13th Australasian Fluid Mechanics Conference
Monash University, Melbourne, Australia
13-18 December 1998

LEADING-EDGE RECEPTIVITY TO OBLIQUE ACOUSTIC WAVES

David FUCIARELLI, Helen REED, and Ian LYTTLE

Department of Mechanical & Aerospace Engineering
Arizona State University, Tempe, Arizona, USA

ABSTRACT

Numerical simulations of leading-edge acoustic receptivity are performed for a flat plate with a modified-super-elliptic (MSE) leading edge. For small freestream amplitude, the agreement between Branch I receptivity coefficients predicted from the DNS and the experiments of Saric & White (1998) for acoustic waves at zero incidence is excellent. The effect of angle of incidence of the impinging wave is investigated and found to produce higher receptivity coefficients than in the symmetric case. The slope of leading-edge receptivity coefficient versus angle of incidence of the impinging wave is found to be less than 1/4 of the slope predicted by zero-thickness flat-plate theory. However, there is excellent agreement between the DNS and finite-nose-radius theory of Hammerton & Kerschen (1996). These results clearly demonstrate the importance of including the effects of the finite nose radius in any receptivity study. Finally, downstream of the leading-edge region, linear stability theory is found to accurately reproduce the characteristics of the instability waves. At higher freestream forcing, an instability wave generated by nonlinear interaction is found at double the frequency of the forcing.

INTRODUCTION

Transition in wall-bounded shear layers occurs because of an incipient instability of the basic flow field, which depends intimately on subtle, and sometimes obscure, details of the flowfield. In other words, the wall-bounded shear layer is an open system. Disturbances in the freestream, such as sound or vorticity, enter the boundary layer as steady and/or unsteady fluctuations of the basic state. This part of the process, called receptivity, provides the vital initial conditions of amplitude, frequency, and phase for the breakdown of laminar flow. The recent progress in this area is summarized in Goldstein & Hultgren (1989) and Saric et al. (1994)

External disturbances common to the flight environment are typically either acoustic or vortical. These types of disturbances are referred to as natural disturbances. In contrast, disturbance environments produced by artificial means such as a vibrating ribbon or suction/blowing are referred to as forced disturbances. Whereas forced disturbances typically contain a broadband of wavelengths, naturally occurring disturbances typically have a narrow wavelength band. The important distinction is that acoustic waves or vortical disturbances generally will not contain a wavelength that coincides with the instability that is generated within the boundary layer. Thus in this case some mechanism for transferring energy from a much longer wavelength wave to a relatively small wavelength instability wave must exist. Therefore a proper understanding of the receptivity process will include proper characterization of the external disturbance environment, as well as an understanding of both the mechanisms for the transfer of energy among different wavelengths and the effects of different disturbance environments. Detailed studies will also show how different geometries affect the process and how fluid properties such as Reynolds number and Mach number affect receptivity. Characterization of the external environment includes the wavelength and frequency spectrum of incoming disturbances. For acoustic waves the orientation of the acoustic wave with respect to the geometry is an important characteristic. For vortical disturbances the orientation, i.e. streamwise, normal, or spanwise or some combination; the size of the vortex core; and the strength of the vortex all play a role in the receptivity process.

Theoretical investigations into acoustic receptivity have identified mechanisms for the transfer of energy among disparate wavelengths. In the research of Lam & Rott (1960), Ackerberg & Phillips (1972), and Goldstein (1983), receptivity to acoustic waves impinging upon a flat-plate geometry at zero angle of incidence was investigated. The primary mechanism for transferring energy is the relatively rapid growth of the mean boundary layer and the associated pressure gradients near the stagnation point. The investigations were performed using the Linearized Unsteady Boundary Layer Equation (LUBLE) which is a parabolic equation and therefore conducive to less costly numerical methods. Energy from the external disturbance environment is transferred to streamwise decaying eigenfunctions whose wavelength decreases as the flow progresses downstream. Thus a process by which long wavelengths become shorter wavelengths is found in these so called Lam-Rott eigenfunctions. The Lam-Rott eigenfunctions match onto solutions of the linear stability theory downstream. Therefore if one knows the amplitude of the Lam-Rott eigenfunction the amplitude of the instability wave downstream is known. Goldstein et al. (1983) and Heinrich & Kerschen (1989) calculated leading-edge receptivity coefficients for various freestream disturbances. Goldstein (1985) and Goldstein & Hultgren (1987) also showed that discontinuities in slope or curvature in the geometry also provide a mechanism for acoustic receptivity. Regions where surface irregularities exist promote small-scale variations of the mean-flow boundary layer and therefore the mechanism is very similar to the flat-plate case with no surface irregularity.

Heinrich et al. (1988) applied the LUBLE to acoustic waves at varying angles of incidence impinging upon a flat plate. The results produced show a strong dependence of initial amplitude of the instability wave on the angle of incidence of the impinging acoustic wave. In addition, the effect of angle of incidence is found to be more pronounced for smaller Mach numbers and singular in the limit as the Mach number approaches zero.

The above results apply to a zero-thickness flat plate. Hammerton & Kerschen (1996, 1997) extend the analysis to account for a finite-thickness and -curvature leading edge by considering a parabolic geometry. The analysis is formulated to provide insight into the effect of nose radius, an effect present in any manufacturable geometry. The Mach number and the amplitude of the impinging acoustic disturbance are considered to be small, so that the mean-flow pressure field can be computed with incompressible theory. The mean flow is two-dimensional and symmetric. Lam & Rott (1993) generalize their eigenfunctions to account for arbitrary streamwise variation in the mean flow, and Hammerton & Kerschen (1996) show that their expressions are in agreement with these more general results. The relevant parameter is the Strouhal number based on the nose radius St_r. The results of this work show that the receptivity coefficient is nearly unity when the nose radius is zero and decreases dramatically with increasing nose radius. The receptivity coefficient is shown to increase with increasing angle of incidence of the acoustic wave. Hammerton & Kerschen (1997) consider the small-Strouhal-number limit. For freestream acoustic waves at zero incidence, the receptivity is found to vary linearly with Strouhal number, giving a small increase in the receptivity coefficient relative to that for the flat-plate case. However, for oblique waves, the receptivity varies with the square root of the Strouhal number, leading to a sharp decrease in the amplitude of the receptivity coefficient compared to the flat-plate case.

A leading edge with finite curvature and thickness has been shown to produce instability waves. Experiments in leading-edge receptivity have shown that great care must be taken in order to produce results that are comparable to theory and numerical simulation (Saric 1994). The very small amplitudes upstream are not detectable in the experiments and the instability waves can only be measured after significant growth has occurred. However, once the amplitude of an instability wave has been determined downstream one can use linear stability theory to provide amplitudes in the leading-edge region.

Numerical simulations of receptivity have provided a database of results that clarify issues in receptivity and can be used to refine experimental procedures. One of the most important results is that the amplitudes of the generated instability waves scale linearly with the acoustic forcing amplitude when the forcing amplitude is less than 1% of the freestream velocity (Lin et al. 1992). Receptivity due to surface irregularities or discontinuities in curvature such as those that exist when a ellipse is joined to a flat plate has been accurately simulated (Lin et al. 1992). When the curvature discontinuity at the juncture of an ellipse and a flat-plate is removed the instability wave amplitudes decrease by more than half. Numerical simulations have also affirmed that smaller nose radii produce larger leading-edge instability-wave amplitudes (Haddad & Corke 1998).

NUMERICAL METHOD

In the research presented here the focus is upon acoustic waves impinging upon an elliptic-type leading edge joined to a flat plate. Numerical simulations are conducted to examine the effect of angle of incidence, frequency, and amplitude of the incoming acoustic wave. The simplified geometry lends itself to comparison with experimental and theoretical investigations while retaining the geometric properties, pressure gradients, and mean-flow boundary-layer adjustment which promote the wavelength conversion necessary for receptivity to occur. The discontinuity at the juncture of the ellipse and the flat-plate is eliminated by using a Modified Super Ellipse (MSE) in order to concentrate on the effects of the finite-curvature and -thickness leading-edge. All of the simulations are for two-dimensional incompressible mean flows and disturbance solutions. The research presented here focuses upon the effects of different disturbance environments by varying the properties of the impinging acoustic wave. It is hoped that by clarifying the effects of these properties that the understanding of the receptivity process is enhanced and that the results provide a basis for further investigation.

The two-dimensional incompressible Navier-Stokes equations are cast in a stream-function/vorticity form and solved in a general-curvilinear-coordinate system using the Modified Strongly Implicit Procedure (MSIP) of Schneider & Zedan (1981). The disturbance equations are formulated by assuming a linear combination of the steady basic state and a time-dependent perturbation.

The geometry of interest is the MSE (as proposed by Lin et al. 1992) attached to a flat plate

$$\left(1-\frac{x}{AR}\right)^{2+\left(\frac{x}{AR}\right)^{2}}+y^{2}=1$$

where lengths have been nondimensionalized with the plate half-thickness. Examining this equation we note that at x=0 (upstream-most point) the exponent is the same as a normal ellipse and becomes cubic as we approach the flat-plate juncture at x=AR. This variation allows for continuous curvature at the juncture. Therefore the receptivity mechanism predicted by Goldstein (1985) and Goldstein & Hultgren (1987) and found by Lin et al. (1992) at a point of discontinuous curvature will not be present in the simulations, thus allowing the focus to be on the leading edge.

Proper selection of boundary conditions is important for the numerical solution of the governing equations. On the solid-surface boundary, the no-slip and no-penetration conditions are imposed. The boundary conditions for the farfield are imposed as velocity boundary conditions. For the basic-state calculations, the inviscid values of velocity are computed from a panel code. For the disturbance calculations a time-periodic velocity fluctuation is imposed. By doing so we model the acoustic wave as an infinite-wavelength disturbance. This near-field condition assumes that the wavelength of the acoustic wave is much longer than the extent of the computational domain.

For the outflow-boundary condition, special care is taken to ensure that waves are not reflected. When traveling waves are present, the solution may be corrupted by the waves reflecting off an ill-posed outflow boundary. The buffer-domain technique similar to the procedure implemented by Mittal & Balachandar (1994) is imposed.

RESULTS

Acoustic leading-edge receptivity on a MSE connected to a flat plate is investigated by direct numerical simulation (DNS). The effects of frequency, forcing amplitude (both linear and nonlinear), and angle of incidence of the impinging acoustic wave are investigated.

Unless otherwise indicated, the simulations are conducted around the baseline case of a

- 6:1 MSE at Reynolds number 2400 based upon the plate half-thickness L and freestream speed U
- non-dimensional frequency of $F = 86 \times 10^{-6}$ (where $F=2\pi\nu f/U^2$ and f is in Hertz).

For the basic-state solution, grid studies were performed to ensure that convergence of the correct solution has been achieved. The location of the far-field boundary has been investigated to ensure that the solution is independent of its location.

In the linear simulations, grid studies were performed and the location of the far-field, the length of the buffer-domain, the parameters in the buffer domain function were all varied to ensure solution independence of these effects. Downstream, the linear calculations compare favorably with solutions of the locally-parallel OSE. The amplitude, wavelength, and eigenfunctions all compare well with the OSE. Therefore, the OSE is used to extrapolate amplitudes and wavelengths downstream beyond the computational boundaries.

In the linear simulations, receptivity coefficients were obtained for a range of frequencies. For example, the values obtained for both a 6:1 and a 20:1 MSE are contained in the table below; the 20:1 geometry is included for comparison with the experiments of Saric & White (1998). The receptivity coefficient (K_I) for these purposes is defined as the amplitude of the instability wave at branch I of the neutral stability curve divided by the amplitude of the incident acoustic wave. The receptivity coefficients have been extrapolated downstream from the numerical results by using linear stability theory. The amplitude of the instability wave was taken at its maximum absolute value within the boundary layer after decomposing using the method presented above. The results of the numerical simulations show no significant variation with frequency. The agreement between the computations and the experiment is excellent, and we conclude that each validates the other.

The results of Haddad & Corke (1998) show that at the leading edge the receptivity coefficient has a value of approximately 0.47 for a Strouhal number based on the nose radius of 0.01 (which is the value in the present numerical simulations). In the theoretical results of Hammerton & Kerschen (1996), the leading-edge receptivity coefficient is found to be nearly unity. The value of the receptivity coefficient at an x-location of 1/2 the wavelength of the associated instability wave (which represents a distance based on the hydrodynamic length scale U/ω of approximately unity) in the present simulations compares well with the theoretical results, predicting a leading-edge value of approximately 0.75. Here U is the speed of the uniform freestream flow and ω is the frequency of the plane acoustic wave.

$F \times 10^6$	K_I 6:1 MSE DNS	K_I 20:1 MSE DNS	K_I 20:1 MSE Experiment
80	0.0030		
82	0.0030	0.048	0.050
84	0.0031	0.048	0.050
86	0.0032	0.048	0.050
88	0.0033		
90	0.0034		

Table 1: Branch I receptivity coefficients for multiple frequencies as predicted by DNS and compared with the experiments of Saric & White (1998).

In the linear simulations, acoustic waves impinging upon the geometry at angles of incidence α_{ac} were investigated. The numerical farfield-boundary condition in the disturbance calculation is modified to include a normal component of the velocity oscillation. This effectively changes the angle of incidence of the impinging acoustic wave. The basic-state solution remains at zero angle of attack. Because the flowfield is no longer symmetric, the full domain must be computed. An oblique acoustic wave causes small-amplitude motion of the stagnation point and thus introduces a small vertical component of velocity at the leading edge.

Non-symmetric forcing of the acoustic wave yielded an increase in the receptivity coefficient with increasing angle of incidence but at a much smaller rate than that predicted by the theory for the zero-thickness flat plate. The slope of the increase in receptivity for the DNS is approximately 0.15 whereas the increase as predicted by Heinrich & Kerschen (1989) is approximately 0.65. Therefore the DNS predicts a slope of less than 1/4 of the slope predicted by this theory.

A comparison with the finite-nose-radius theoretical results of Hammerton & Kerschen (1996) shows much more encouraging results.

α_{ac} (degrees)	C_I DNS	C_I Theory
0	0.75	1.0
5	1.3	1.8
10	2.1	2.6
15	3.2	3.4

Table 2: Leading-edge receptivity coefficients for various incidence angles as predicted by DNS and finite-nose-radius theory of Hammerton & Kerschen (1996).

The agreement is excellent and clearly demonstrates the importance of including the effects of the finite nose radius in any receptivity study.

In the nonlinear simulations, at 1% freestream forcing, an instability wave generated by nonlinear interaction was found at double the frequency of the forcing. This superharmonic was found to grow at the Branch I location predicted by LST for the doubled frequency. The amplitude of the superharmonic was $O(10^{-4})$ which corresponds to the square of the forcing amplitude. However, because the fundamental-frequency solution decays in the computational domain and the superharmonic grows, the superharmonic dominated the fundamental-frequency solution in some region downstream of the leading edge. Further downstream, for these experimental conditions, the superharmonic decayed again and had negligible amplitude at the Branch I location of the fundamental-frequency instability wave.

CONCLUSIONS

Numerical simulations of leading-edge acoustic receptivity are performed for a flat plate with a modified-super-elliptic (MSE) leading edge. For small freestream amplitude, the agreement between Branch I receptivity coefficients predicted from the DNS and the experiments of Saric & White (1998) for acoustic waves at zero incidence is excellent. The effect of angle of incidence of the impinging wave is investigated and found to produce higher receptivity coefficients than in the symmetric case. The slope of leading-edge receptivity coefficient versus angle of incidence of the impinging wave is found to be less than 1/4 of the slope predicted by zero-thickness flat-plate theory. However, there is excellent agreement between the DNS and finite-nose-radius theory of Hammerton & Kerschen (1996). These results clearly demonstrate the importance of including the effects of the finite nose radius in any receptivity study Finally, downstream of the leading-edge region, linear stability theory is found to accurately reproduce the characteristics of the instability waves.

Nonlinear interactions for the particular freestream and geometry conditions simulated appear at forcing amplitudes greater than about 1%. The 1% forcing condition corresponds to nearly 120dB for the experiments performed in the ASU Unsteady Wind Tunnel. It is hard to imagine a flight condition where acoustic levels of this magnitude would be present. However, these (and much higher) levels are present in turbine engines and the results may indicate trends for use in investigating transition in these environments. Moreover, at the higher Reynolds numbers of flight, Branch I for the various frequency solutions may be located in adverse-pressure-gradient regions near the leading edge and disturbances created by lower-amplitude forcing may provide a richer content for the transition process. This plus the enhanced receptivity at incidence suggests a possible role for nonlinearity at the lower-amplitude forcing characteristic of flight.

ACKNOWLEDGMENTS

This work was supported at various stages by the Air Force Office of Scientific Research and NASA Langley Research Center.

REFERENCES

ACKERBERG, R. and PHILLIPS, J. 1972 The unsteady laminar boundary layer on a semi-infinite flat plate due to small fluctuations in the magnitude of the free-stream velocity. *J. Fluid Mech.* 51, 137-157.

GOLDSTEIN,M.1983 Evolution of Tollmien-Schlichting waves near a leading edge. *J. Fluid Mech.* 127, 59-81.

GOLDSTEIN, M. 1985 Scattering of acoustic waves into Tollmien-Schlichting waves by small streamwise variations in surface geometry.*J.FluidMech.*154,509-529.

GOLDSTEIN, M. and HULTGREN, L. 1987 A note on the generation of T-S waves by sudden surface-curvature change. *Ann. Rev. Fluid Mech.* 181.

GOLDSTEIN, M. and HULTGREN, L. 1989 Boundary-layer receptivity to long-wave freestream disturbances. *Ann. Rev. Fluid Mech.* 21, 137-166.

GOLDSTEIN, M., SOCKOL, P. and SANTZ, J. 1983 The evolution of Tollmien-Schlicting waves near a leading edge. Part 2. Numerical determination of amplitudes. *J. Fluid Mech.* 129, 443-453.

HADDAD, O. and CORKE, T. 1998 Boundary layer receptivity to freestream sound on parabolic bodies. Submitted to *J. Fluid Mech.*

HAMMERTON, P. and KERSCHEN, E. 1996 Boundary-layer receptivity for a parabolic leading edge. *J. Fluid Mech.* 310, 243-267.

HAMMERTON, P. and KERSCHEN, E. 1997 Boundary-layer receptivity for a parabolic leading edge. Part 2. The small Strouhal number limit. *J. Fluid Mech.* 353, 205-220.

HEINRICH, R., CHOUDHARI, M. and KERSCHEN, E. 1988 A comparison of boundary-layer receptivity mechanisms. *AIAA Paper* 88-3758.

HEINRICH, R. and KERSCHEN, E. 1989 Leading-edge boundary layer receptivity to various free-stream disturbance structures.*Z.Angew.Math.Mech.*69,T596-598.

LAM, S. and ROTT, N. 1960 Theory of linearized time-dependent boundary layers. *Cornell Univ. Grad. School of Aero. Engineering Rep.* AFOSR TN-60-1100.

LAM, S. and ROTT, N. 1993 Eigen-functions of linearized unsteady boundary layer equations. *ASME I: J. Fluids Engrg* 115, 597-602.

LIN, N., REED, H. and SARIC, W. 1992 Effect of leading edge geometry on boundary-layer receptivity to freestream sound. *Stability, Transition, Turbulence,* eds. M. Hussaini, A. Kumar, C. Streett, Springer Verlag, NY.

MITTAL, R. and BALACHANDAR, S. 1994 Direct numerical simulation of flow past elliptic cylinders. *Submitted to Journal of Computational Physics.*

SARIC, W. 1994 Physical description of boundary layer transition: experimental evidence. *Special Course on Progress in Transition Modeling. AGARD Rep. 793.*

SARIC, W., REED, H. and KERSCHEN, E. 1994 Leading-edge receptivity to sound: experiments, DNS, theory. *Invited Paper, AIAA-94-2222,* Colorado Springs.

SARIC, W. and WHITE, E. 1998 Influence of high-amplitude noise on boundary-layer transition to turbulence. *AIAA-98-2645.*

SCHNEIDER, G. and ZEDAN, M. 1981 A modified strongly implicit procedure for the numerical solution of field problems. *Numerical Heat Transfer* 4, 1-19.