: Solitary wave running up a wall. The tank is 1.68m long with the same scale being employed in the vertical direction. Particles are shaded according to their speed with lighter shades denoting higher speeds. The times of each of the above frames are 0.0s, 0.28s and 0.55s from top to bottom respectively. 25,875 particles were used in the simulation, with the computation time being 6.5 hours for the 6,400 steps to maximum run up. All computations in this paper were performed using an SGI O2.

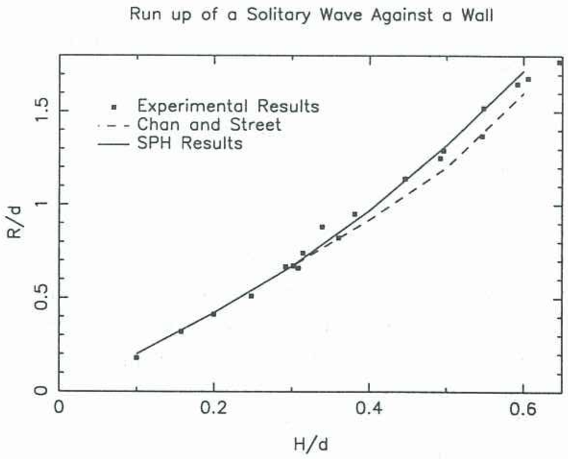

Figure 2: Comparison of SPH results for the run up against a wall to those of Chan and Street (1970), and the experimental results of Camfield and Street (1968).

at the constant velocity V for a time $t = \frac{1}{2}t_s$. Different amplitude waves were produced by altering the parameter t_* between 1 and 5. This provides a more realistic way of initiating waves because it is analogous to the experiments.

Measurements showed that given an amplitude, waves generated by this method had comparable cross sectional areas and speeds to a wave of the form (5), although the half-widths were different. So the waves were of a similar form of those in the previous section although not strictly the same.

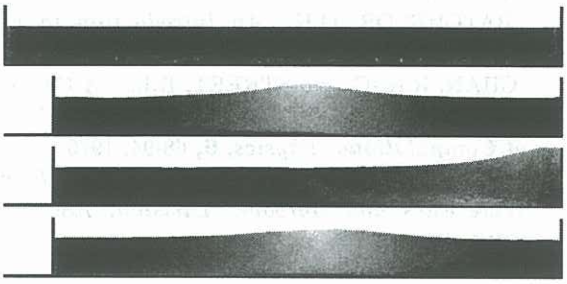

Figure 3: Wave Generated by Moving Wall in a 2.4m long tank. The times of the frames are 0.0s, 1.32s, 2.19s, 2.89s from top to bottom respectively. 14,533 particles were used in the computation which took 32 minutes per 1,000 steps. With 19,000 steps in the computation.

The next step is to study how the wave after reflection off the wall compares to the incident wave. Figure 3 illustrates the various stages of a wave that is generated by a moving left hand wall in a 2.4m long tank. It was found that when the motion of the wall was instantaneously stopped from constant velocity, the water close to the wall retained the velocity of the wall for a time after the wall had stopped moving. This led to the fluid moving away from the wall and the production of a smaller secondary wave as the water curled back towards the wall. The simulation illustrated in Figure 3 overcame this by decelerating the wall back down to zero velocity before the motion of the wall was halted.

Figure 3 illustrates the wave at four important times: the initial setup before the wall motion began, the incident wave at midtank, the wave running up the wall and finally the reflected wave at midtank. One notices that as in the previous section the small amplitude wave $H/D = 0.22$ is reflected without significant change. The only major difference between the reflected and incident waves is a small amount of dispersion.

We also found agreement within experimental error with the results presented in Figure 2 for the relation between wave amplitude and runup height. These results are shown in Figure 4 and the agreement is pleasing as it shows how three distinct ways of modelling the run up of waves on a wall (experiment, computer simulation with a given initial condition, and a computer simulation which generates the wave) all produce similar results.

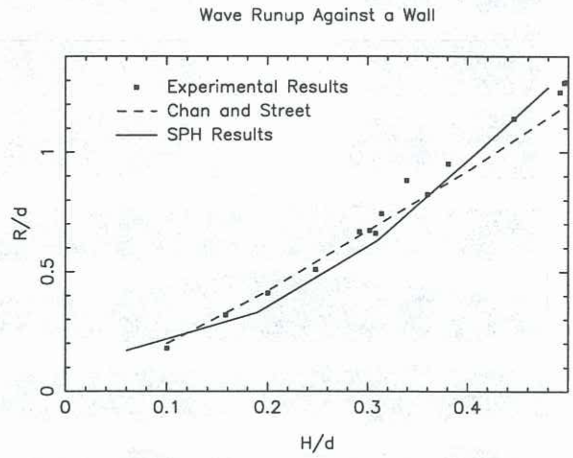

Figure 4: Comparison of SPH results for the run up against a wall for a computationally generated wave to those of Chan and Street (1970), and the experimental results of Camfield and Street (1968).

Volcanoes - Caldera Subsidence

As this paper is concerned with fluid mechanics only a brief overview of the phenomena of subsidence calderas will be presented here. The interested reader being refered to the review article of Lipman (1997) for further information.

A volcanic eruption involves the release of magma from an underground chamber. When the released magma is not replaced from some source, it is possible for the magma chamber to become partially evacuated and no longer be able to support the overlying rock and earth, which can then subside into the magma chamber.

In the case of the Bronze Age eruption of the island of Santorini, near Crete, water from surrounding sea was able to flow into the crater formed by the eruption. The waves produced in this type of event are the focus of the model in the following section.

Model of a Caldera

A simple model of the waves being produced in a body of water surrounding the eruption of a caldera volcano is illustrated in Figure 5. A 2D model is presented as we eventually wish to be able to compare our results with a 2D experiment. Indeed this is a simplification of the real case. There is no great obstacle preventing a 3D SPH simulation, although it would be computationally expensive.

The model consists of two bodies of water either side of a cavity. The cavity represents the volcano before

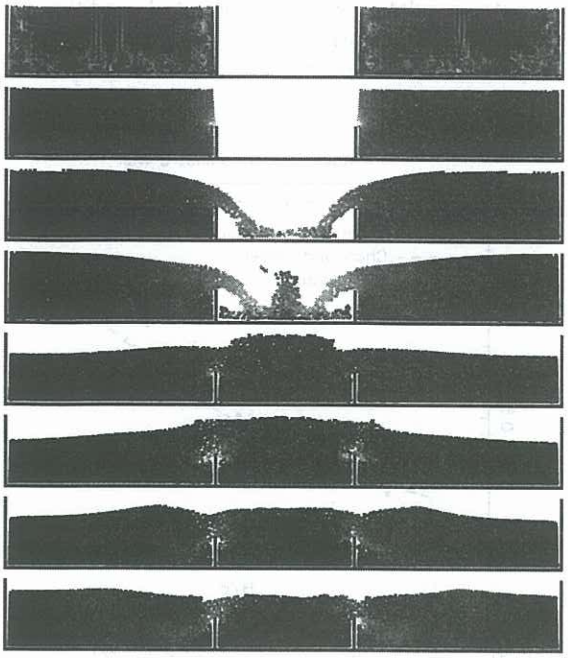

Figure 5: Model Of Caldera Subsidence. The initial frame corresponds to the end of damping, with other frames refering to times 0.02s, 0.22s, 0.36s, 0.85s, 1.09s, 1.29s and 1.43s after damping. The length of the tank was 1.6m, with the cavity being 0.4m wide. The total water depth was 0.20m with the height of the removable wall being 0.1m. 5,602 particles were used in the computation. 1,000 time steps were computed in 8 minutes, with 12,900 steps required for the calculation.

eruption. The model uses damping for the first 2,000 time steps to reduce the excess energy that is inherent in the initial (regular) particle setup.

After the damping had been completed the collapse of the rock over the top of the magma chamber into the chamber was simulated by removing the top half of the two inner boundary walls. This leaves behind a cavity into which water is allowed to flow.

Water proceeds to flow into the middle of the cavity splashing up to form a large central column. One can notice a few particles breaking away from the bulk of the fluid in the fourth frame of Figure 5. This assymetry is more than likely due to noise being produced at the boundaries or the fact that the system had not undergone enough damping to reduce all of the excess energy in the initial state before the walls were removed.

The central column continues to grow, filling up the cavity and increasing in height until the height of the column is greater than that of the surrounding water. The column then breaks up into three parts, two waves emerge travelling left and right, with the central column over the cavity dispersing away over time.

CONCLUSION

The SPH method was used to simulate the motion of solitary waves and was found to compare favourably both with experiment and a different numerical scheme. SPH was then used to simulate the generation of solitary waves, it was found that this could be done without difficulty. Finally the method was applied to a simple model of a caldera volcano and was found to produce sensible results. Further work is required to validate these results.

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