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An Improved Theoretical Model for the Resistance of a Vessel with a Transom Stern

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ABSTRACT

In this paper is presented a more sophisticated model of the water flow in the zone near the transom stern of a vessel travelling at a constant speed. The model takes into account the drop in the water level behind the stern at low speed, so that a practical and accurate prediction of the resistance can now be computed over the complete speed range.

INTRODUCTION

Background

Previous work on the subject of prediction of resistance of marine vehicles, such as monohulls and catamarans, has shown that the *trends* in the curve of total resistance with respect to speed can be predicted with excellent accuracy, using the traditional Michell (1898) wave-resistance theory. One also requires a suitable formulation for the component of frictional resistance. There have been further enhancements to this theory, namely the inclusion of the influences of finite depth and finite width of the canal by Lunde (1951) and Sretensky (1936). These theoretical effects, also, properly reflect the experimental evidence.

The methodology being promoted here is to utilize the traditional linear-wave-resistance theory in conjunction with correction factors which are obtained experimentally. It has been shown that the linear theory predicts the impact of changes to the hull geometry with a very high degree of accuracy. This point was demonstrated by Doctors and Renilson (1992), in which water-depth effects were the point of emphasis. Finally, the importance of hull-form parameters, such as prismatic coefficient, was the subject of work by Doctors (1995a and 1995b). In all these cases, extremely high levels of agreement between the theoretical predictions and the experimental data was obtained, provided the theory was "anchored" at one or two reference positions in the data.

These principles were advanced on two fronts in the research recently presented by Doctors and Day (1997). Firstly, transom-stern effects were included in the theory by accounting for the hollow in the water behind the vessel. This work was essentially a development of the ingenious and practical approach first presented by Molland, Wellicome, and Couser (1994). Also importantly in the present context, it was proven that by using only two experimentally determined factors, it was possible to obtain a very high level of correlation between the predictions and the experimental data for a large set of conditions. These were in terms of displacement, trim, and speed of the towing-tank catamaran model. These two factors were the (traditional) frictional-resistance form factor and a (new) wave-resistance form factor.

Current Work

The computations also show that the predictions for resistance are somewhat too high at very low speeds. This problem has been related back to the transom-stern model, which assumes that the transom is fully dry at all speeds. Clearly, this approximation is not true in the low-speed régime.

In the current work, the approach is to be taken another step forward by employing a better model for the water flow in the zone near the transom stern. Potential-flow theory is used to estimate the suction created at the bottom of the transom stern and hence the approximate drop in the water level. In this way, the forward hydrostatic force acting on the transom will be computed far more accurately in the low-Froude-number régime and the abovementioned overprediction of the resistance can be corrected.

Theory

Definition of the Problem

Figure 1(a) shows the mesh defining a typical hull. A second meshing, consisting of "pyramids" or "tents" with a rectangular base, is employed for the purpose of the numerical calculation of the wave resistance.

We consider that the ship model of length L, beam B, and draft T, is operated in a towing tank, with a width H and a depth d. The model is towed at a constant speed U. The x, y, z coordinate system is also shown in Figure 1(a).

Resistance of the Vessel

We will utilize the previously mentioned linearized (thin-ship) theory, as described by means of the tent

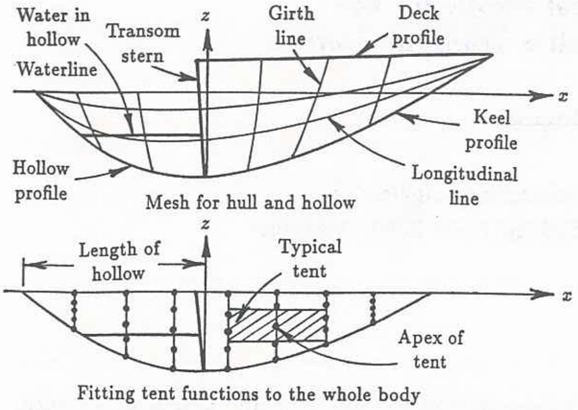

Figure 1: Definition of the Problem
(a) Fitting Mesh to the Vessel

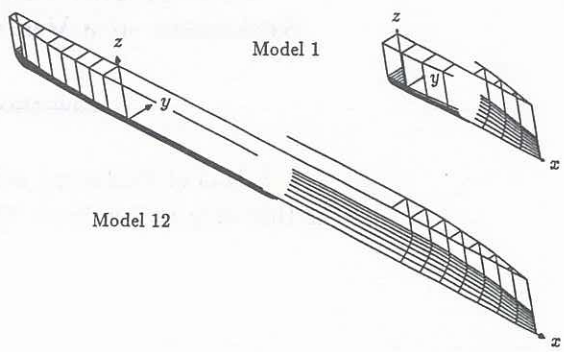

Figure 1: Definition of the Problem
(b) Pictorial Views of Two Models

functions. The relevant equations were provided by Day and Doctors (1996).

The total resistance of the vessel is computed on the basis of the sum of the components of resistance, as follows:

$$R_T = f_W R_W(f_g, f_L) + R_H + f_F R_F + + R_A + R_{\text{aero.}} . \quad (1)$$

Here, a wave-resistance form factor f_W has been applied to the computed linearized wave resistance, an idea which is analogous to that behind the traditional (frictional-resistance) form factor. The second term in Equation (1) is the hydrostatic resistance. The third term is the frictional resistance R_F, estimated using the 1957 International Towing Tank Committee (ITTC) formula. The usual frictional resistance form factor f_F has been used here. Finally, the fourth term is the correlation allowance R_A, which is zero for a model. We will also neglect the fifth term, this being the aerodynamic resistance $R_{\text{aero.}}$.

Model of Flow behind the Transom Stern

The effective extension to the vessel due to the transom stern was provided in detail by Doctors and Day (1997). In the current work, we introduce an additional element of sophistication to the model, by estimating the suction created at the bottom of the transom, using the formulas:

$$\boldsymbol{u} = \int_{x_1}^{x_2} \frac{\sigma(\boldsymbol{x}')[\boldsymbol{x} - \boldsymbol{x}']}{4\pi r^3} \, \mathrm{d}x' , \quad (2)$$

$$\sigma = -2U \frac{\mathrm{d}A}{\mathrm{d}x} , \quad (3)$$

$$r = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2} , \quad (4)$$

$$\boldsymbol{q} = -U\boldsymbol{i} + \boldsymbol{u} , \quad (5)$$

$$\zeta_{\text{holl.}} = (U^2 - q^2)/2g . \quad (6)$$

Here, $\boldsymbol{u}$ is the perturbation velocity induced by the slender-body source distribution $\sigma(\boldsymbol{x})$ along the axis of the ship plus the hollow on the free surface, $\boldsymbol{x}$ is the coordinate at the bottom of the transom, A is the local cross section of the vessel or hollow, r is the radial distance, q is the velocity at the bottom of the transom, $\zeta_{\text{holl.}}$ is the elevation of the free-surface behind the transom, and g is the acceleration due to gravity.

RESULTS

Modified Wigley-Hull Series

The ship models were constructed from up to seven segments. The bow and stern segments have parabolic waterplanes. The bow segments, stern segments, and the parallel middle-body segments all possess parabolic cross sections. Figure 1(b) shows pictorial views of two of the test models. Table 1 lists the details of these so-called Lego series. Each model had a beam B of 0.150 m and a draft T of 0.09375 m.

Original Thin-Ship Theory

We start by illustrating the effectiveness of the straightforward Michell theory together with the ITTC estimate of the frictional resistance, with form factors f_W and f_F of unity, indicated by Func. = 0 in Figure 2.

Model 2 represents a relatively extreme case in terms of the prismatic coefficient C_P and the slenderness coefficient $L/\nabla^{1/3}$. Here, L is the vessel length and ∇ is its displaced volume. The graphs show a plot of the specific resistance R/W against the Froude number $F = U/\sqrt{gL}$.

The agreement is qualitatively reasonable, in Figure 2(a), except at low speeds, where the hydrostatic drag due to the hollow is clearly grossly estimated, since the transom is assumed to be dry. There is a marked improvement in the accuracy in Figure 2(b),

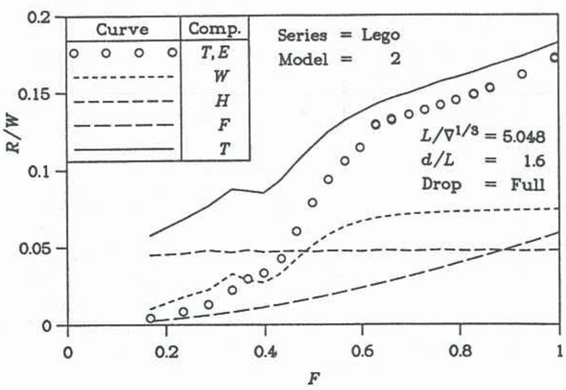

Figure 2: Components of the Resistance (a) Full Drop in Transom Hollow

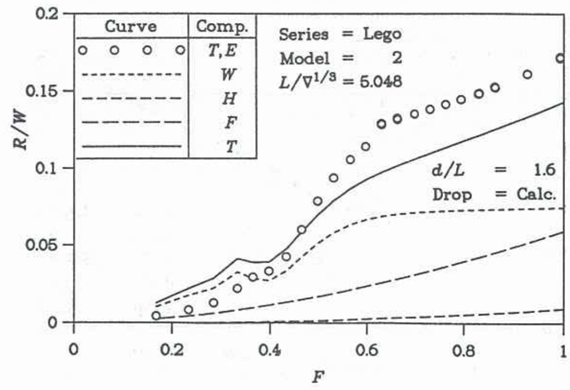

Figure 2: Components of the Resistance (b) Calculated Drop in Transom Hollow

Ship Model	Segments	Length L (metres)	Prismatic Coefficient C_P
1	1	0.7500	0.6666
2	15	0.9375	0.7290
3	156	1.1250	0.7499
4	1567	1.3125	0.7290
5	12	1.5000	0.8332
6	125	1.6875	0.8494
7	1256	1.8750	0.8499
8	12567	2.0625	0.8275
9	1234	2.2500	0.8888
10	12345	2.4375	0.8957
11	123456	2.6250	0.8928
12	1234567	2.8125	0.8735

Table 1: The Twelve Ship Models

Function Code Func.	Root-Mean-Square Error in Total Specific Resistance e_{RMS}	
	Drop = Full	Drop = Calc.
0	1.748×10^{-2}	2.328×10^{-2}
1	1.070×10^{-2}	5.482×10^{-3}
3	8.823×10^{-3}	4.929×10^{-3}

Table 2: Summary of Results

where the drop in the water level has been calculated by the new method.

Use of Form Factors

We now examine the greatly improved accuracy that can be obtained by using appropriate form factors. The results from Figure 2 are now re-calculated and presented in Figure 3. The form-factor function Func. = 1 implies the use of a constant form factor, while Func. = 3 implies a form factor which is a linear function of the beam-to-length ratio B/L (that is, it can be expressed as $k_1 + k_2(B/L)$).

The two parts of Figure 3 demonstrate the advantage of a judicious choice of form factors.

Comparison of Models for the Hollow

We now present Figure 4 which compares the new sophisticated low-speed model for the flow in the transom-stern zone with the previous theory, for two different models. The current theory clearly provides greater accuracy in these cases.

Finally, Table 2 shows a complete set of test cases for the complete set of twelve Lego models. The first case with Func. = 0 is, of course, the original theory. The other two cases show successive improvements in terms of the root-mean-square error in the values of R/W, showing a factor of reduction in the error to about 0.88% using the simple transom-flow model and a reduction in the error to about 0.49% using the present more sophisticated transom-flow model.

CONCLUSIONS

The research presented here demonstrates the very worthwhile improvement that can be achieved by calculating the estimated drop in the water level in the transom stern — as opposed to assuming that the transom is completely dry.

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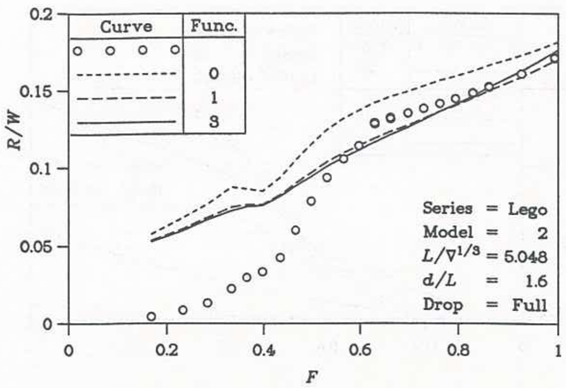

Figure 3: Use of Form Factors
(a) Full Drop in Transom Hollow

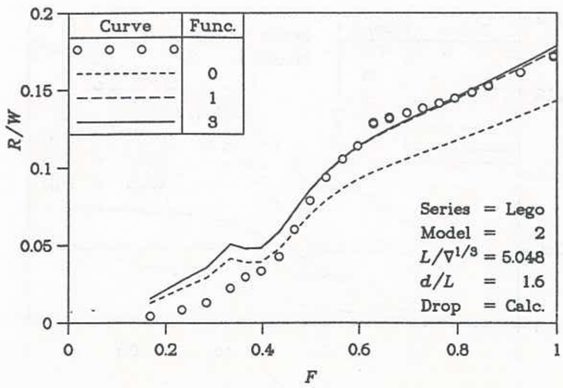

Figure 3: Use of Form Factors
(b) Calculated Drop in Transom Hollow

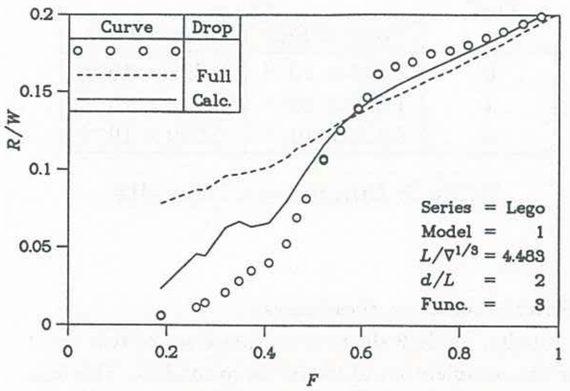

Figure 4: Effect of Transom-Stern Models
(a) Lego Model 1

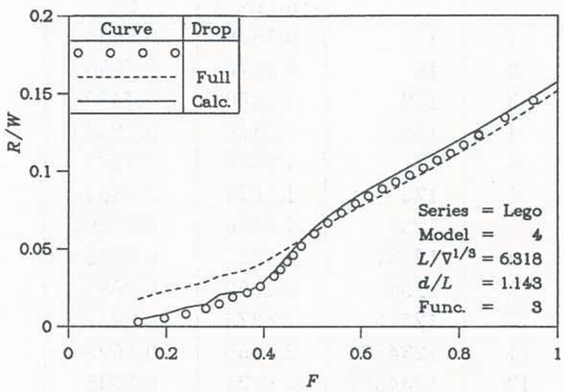

Figure 4: Effect of Transom-Stern Models
(b) Lego Model 4

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