13th Australasian Fluid Mechanics Conference
Monash University, Melbourne, Australia
13-18 December 1998

DIRECT NUMERICAL SIMULATION OF MIXING IN ISOTROPIC TURBULENCE WITH SCALAR INJECTION

Marc ELMO, Jean Pierre BERTOGLIO and Vladimir SABELNIKOV(*)

Laboratoire de Mecanique des Fluides et d' Acoustique UMR 5509
Ecole Centrale de Lyon, 36 Av. Guy de Collongue, 69130 Ecully, FRANCE
(*)TSAGI, Zhukovsky, Moscow Region 140160, RUSSIA

ABSTRACT

The mixing of a passive scalar is investigated in a DNS of homogeneous isotropic turbulence. A new technique to force the scalar field is developed. This technique permits to obtain statistically steady states associated with different levels of mixing. The probability density function of the scalar and the conditional scalar dissipation are studied. Both quantities are found to strongly depend on the level of mixing. It is also shown that the ratio between the integral scalar length scale and the integral velocity length scale affects the shape of the probability density function and the conditional scalar dissipation.

INTRODUCTION

The probability density function (p.d.f) approach of turbulent mixing of a scalar is of high interest since it allows to take into account a chemical reaction in a closed form. For several practical problems, such as combustion, it is important to understand situations in which the mixing is not complete. In particular there is a need of information about the conditional scalar dissipation, $< \varepsilon_c/c = \Gamma >$ previously investigated by ESWARAN & POPE (1988), to make a practical use of the p.d.f method, since it is the unclosed term in the p.d.f equation

$$\frac{\partial}{\partial t}P(\Gamma,t) = -\frac{\partial^2}{\partial \Gamma^2}[< \varepsilon_c/c = \Gamma > P(\Gamma,t)]$$

Another important issue is that the statistical properties of the scalar seem to be strongly dependent on the way the fluctuations are "injected" in the fluid (JABERI et al. 1996) or on the way they are produced by the fluid motion in the case of the presence of a mean scalar gradient (OVERHOLT & POPE 1996, JAYESH & WARHAFT 1992). One can also suspect that the ratio between the scalar and velocity integral length scales may play a major role in the form adopted by the p.d.f . These considerations had led us to develop a new technique to force a scalar field in a DNS of isotropic turbulence. This technique has the advantage of providing a way to control the integral length scale ratio and to generate statistically steady states whose scalar p.d.f can continuously be varied from nearly bi-modal ditributions associated with a high level of unmixing to roughly Gaussian distributions corresponding to fully mixed situations.

The results obtained with this forcing technique applied to incompressible isotropic turbulence at a Reynolds number (based on the integral length scale) equal to 90 and a Schmidt number equal to 1 are presented in this paper.

NUMERICAL METHOD-INJECTION TECHNIQUE

The Navier Stokes equation and the convection diffusion equation for the scalar c are integrated using a pseudo-spectral method. These equations are solved in a three dimensional cubic domain of size L with periodic boundary conditions on the velocity and scalar fields. The time stepping scheme is a second-order Runge Kutta method and the simulations are performed at resolution of 128^3 grid points. A random forcing is applied in the low-wave number range of the spectrum in order to maintain a statistically stationary velocity field. This stochastic forcing can be alternatively white-noise or time correlated (generated using a Langevin equation).

The scalar forcing technique consists in refreshing the field by operating an injection of fluctuations in physical space. This operation is repeated periodically in time, with a period T_i. The basic outline of the forcing can be described as follows. In the computational domain (of size L), n subboxes of size l are randomly selected. In one half of these subboxes (n/2), $c(x) = +1$ is imposed, whereas $c(x) = -1$ is imposed on the other half. This procedure essentially leads to a forcing function whose p.d.f is bi-modal. The choice of l/L allows to keep control upon the integral scalar length scale L_c and correlatively on the ratio $R_l = \frac{L_c}{L_u}$ where L_u is the velocity integral length scale. L_u and L_c are respectively defined by:

$$L_u = \frac{3\pi}{4}\frac{\int_0^\infty \frac{E_u(k)}{k}dk}{\int_0^\infty E_u(k)dk}$$

$$L_c = \frac{\pi}{2}\frac{\int_0^\infty \frac{E_c(k)}{k}dk}{\int_0^\infty E_c(k)dk}$$

where $E_u(k)$ and $E_c(k)$ are respectively the velocity and scalar spectra. A characteristic time of injection is $T_{res} = T_i * (\frac{V_t}{nv_f})$, where V_t is the volume of the computational domain and v_f is the volume of a forced subbox. The choice of the time scale ratio $R_t = T_{res}/T_{turb}$ (T_{turb} being the eddy turnover time of the turbulent field) governs the level of mixing.

RESULTS

Fig.1 shows the time evolution of the scalar r.m.s value $\sigma = \sqrt{< c^2 >}$ for two time scale ratios $R_t = 0.35$ and $R_t = 2.2$ (R_l being fixed to 0.71). It is seen that a statistically stationary state is reached after a transient behaviour observed on a time of order $R_t * T_{turb}$.

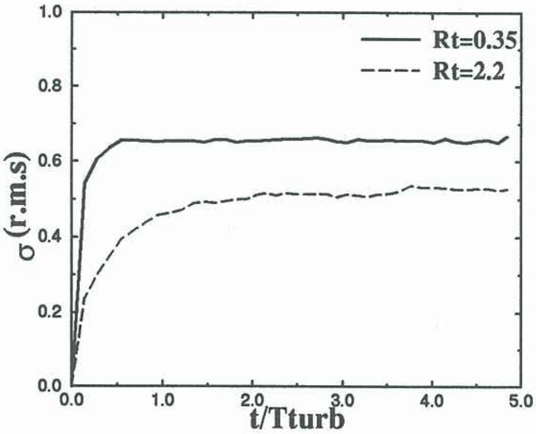

Figure 1: Time evolution of the scalar r.m.s value for two time ratios, $R_t = 0.35$ and $R_t = 2.2$ ($R_l = 0.71$).

A larger r.m.s value is obtained for $R_t = 0.35$ which can be explained by the fact that more unmixed scalar fluctuations are injected in this case. Fig.2 shows the time evolution of the scalar r.m.s for a given time scale ratio ($R_t = 2.2$) and two length scale ratios $R_l = 0.71$ and $R_l = 0.47$. It can be observed that the stationary level of σ is smaller for $R_l = 0.47$ than for $R_l = 0.71$, indicating that the mixing is improved when the scalar is injected at smaller scales. This result is consistent with the study of WARHAFT & LUMLEY 1978 who have shown that the length scale ratio affects the exponent of the power law of a decaying scalar.

The statistical results presented below are for stationary states. They are averaged in time to improve the statistical sampling. In Fig.3, the spectrum of the scalar fluctuation $E_c(K)$ is plotted for $R_l = 0.71$ and $R_l = 0.47$ ($R_t = 2.2$). It can be observed that a $K^{-\frac{5}{3}}$ range is present for $R_l = 0.71$. This "scaling" is in agreement with the analysis of CORRSIN 1951 but differs from the K^{-1} behaviour observed by METAIS & LESIEUR (1992) in a large eddy simulation of isotropic turbulence.

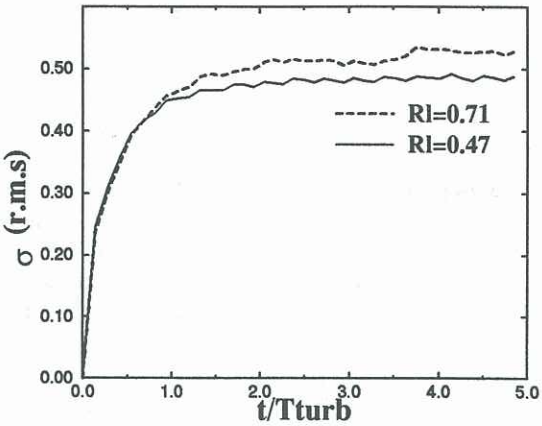

Figure 2: Time evolution of the scalar r.m.s value for two scale ratios, $R_l = 0.71$ and $R_l = 0.47$ ($R_t = 2.2$).

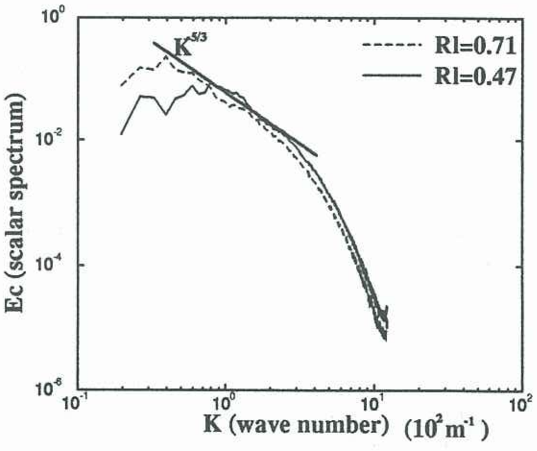

Figure 3: Scalar spectrum, $R_l = 0.71$ and $R_l = 0.47$ ($R_t = 2.2$).

Fig.4 shows the stationary p.d.f of the scalar for three time scale ratios, $R_t = 0.35$, $R_t = 1.1$ and $R_t = 2.2$. For $R_t = 0.35$ two peaks are clearly observed close to $\pm\frac{\Gamma}{\sigma} = 1.4$, which correspond to the extremal fluctuations $c = \pm 1$. Such a behaviour is associated with a high level of unmixing. The peaks are much smaller when $R_t = 2.2$ and the p.d.f of small fluctuations is higher indicating a better mixed situation. The p.d.f for $R_t = 1.1$ is also shown, giving an example of an intermediate situation of mixing in which the peaks at $c = \pm 1$ are still pronounced but the p.d.f is nearly uniform. In Fig.5, the p.d.f for $R_t = 2.2$ is replotted in semi-logarithmic axis and compared to a Gaussian law. The agreement with a Gaussian distribution is good over a large range of scalar fluctuations ($-\sigma < c < +\sigma$).

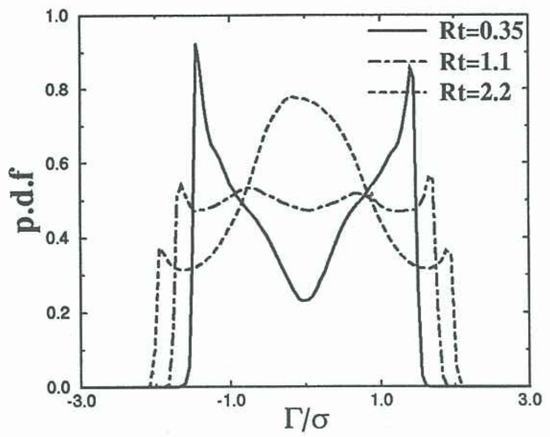

Figure 4: Probability density function of the scalar for three time ratios, $R_t = 0.35$, $R_t = 1.1$ and $R_t = 2.2$ ($R_l = 0.71$).

Fig.6 shows the conditional scalar dissipation $< \varepsilon_c/c = \Gamma > = < D\frac{\partial c}{\partial x_i}\frac{\partial c}{\partial x_i}/c = \Gamma >$ normalized by the mean dissipation rate $< \epsilon_c > = < D\frac{\partial c}{\partial x_i}\frac{\partial c}{\partial x_i} >$ where D is the molecular diffusion coefficient. It can be observed that the conditionnal scalar dissipation strongly departs from a uniform distribution for unmixed situations ($R_t = 0.35$ and $R_t = 1.1$) whereas in the case of the lowest forcing ($R_t = 2.2$) it tends to become roughly constant over a large plateau corresponding to small and moderate scalar fluctuations. It is interesting to point out that high values of the p.d.f of the scalar, corresponding to the effect of injection at $c = \pm 1$, are associated with low values of the conditional dissipation.This suggests the existence of blobs of nearly uniform scalar concentration in the field. These blobs are reminiscent of the injection subboxes. It is also interesting to notice the correspondence between the flat shape of the conditional scalar dissipation (for $R_t = 2.2$) and the Gaussian distribution of the scalar p.d.f already observed in Fig.5. This is in agreement with the theoretical result of SINAI & YAKHOT 1989 which shows that if the p.d.f is Gaussian then the conditional dissipation is uniform.

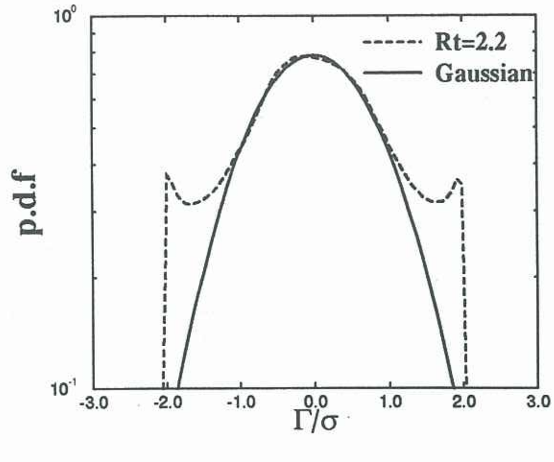

Figure5: Comparision of the probability density function with a Gaussian distribution, $R_t = 2.2$ and $R_l = 0.71$.

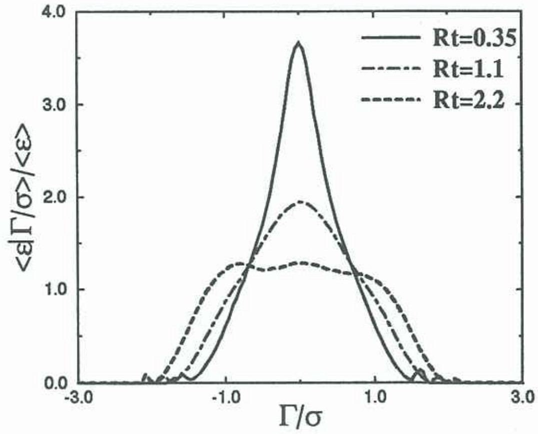

Figure 6: Normalized conditional scalar dissipation for three time ratios $R_t = 0.35$, $R_t = 1.1$ and $R_t = 2.2$ ($R_l = 0.71$).

Fig.7 shows the p.d.f of the scalar for two scale ratios, $R_l = 0.71$ and $R_l = 0.47$ ($R_t = 2.2$). It is seen that the two peaks close to $\pm\frac{2\Gamma}{\sigma}$ are lower when $R_l = 0.47$ indicating that the p.d.f is associated with a better mixed situation than for $R_l = 0.71$. This result is in agreement with the fact that the mixing is better when the fluctuations are injected at smaller scales observed previously for the r.ms value (Fig.2). In Fig.8, it can be observed that the conditional dissipation is also affected by the length scale ratio.

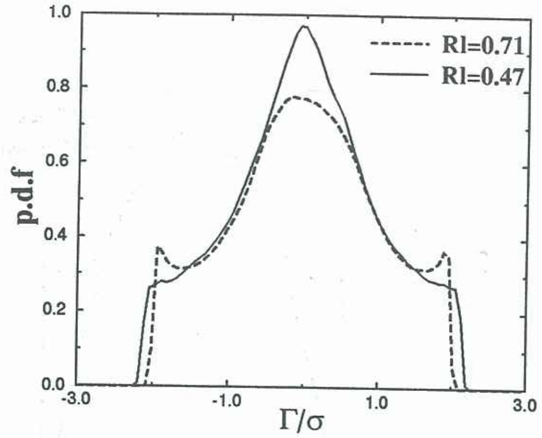

Figure 7: Probability density function of the scalar for two scale ratios, $R_l = 0.71$ and $R_l = 0.47$ ($R_t = 2.2$).

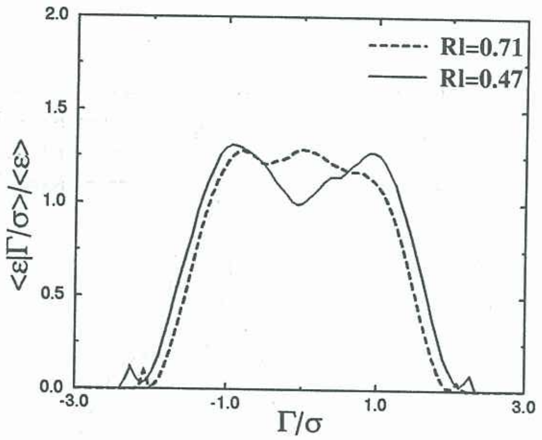

Figure 8: Normalized conditional scalar dissipation for two scale ratios, $R_l = 0.71$ and $R_l = 0.47$ ($R_t = 2.2$).

CONCLUSION

The new scalar injection technique proposed in the paper has been used to study turbulent mixing in situations where the mixing is not complete. Both the scalar p.d.f and the conditional dissipation were found to strongly depend on the level of mixing. The length scale at which the scalar fluctuations are injected in the turbulent field also appears to affect the statistical results.

More results are necessary before attempting to close the p.d.f equation by expressing the conditional dissipation. It is also important to notice that the influence of the Reynolds and Schmidt numbers should be investigated.

REFERENCES

ESWARAN, V. and POPE, S.B., "Direct numerical simulations of the turbulent mixing of a passive scalar", *Phys. Fluids,* **31**, 506-520, 1988.

JABERI, F.A., MILLER, R.S., MADNIA, C.K. and GIVI, P., "Non-Gaussian scalar statistics in homogeneous turbulence", *J. Fluid Mech.* **313**, 241-282, 1996.

OVERHOLT, M.R. and POPE, S.B., "Direct numerical simulation of a passive scalar with imposed mean gradient in isotropic turbulence", *Phys. Fluids,* **8**, 3128-3148, 1996.

JAYESH and WARHAFT, Z., "Probability distribution, conditional dissipation and transport of passive temperature fluctuations in grid-generated turbulence", *Phys. Fluids,* **A 4**, 2292-2307, 1992.

WARHAFT, Z. and LUMLEY, J.L., "An experimental study of the decay of temperature fluctuations in grid generated turbulence", *J. Fluid Mech.* **88**, 659-684, 1978.

CORRSIN, S., "On the Spectrum of Isotropic Temperature Fluctuations in an Isotropic Turbulence", *Journal of Applied Physics,* **22**, 460-473, 1951.

METAIS, O. and LESIEUR, M., "Spectral large-eddy simulation of isotropic and stably stratified turbulence", *J. Fluid Mech.* **239**, 157-194, 1992.

SINAI, Ya.G. and YAKHOT, V., "Limiting Probability Distributions of a Passive Scalar in a Random Velocity Field", *Phys. Rev. Letters,* **63**, 1962-1964, 1989.