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THE BOUNDARY-LAYER FLOW OF A MICROPOLAR FLUID OVER A STRETCHING POROUS SHEET

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ABSTRACT

We consider the boundary-layer flow of a micropolar fluid over a porous stretching sheet. The results obtained from a perturbation technique are presented. These results, other numerical solutions obtained with a quasilinearization scheme and some previous data are in very good agreement even in the case of moderate values of the material coefficient and Eckert numbers.

INTRODUCTION

Eringen [1] first derived the constitutive equations for fluids with micro-structures. This theory can be used to explain the flow of colloidal fluids, liquid crystals, animal blood, etc. This is discussed by various authors (eg. [2]). Within the boundary layer approximations, we consider the governing equations of a steady, incompressible micropolar fluid over a continuous porous sheet. Continuous surfaces introduced by Sakiadis [3] are surfaces such as polymer sheets or filaments continuously drawn from a dye. Numerical data have been tabulated for various Prandtl numbers by Hassanien and Gorla [4]. The case of a continuously moving plate has been studied by Soundalgekar and Takhar [5]. More recently, Hady [6] has presented a solution using the method of successive approximations. In the present work, we complete the previous works outlined in references [4] and [6]. We propose to analyse the effects of the coupling constant between the physical parameters of the fluid, the normal velocity at the porous boundary and the Eckert number on velocity and temperature profiles.

MATHEMATICAL ANALYSIS

Governing equations

Within the framework of the boundary-layer theory, the governing equations may be written as follows :-

$$\frac{\partial \hat{u}}{\partial \hat{x}} + \frac{\partial \hat{v}}{\partial \hat{y}} = 0$$

$$\hat{u}.\frac{\partial \hat{u}}{\partial \hat{x}} + \hat{v}.\frac{\partial \hat{u}}{\partial \hat{y}} = \nu.\frac{\partial^2 \hat{u}}{\partial \hat{y}^2} + \frac{k}{\rho}.\frac{\partial \hat{N}}{\partial \hat{y}}$$

$$G.\frac{\partial^2 \hat{N}}{\partial \hat{y}^2} - 2.\hat{N} - \frac{\partial \hat{u}}{\partial \hat{y}} = 0$$

$$\hat{u}.\frac{\partial \hat{T}}{\partial \hat{x}} + \hat{v}.\frac{\partial \hat{T}}{\partial \hat{y}} = \frac{k}{\rho.c_p}.\frac{\partial^2 \hat{T}}{\partial \hat{y}^2} + \frac{\nu}{c_p}\left(\frac{\partial \hat{u}}{\partial \hat{y}}\right)^2$$

In the above equations, $\hat{u}$ and $\hat{v}$ are the dimensional velocity components in $\hat{x}$ and $\hat{y}$ directions, $\hat{N}$ and $\hat{T}$ represent the microrotation and the temperature, G the microrotation constant. The last term in energy equation must be taken into account when the temperature difference $\hat{T}_0 - \hat{T}_\infty$ is small.

We assume that the velocity of a point on the porous plate is proportional to its distance from the leading edge ; it will be supposed that the no-slip condition of viscous flow continues to apply at the surface of the sheet. Furthermore, velocity $\hat{V}_0$ normal to the sheet specifies the mass injection or withdrawal rate :-

$$\hat{y} = 0 \ : \ \hat{N} = 0 \ ; \ \hat{T} = \hat{T}_0$$

$$\hat{u} = c.\hat{x} = \hat{U}_w \ ; \ \hat{v} = \hat{V}_0$$

Far away from the boundary layer, conditions are :-

$\hat{y} \to \infty$: $\lim \hat{N} = 0$; $\lim \hat{T} = \hat{T}_\infty$
$\lim \hat{u} = 0$; $\lim \hat{v} = 0$

Similarity solution

For a shape-preserving boundary-layer profile, we stipulate that there exists a stream function ψ which depends on a similarity variable $\eta = \sqrt{c/\nu}.\hat{y}$. Using the next definitions :-

$$\psi = \sqrt{c\nu}.\hat{x}f(\eta) \; ; \; \hat{N} = \sqrt{c^3/\nu}.\hat{x}g(\eta) \; ;$$

$$\hat{T} = T_\infty + A.\hat{x}^2.\theta(\eta)$$

(a quadratic power law is chosen to obtain a constant for Eckert number - cf Schlichting [7, chap. XII]), the different equations are, respectively transformed into :-

$$f''' + f.f'' - f'^2 + \frac{k}{\rho.\nu}.g' = 0 \quad (1)$$

$$\frac{G.c}{\nu}.g'' = 2.g + f'' \quad (2)$$

$$\theta'' = Pr.\left\{\left(2.\theta f' - \theta' f\right) - Ec.\left(f''\right)^2\right\}$$

The transformed boundary conditions are

$$f(0) = \frac{-\hat{V}_0}{\sqrt{c.\nu}} = V \; ; \; f'(0) = 1$$

$$g(0) = 0 \; ; \; \lim_{\eta\to\infty} f' = 0 \; ; \; \lim_{\eta\to\infty} g = 0$$

$$\theta(0) = 1 \; ; \; \lim_{\eta\to\infty} \theta = 0$$

(a negative value for V signifies injection).

Perturbation method

According to Pipkin [8, page 145], for small values of $\varepsilon = k/(\rho.\nu)$, the similarity functions may be expanded into regular perturbation expansions (different from the series used by Gorla and Ameri [9]) :-

$$f = f_0 + \varepsilon.f_1 + \varepsilon^2.f_2 + \dots \quad (3)$$
$$g = g_0 + \varepsilon.g_1 + \varepsilon^2.g_2 + \dots$$

We substitute developments (3) into the boundary-layer equations to analyse the effect of the ratio of the material properties $k/(\rho.\nu)$. By collecting terms in equal powers of ε, we obtain :

$$\frac{G.c}{\nu}.g_k'' - 2.g_k = f_k'' \; ; \quad k \geq 0 \; (4)$$

The equation for the first order dimensionless stream function is :-

$$f_0''' + f_0.f_0'' - f_0'^2 = 0$$

For the next orders, equations are of the same type :-

$$f_k''' + f_0 f_k'' - 2.f_0' f_k' + f_k f_0'' = \dots \quad (5)$$

the right hand side depends on the order k. For the first two ones, we get :-

$$k = 1 : \quad -g_0'$$

$$k = 2 : \quad f_1'^2 - f_1.f_1'' - g_1'$$

These ordinary linear differential equations are subjected to the following conditions :-

$$f_0(0) = V \; , \; f_0'(0) = 1 \; , \; g_0(0) = 0$$

$$f_k(0) = f_k'(0) = g_k(0) = 0 \; ; \; k \geq 1$$

$$\lim_{\eta\to\infty} f_k' = 0 \; ; \; \lim_{\eta\to\infty} g_k = 0 \; ; \; k \geq 0$$

Analytical solutions

According to the previous conditions, a solution for f_0 is found in [10] :-

$$f_0 = \beta + exp(-\beta.\eta)/\beta \quad (6)$$

with $\beta = \left((V^2+4)^{1/2} + V\right)/2$

Using this first order solution, defining a new variable :-

$$\zeta = -\pi.exp(-\beta.\eta) \; ; \pi = -Pr.\beta^2$$

for the temperature, we get :-

$$\zeta.\theta^{\bullet\bullet} + \theta^{\bullet}.\left(1 - Pr - \zeta\right) + 2.\theta = -\zeta\frac{Ec}{\pi}\beta^2$$

An analytical solution is found :-

$$\theta = \frac{Pr.Ec}{4-2Pr}\left(\frac{\zeta}{-\pi}\right)^2 + \dots$$

$$\left(1 - \frac{Pr.Ec}{4-2Pr}\right)\frac{\Phi(Pr-2, Pr+1, \zeta)}{\Phi(Pr-2, Pr+1, -\pi)}\left(\frac{\zeta}{-\pi}\right)^2$$

It is a linear combination between a quadratic polynomial and a Kummer's hypergeometric function [11].

NUMERICAL PROCEDURE

In this section, we consider equations (4) and (5) which give the successive functions g_k (k≥0) and f_k (k≥1). All these equations are linear. So, we may construct a solution

$$S_k = \left[f, f', f'', g, g' \right]^t$$

with a linear combination (depending on a parameter ϖ) of two independent solutions which are :-

i : a particular solution S_p

ii : a solution for the homogeneous part :-

$$f_k''' + f_0 f_k'' - 2.f_0' f_k' + f_k f_0'' = 0$$

$$\frac{G.c}{\nu}.g_k'' - 2.g_k = 0$$

with initial conditions :-

$$S_h(0) = \left[0 , 0 , 1 , 0 , 1\right]^t$$

The combination $S = S_p + \varpi . S_h$ can be split up into two coefficients. These coefficients are calculated, in a single step, with equality deduced from conditions at infinity :-

$$f_p' + \omega_f . f_h' \Big|_{\eta=\eta_{max}} = 0$$

$$g_p + \omega_g . g_h \Big|_{\eta=\eta_{max}} = 0$$

Here η_{max} is "sufficiently large" to ensure that $exp(-\beta.\eta_{max})$, $f_0'(\eta_{max})$ and $g_0(\eta_{max})$ are less than a typical value (eg. 10^{-4}).

One restriction arises in this method. It is possible to obtain a poor estimation of f_0 using large rates of injection and the Runge-Kutta numerical integration scheme to evaluate $S_h(\eta)$. For example, V =-5.0 and $f_0'(\eta_{max})<10^{-4}$ need $\eta_{max}>48$. Then a non precise value of the last initial condition may imply an overflow error.

RESULTS

In figure 1, we analyse the influence of the different orders in the perturbation development and compare these results with a complete solution obtained with a quasilinearization method [12](subscript 'q'). We can see the rapid convergence of the development even in the case of a large value of ε (the numerical value used in [6] is 0.2)

A comparison between the first two approximations f_0' and $f_0' + \varepsilon.f_1'$ shows that an increase in $k/(\rho.\nu)$ leads to a decrease in the skin friction coefficient : $f_0''(0) = -0.618$, $f''(0)\Big|_{k=1} = -0.571$ and $f_q''(0) = -0.569$.

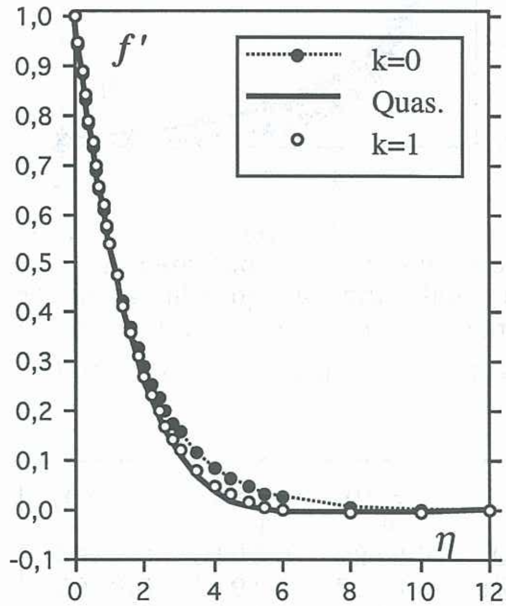

Figure 1 : Comparison between velocity profiles obtained with a perturbation method (black and white dots) and a quasiliearization scheme. Injection rate V=-1.0 ; $(G.c)/\nu = 2.0$; $k/(\rho.\nu)$=1.0

In practice, beginning with moderate injection rates, we have found that the analytical solution f_0'' may be used for description of heat transfer. Figure 2 represents the temperature field found for

Eckert numbers, analysing if the wall temperature is hotter or not than outside the boundary layer. The differences in the rates of heat transfer are negligible using f''_0 or a complete solution f''_q (cf Table 1). Eckert number effect is negligible upon the temperature profiles and the heat transfer.

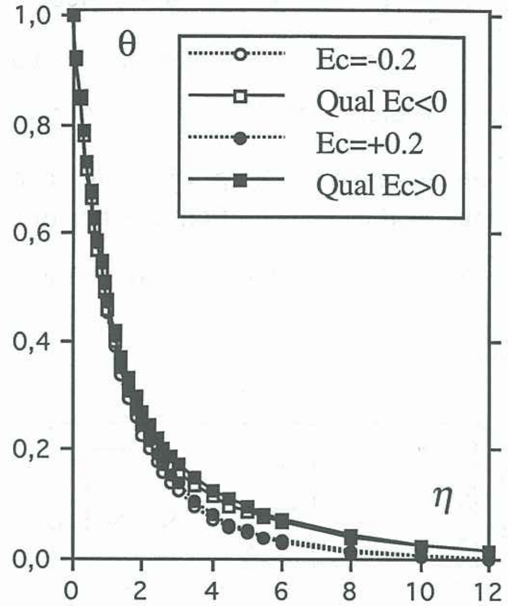

Figure 2 : Comparison between temperature profiles obtained with an analytical and a quasilinearization solutions for Eckert numbers $Ec = \pm 0.2$; V=-1.0 ; $(G.c)/\nu = 2.0$; $Pr = 0.7$

ε	$-f''_q(0)$	$g'_q(0)$	$-\theta'_q(0)$
0.0	0.6180	0.1910	0.8564
0.25	0.6062	0.1912	0.8565
0.50	0.5941	0.1915	0.8565
0.75	0.5817	0.1918	0.8563
1.0	0.5691	0.1923	0.8554

Table 1 : Initial values : Quasilinearization scheme : parameters V=-1.0 ; $Pr = 0.7$; $Ec = -\ 0.2$

CONCLUSION

The velocity and the micro-rotation distribution increase with increasing ε. An increase in ε leads to a decrease in the skin friction and the rate of heat transfer.

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