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THE VORTICITY OF A VORTEX RING CORE

John CATER[1], Julio SORIA[1] & T.T. LIM[2]

1. Department of Mechanical Engineering
Monash University, Victoria, AUSTRALIA

2. Department of Mechanical & Production Engineering
National University of Singapore, SINGAPORE

ABSTRACT

This paper presents high spatial resolution cross-correlation Particle Image Velocimetry measurements of the instantaneous in-plane velocity field of orifice generated vortex rings. Post–processing of this data has resulted in vorticity distributions with high spatial resolution. The vorticity and circulation of the core region of developed and developing laminar vortex rings has been investigated.

INTRODUCTION

The distribution of vorticity across the core of a vortex ring has been a matter of speculation for some considerable time. Several analytical solutions of the governing equations have been developed for special cases such as an inviscid ring (Lamb(1932)) or Hill's spherical vortex. Mathematical models are also available for the formation of rings (Saffman(1995)), however due to various simplifications, these do not adequately explain the observable behaviour of physical viscous rings at the later stages of their evolution.

Traditionally dye visualisation techniques have been used to gather information about the behaviour of vorticity. These methods are limited because a dye tracer fails to capture three important aspects of vorticity. These are stretching, diffusion and annihilation of vorticity by cancellation.

It is difficult to measure the velocity fields and resultant vorticity fields using single point measurement techniques such as Laser Doppler Velocimetry (LDV) due to the unsteady nature of the flow (Glezer & Coles(1990)). More recent studies using digital particle image velocimetry (DPIV), such as that by Fabris & Liepmann(1997), have hinted that the structure of a vortex ring is more complex than the simple models proposed over twenty five years ago (Saffman(1970)), but as yet a systematic study using multi-point techniques has not been completed.

In this investigation the core region of vortex rings has been studied. High spatial resolution measurements of velocity have been obtained using an adaptive cross-correlation digital particle image velocimetry (CCDPIV) technique (Soria(1996)).

EXPERIMENTAL APPARATUS

The vortex rings considered here were formed by discharging water from a circular tube of 50 mm inner diameter through an orifice of between 10 mm and 50 mm diameter into a transparent enclosed perspex tank of internal dimensions 1000 mm × 500 mm × 500 mm. The tank was filled with filtered water initially at rest. The fluid was discharged using a piston mechanism. The piston was driven by a computer controlled stepper motor with a directly coupled optical encoder. The piston velocity for the experiments could be varied along with the stroke length. The ratio of orifice diameter to stroke length is known as the Impulse Ratio (L_D). The final position of the piston face was also controllable. A schematic diagram of the apparatus is shown in Fig. 1.

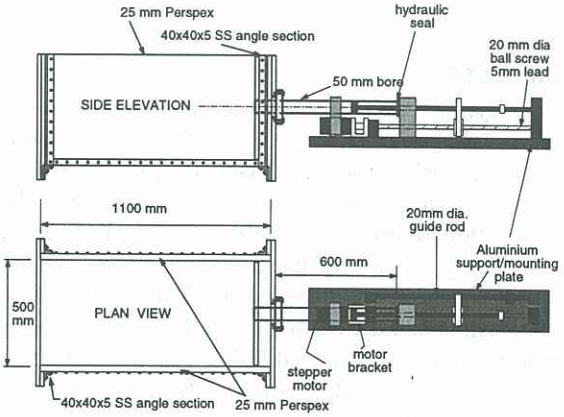

Figure 1: Vortex Ring Generator

The experiments were carried out in water seeded with 25 μm polyethylene spheroids with

a nominal S.G. of 0.9. The particles were illuminated with two co-planar laser sheets. The laser sheets were generated by two frequency doubled Nd:YAG lasers producing 400 mJ at 532 nm in 6 ns pulses.

Two systems of image acquisition were used. The first was conventional digital acquisition of images in pairs at 8 Hz with a pulse separation of 34 ms using a Kodak Megaplus CCD camera. The CCD array has a resolution of 1018 × 1018 pixels.

An image shifting technique that permits cross-correlation PIV of non-overlapping images recorded on photographic media has also been used for this analysis (Cater *et al.* (1998)). The technique uses half-frame image shifting (Soria *et al.* (1998)) and yields two independent images. The scanned resolution of the film is 100 px/mm, which results in two images of size 1750 × 2700 pixels for standard 35 mm film and 8500 × 9000 pixels for large format 5" × 4" film plates. The system also allows the capture of images with a small time delay (of the order of 1 ms). However the maximum rate of image acquisition of pairs is limited by the framing rate of the acquisition device.

During CCDPIV analysis only data with a normalised cross-correlation peak greater than 0.9 and a displacement less than $0.35 \times SWS$ (sampling window size) was accepted as a valid measurement. The resultant velocity fields have a high dynamic range and are used to calculate high resolution out-of-plane vorticity fields that allow the location and magnitude of peak regions of vorticity to be determined.

The Reynolds number was to characterise the flow was based on the orifice diameter and the mean piston velocity (U_0). ie.

$$Re_0 = \frac{U_0 D}{\nu} \qquad (1)$$

RESULTS

A typical CCDPIV velocity field of a laminar vortex ring is presented in Fig. 2. This data was acquired using the film-based image acquisition technique at a resolution of $88 \pm 1 \mu m/px$. The resolution of the film enables the entire vortex ring image to be captured in each image. An interrogation window spacing of 16 pixels was used. The orifice is located at $x = 0$. The vorticity distribution was calculated using the χ^2_{13} vorticity calculation method (Fouras & Soria(1998)) and is presented in Fig. 3.

An example of the vorticity distribution across the center of a laminar vortex ring is presented in Fig. 4. This data was acquired for half a vortex ring using the digital camera for a vortex ring

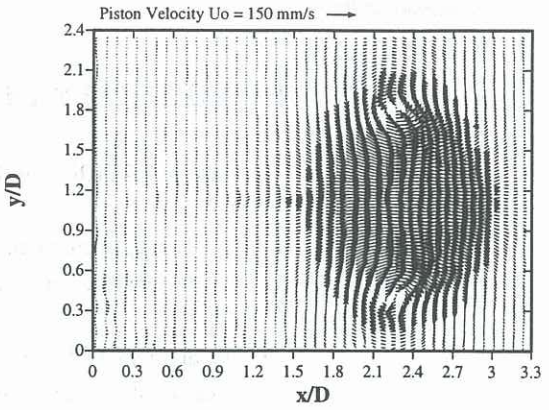

Figure 2: High resolution velocity field of a laminar ring using film acquisition, only every 2nd vector is displayed ($Re_0 = 7600$). Orifice at $x = 0$, centerline $y = 1.4$. SWS = 32 px with 50% overlap. Flow is from left to right.

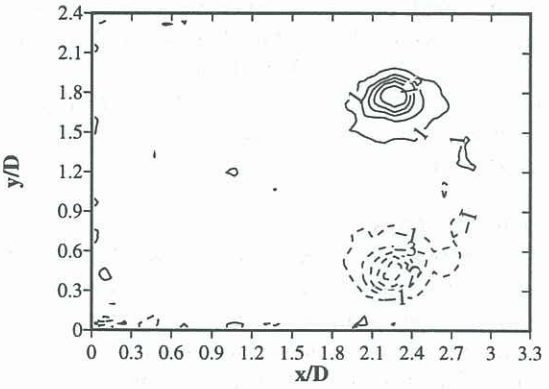

Figure 3: Normalised vorticity field of laminar ring calculated using χ^{13}_2 method ($Re_0 = 7600$). SWS = 32 px.

with Reynolds number of 933. The peak vorticity location was determined to be at $x_0/D = 0.87$ and $y_0/D = 0.72$ using the CCDPIV data analysed with a sampling window size (SWS) of 32 pixels and an overlap of 50%. The digital images were acquired with a scale of $77.7 \pm 1 \mu m/px$.

Figure 4 displays the normalised vorticity distribution in the stream-wise direction for half a laminar vortex ring, passing through the point of peak vorticity found to be located at $x_0/D = 0.87$ and $y_0/D = 0.72$. The vorticity was normalised by the peak vorticity in this case and the (x, y) coordinates have been appropriately shifted and then normalised with a length scale $L = 115.7$ pixels. The corresponding normalised Oseen vortex vorticity distribution is also plotted for comparison.

DISCUSSION

An immediate observation from the vorticity maps is that the core of the vortex ring flow is

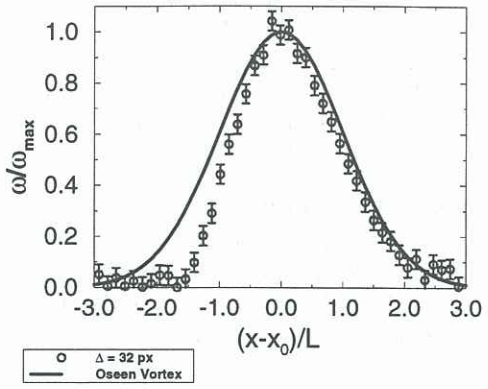

Figure 4: Normalised vorticity distribution in the stream-wise direction at peak vorticity location ($x_0/D = 0.87$ and $y_0/D = 0.72$) of a half of a laminar ring ($Re_0 = 933$) acquired using CCD array. The distributions for SWS = 32 px with 50% is shown. The Oseen vortex vorticity distribution with characteristic length $L = 115.7px$ is also shown for comparison.

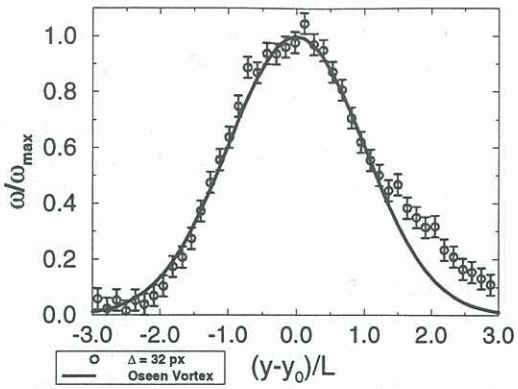

Figure 5: Normalised vorticity distribution in the radial direction at peak vorticity location for half of a laminar ring. ($x_0/D = 0.87$ and $y_0/D = 0.72$). $Re_0 = 933$, SWS = 32 px with 50% overlap. $L = 115.7$

not circular as assumed by some models, but is stretched in the vortex ring propagation direction and contracted normal to this direction. This elliptically shaped distribution has been previously noted by Maxworthy(1972).

However, other published DPIV measurements of circulation may have underestimated the peak vorticity by up to 50% (Soria & Fouras(1995)). This is due to low spatial resolution of the vortex core region of these experiments. The out-of-plane vorticity measurement accuracy is also dependent on the computational method used for its determination from measured in-plane velocity vector field data.

The circulation (Γ) around the peak vorticity location was computed using the velocity line integral expressed in Eqn. 2 and the vorticity surface integral shown in Eqn. 3. The symbol ω_ϕ represents the out-of-plane vorticity component. The symbol dA is the infinitesimal area.

$$\Gamma = \oint \underline{u} \cdot d\underline{x} \tag{2}$$

$$\Gamma = \int \omega_\phi \cdot dA \tag{3}$$

The circulation was calculated for an area containing the core and circumscribed by a square box on one side of the axis of symmetry of the vortex ring. Both these integrals were computed numerically using the extended trapezoidal rule. The integrand was computed using local χ^2_{13} velocity interpolation in the numerical computation of Eq. 2 and the χ^2_{13} vorticity calculation method was employed in the numerical computation of Eq. 3. The Γ integrals were deemed to have converged when the relative change in the integral between 2 stages did not vary by more than 10^{-2}.

The maximum calculated circulation (Γ_{max}) along with an estimate for the maximum vorticity (ω_{max}) can be used to estimate a value for the length scale L, based on the maximum vorticity formula of the Oseen vortex vorticity distribution shown in Eqn. 4 as suggested by Fouras & Soria(1998).

$$L = \sqrt{\frac{-\Gamma}{2\,\pi\,\omega_\phi}} \tag{4}$$

Substituting Γ_{max} for $-\Gamma$ and ω_{max} for ω_ϕ in this equations resulted in $L = 115.7$ pixels. The estimate of the peak vorticity and its location were determined from the CCDPIV data analysed using an SWS of 32 pixels. After the calculation of ω_ϕ using the χ^2_{13} vorticity calculation method, the peak vorticity and its location were found using a linear search. A local two-dimensional Gaussian least square fit was then employed to determine the peak vorticity and its location more precisely. For measurements presented $\Gamma_{max} = -0.00106156 m^2/s$ from the velocity integral for SWS = 32 px and with a velocity sampling interval of 32 px. This method gives a vorticity bias error of 1.5%. Using the ratio between the random velocity error and random vorticity error (λ_0), the uncertainty due to random error in the vorticity is estimated to be 5.0%.

The measurements presented in Fig. 4 and

Fig. 5) demonstrate that the Gaussian vorticity distribution of the Oseen vortex is a reasonably good model for the experimentally determined peak vorticity region. However, significant deviation from the model occurs behind the ring (when $x < x_0$). Near the center of the ring ($y > y_0$ in Fig. 5), vorticity diffusion across the centerline leads to an asymmetric widening of the distribution.

CONCLUSIONS

The image acquisition techniques used in this work coupled with an innovative adaptive cross-correlation analysis method has provided information about the distribution of vorticity in the core region of a vortex ring at a resolution that has not previously been available. An asymmetry in the structure of the core region has been identified.

ACKNOWLEDGMENTS

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