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BARS FORMATION ON A SLOPING ERODIBLE BED : A LABORATORY STUDY

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ABSTRACT

The work presented here is a laboratory study on the formation of bars under the action of a partially-standing wavefield on an initially flat sloping erodible bed. It has been motivated by addressing the role of the different length scales of the wave envelope in the bars formation via the Bragg resonance. These experiments were conducted with weakly non linear surface waves. They demonstrate that the bars formation is controlled by the fundamental of the wave envelope, even if perturbated by a free wave.

INTRODUCTION

The investigation of dynamic interactions between surface waves and erodible beds is of practical importance in oceanography and in coastal engineering. Patches of shore-parallel bars are often observed off beaches with height of the order of 1m and with spacing in the range of 10 to 100 m. When a partially-standing monochromatic wavefield is caused by reflection from a beach, a possible explaination is that the formation of these bars is the result of spatial non-uniformities in the net transport of sediment (Carter *et al.*, 1972). The formation of such bars for constant mean water depths has been extensively investigated in both the field and the laboratory (see Rey et al, 1995 for a detailed bibliography). Bars formed in this way are then expected to have spacing equal approximately to half the surface wavelength leading to resonant or Bragg reflexion of the incident waves.

Here the investigations have been extended by addressing the case of an initially sloping bed, i.e. when the wavelength changes with the depth. The experiments for various wave heights, including non-linear and beach reflection effects were carried out in a small wave tank.

EXPERIMENTAL TECHNIQUES

The experiments were conducted in a glass-walled wave tank 4.7 m long and 0.39 m wide with a maximum water depth of 0.15 m. At one end of the tank a paddle wave maker produced the gravity wave which was either reflected or absorbed by a sloping beach located at the other end (see figure 1). The initial non-cohesive sand bed profile was a sloping plane extending over a length of typically 200 cm, from a water depth up-wave h = 6 cm, to a water depth down-wave h = 4 cm.

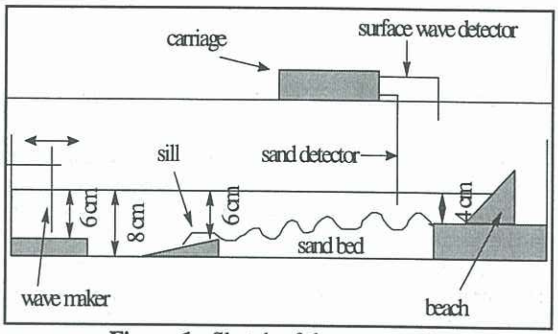

Figure 1 : Sketch of the wave tank.

In the present experiments, the generated wave frequencies and heights were respectively in the range 1 to 2 Hz and 1 to 2 cm. Bed erosions where carried out for f_0=1.5 Hz. A free wave having the frequency $2f_0$ was observed beating with the first harmonic, due to the paddle system and the rapidly varying water depth (Madsen, 1971 ; Hulsbergen, 1974). Depending upon its shape, a sill of sand extending over 20 cm up-wave of the sand bed allowed to reduce the free wave amplitude generated by the wave board.

The reflecting beach or the absorbing beach consisted in a plane board of reflection coefficient depending upon its length and slope. The weaker reflection coefficient available was less than 5% in the range of wave frequencies and heights of interest.

The water surface displacement due to waves and the bed profile measurements were performed by using ultra sonic sensors mounted on a carriage which could be translated along the center line of the tank by a stepping motor ensuring a great reproducibility of the measuring position.

NUMERICAL MODEL

The numerical model is based on the linearized potential theory of gravity waves (see for details Rey, 1992). The bed was considered to be fixed, one-dimensional, impermeable and discretized into N steps of constant water. For each region of constant depth, the general expression of the velocity potential is the sum of two propagative modes and an infinity of non propagative or evanescent modes, truncated at an order P for the numerical solution. The integral matching along vertical boundaries between successive rectangular regions for both fluid velocity and pressure gives the 2N(P+1) linear equations for the expression of the potential for the whole domain. The velocity potential and the reflection and transmission coefficients are then know.

The physical limitations of the computations arise both from approximations of the use of the linear potential theory of gravity waves with no absorbtion, and from the truncation of the evanesccent modes in the model.

RESULTS

The experiments focussed on the dependance of the bar formation upon two parameters : the beach reflection and the rate of free harmonic wave. Wave heigths were chosen just above the threshold conditions for sediment motion (wave height of 1.2 cm for f=1.5 Hz, R=0.16 and h=5 cm).

Strong Beach Reflection, Weak Rate of Free Harmonic

The initial reflection coefficient, due to the beach, was R = 0.315. The wave height was of 1.6 cm for h=6cm.
At short times, a formation of ripples was observed due to the local near-bed water oscillation under the waves (ripples length of the order of the fluid orbital amplitude). The long time evolution is presented on figure 2 : a modulation with spacing of the order of half the local wavelength of the fundamental wave is observed under the partially-standing wave.

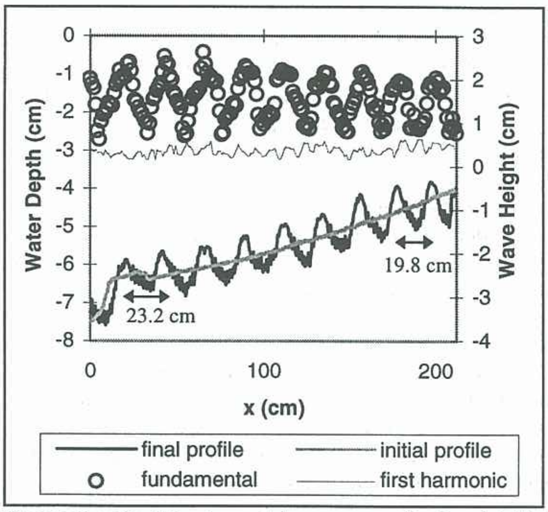

Figure 2 : Final fundamental wave over final and initial bathymetric profiles. H = 1.6 cm and R = 0.31 .

This long-term sediment transport process has been observed for constant water depth (see a. g. Davies et al, 1983 ; Brooke-Benjamin et al, 1987 ; O'Hare et al, 1992 ; Rey et al, 1995). Here, the distance between two consecutive nodes or antinodes of the surface wave increases smoothly with the water depth, what is also observed for the sand bars.
The figure 3 shows the fast Fourier transform (FFT) of the modulated part of the bottom, and that of an hypothetic pseudo-sinusoidal bottom of the same amplitude and local wavenumber of twice the wavenumber of the fundamental wave of frequency 1.5 Hz.

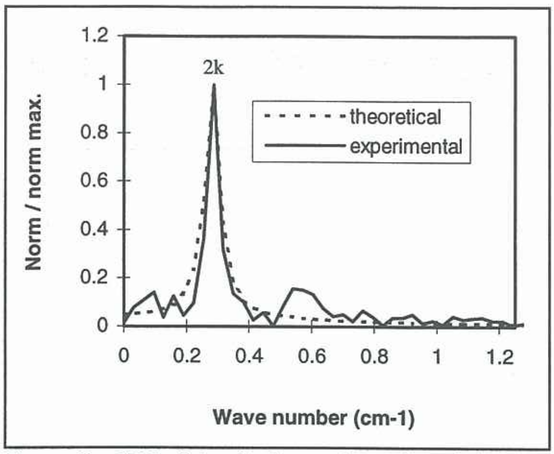

Figure 3 : FFT of the final experimental and theorical bathymetric profiles.

The good agreement confirms that the bars formation process is mainly due to the modulated fundamental wave. Hence, the bottom spacing corresponds to the local resonant Bragg condition, at the origin of strong reflection in addition to those due to the beach. This is confirmed by a final reflection coefficient of order of 50%. The analysis of the bottom reflection versus the frequency measured for low wave amplitude with an absorbing beach, confirms a maximum of reflection when the Bragg condition is satisfied. As shown on figure 4, the results are in good agreement with those of a numerical model based on the full potential theory (Rey, 1992).

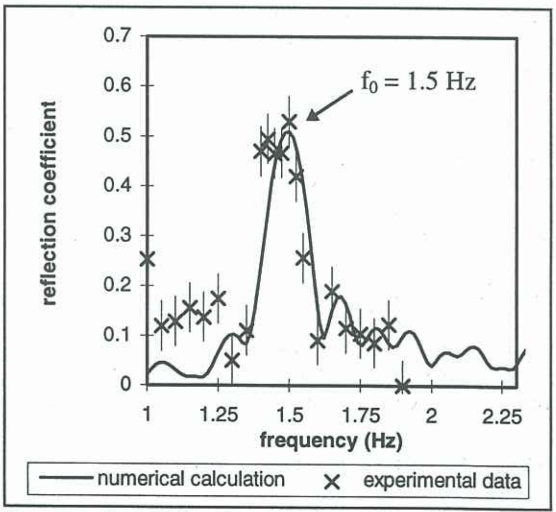

Figure 4: Reflection study with the purely monochromatic wave.

Strong Beach Reflection, Strong Rate of Free Harmonic

The initial reflection coefficient and wave height were respectively of 0.425 and 1.9 cm. A free wave component was superimposed on the Stokes'wave, and a beating observed for the first harmonic components of the surface wave (see figure 5). Indeed, if k and K are the wavenumbers for respectively 1.5 Hz and 3.0 Hz, the beating length for h ≈ 5 cm is $\lambda_{beating} = 2\pi/(K-2k) \approx 70$ cm (see figure 5).

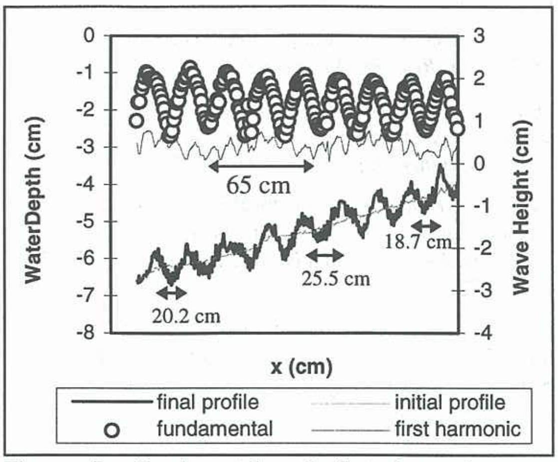

Figure 5: Fundamental and first harmonic wave components over initial and final profiles (presence of the free wave).

On figure 5, we can observe that the bars formation is altered by the non-linear wave-wave interaction, and that bars spacing no more regularly increases with the depth, like the envelope of the fundamental. The length characteristics of the modulated bed are more evidenced by the FFT on figure 6. Indeed, not only the peak corresponding to the Bragg resonance discussed in the paragraph III.1 is observed, but also a peak at higher wavenumber, of order of $(16.5\text{ cm})^{-1}$, which could be related to the 2^{nd} order Bragg resonance for the free wave (wavelength = 16.5 cm for h = 5 cm).

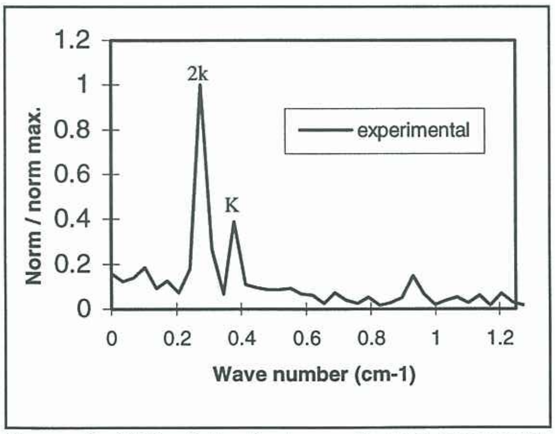

Figure 6: FFT of the final experimental bathymetric profile (presence of the free wave).

The analysis of the reflection versus the frequency wave over the modulated bottom for an absorbing beach, and its comparison with numerical calculations confirms this second peak (figure 7), with a maximum of reflection corresponding to the bottom wavelength 16.5 cm, which appears here as a first Bragg resonance for waves of frequency ≈ 1.83 Hz (wavelength of order 32.4 cm).

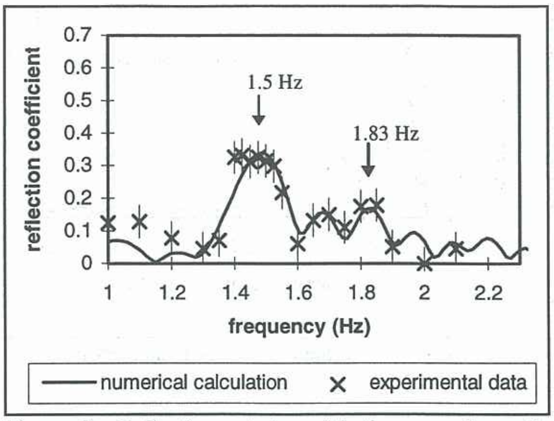

Figure 7: Reflection study with the monochromatic wave polluted by a free wave.

Weak Beach Reflection, Strong Rate of Free Harmonic

The initial reflection coefficient and wave height were respectively of 0.10 and 1.65 cm. The figure 8 reveals a slow oscillation in the long-term bottom shape, corresponding to the modulation of the first harmonic components of the wave.

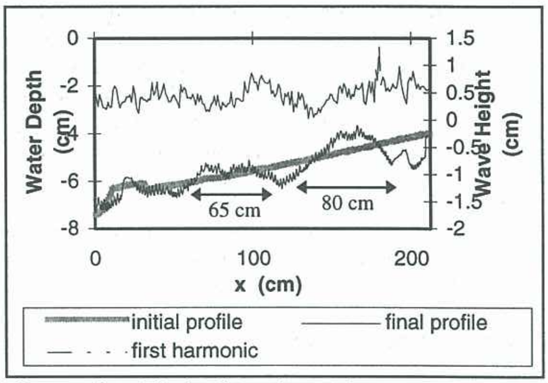

Figure 8: Final first harmonic component and bathymetric profiles. H = 1.65cm and R = 0.1 .

The Bragg quasi-periodic modulation is no more observed for this weakly partially-standing wave. The processes leading to the final bars are then completely governed by the non-linear wave behaviour.

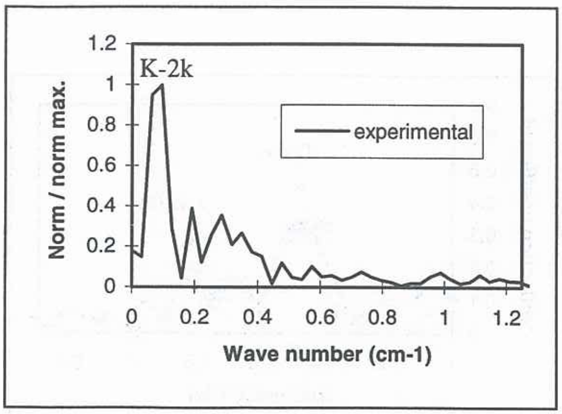

Figure 9 : FFT of the final experimental profile.

CONCLUSION

Each of the experiments presented were repeated and reproducible results concerning the bars formation were obtained.

These results confirm that the quasi-periodic bars, formed under the action of partially standing waves, can be explained by the modulation in the fluid motion predicted by simple linear wave theories. These bars will enhance the beach reflection through Bragg resonance. For coastal engineering purposes, many submerged obstacles taking advantage of such reflecting power were proposed for shore protections.

The larger scale bottom oscillations observed for quasi-progressive waves are related to a beating between the two first non-linear components of the wave. Long scale bars are commonly observed in the field where they are explained by non-linear wave changes during shoaling. They are of interest for beach profile predictions, and for long-term sediment transport behaviour-oriented models calibrations.

From a fundamental point of view, these results may be seen as an example of non linear wave behaviour with an evolutive bottom as boundary condition.

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