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MIXING BY A TURBULENT FOUNTAIN IN A CONFINED STRATIFIED REGION

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ABSTRACT

We present a combined experimental and theoretical investigation of the mixing produced by a turbulent fountain when dense fluid is injected from below into a confined stratified fluid. The experiments show that the injected fluid rises to a maximum height before the flow reverses direction, and then intrudes either along the base of the tank or at an intermediate height in the environment. We determine the initial and steady-state heights of the fountain, the height of intermediate intrusion and the critical condition for spreading to occur along the base.

With the continued injection of fluid, both the fountain and the environment are observed to evolve with time. We determine expressions for the increase in the height of the fountain, and for the motion of the ascending and descending 'fronts' that mark the vertical extent of the spreading layer of mixed fluid.

INTRODUCTION

Turbulent fountains and plumes arise in a variety of environments, including the interiors of buildings, large magma chambers in the Earth's crust, and the Earth's oceans and atmosphere. In all of these examples, both the flow and environment evolve with time as the presence of confining boundaries results in the accumulation of injected fluid.

The continuous flow of a plume into a confined region containing an initially homogeneous fluid was first analysed by Baines and Turner (1969). They determined the changes to the environmental density profile resulting from the continuous addition of buoyant fluid from both point and line sources. This problem subsequently became known as a 'plume filling box' model. Similar filling box models have since been applied to axisymmetric plumes in an initially stratified fluid (Cardoso and Woods, 1993) and to fountains in initially homogeneous surroundings (Baines, Turner and Campbell, 1990).

In this paper, we summarize our investigations of axisymmetric turbulent fountains in a stratified fluid (Bloomfield and Kerr, 1998a,b). We first examine the initial flow of the fountain, and quantify how the strength of the stratification determines whether the falling fluid spreads along the base of the tank, or whether it intrudes at an intermediate height in the environment. We then develop a 'stratified filling box' model which quantifies the subsequent evolution of the fountain and the environment when either basal or intermediate spreading occurs.

EXPERIMENTAL METHOD

The experiments were carried out in an acrylic tank which was 38 cm $\times$ 38 cm in internal cross section and 80 cm deep. The ambient linear density gradient was established with NaCl solutions using the double bucket method, and was measured to $\pm$ 1%.

The source fluid was placed in a 20 l bucket which was raised 1.5 m higher than the main tank. The flow rate resulting from this gravitational head, which was kept constant throughout an experiment, was measured with a flow meter (to $\pm$ 4%). The source fluid was injected upwards from the base of the tank through a tube with an 8.8 mm inner diameter. Two sets of cross hairs of 0.5 mm diameter were positioned 3 mm and a further 44 mm from the tube outlet to ensure that the flow was turbulent from the source. Using a method outlined by Baines *et al.*, (1990), measurements of the position of the descending front formed by a weakly buoyant jet indicated that the position of the virtual point source was a distance $z_v = 1.0 \pm 0.2$ cm below the base of the tank, and the effective source radius was $r_e = 4.16 \pm 0.23$ mm.

The flows were observed using the shadowgraph method, and dye was introduced into the input fluid to mark the extent of the spreading layer. Recording the flows on video enabled the mean fountain height to be measured to within 0.5 cm.

QUALITATIVE OBSERVATIONS

In our experiments, the dense injected fluid entrains environmental fluid and rises until gravity brings it to rest at an initial height. This height is then reduced to a lower, final value as the flow reverses direction and the downflow interacts with the continued upflow.

In a weakly stratified environment with a sufficiently large buoyancy flux at the source, the subsequent behaviour is qualitatively similar to that observed in a homogeneous environment (Baines *et al.*, 1990). The downflow spreads along the base until it reaches the walls, and an *ascending front* is formed that rises as ambient fluid from above it is entrained into the downflow. At the same time, the presence of the dense layer reduces the density difference between the source fluid and its immediate environment, and thus causes the fountain height to rise. The front rises faster than the fountain height, so that, eventually, it overtakes the top of the fountain. After this point, the fountain interacts only with the stratified layer, and the rise of the front is controlled only by the rate at which source fluid is added.

In a strongly stratified environment with a sufficiently small buoyancy flux at the source, the downflow spreads above the base of the tank (figure 1). As a result, an additional *descending front* at the bottom of the spreading layer moves towards the base of the tank as fluid from below it is entrained into the upflow of the fountain. The formation of a second front in a stratified fluid is analogous to the 'plume filling box' models in which one front is observed in a homogeneous environment (Baines and Turner, 1969) while two form in a stratified fluid (Cardoso and Woods, 1993).

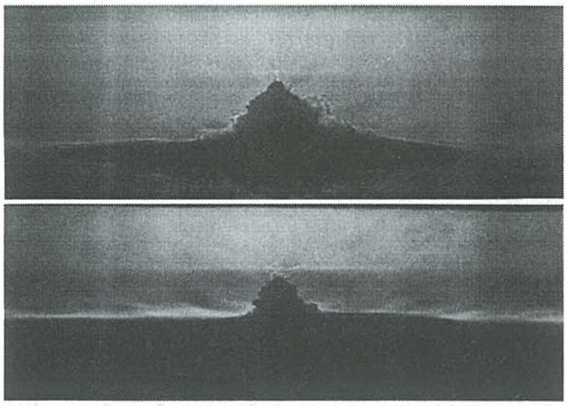

Figure 1: Photographs of an axisymmetric fountain where the density of the input fluid is equal to that at the base of the stratified environment. The flow rises like a jet and then falls before intruding into the environment at an intermediate height (upper photo). With the continued input of fluid, ascending and descending fronts are created in the environment (lower photo).

INITIAL, FINAL AND SPREADING HEIGHTS

During the first stages of the flow in a stratified fluid, dimensional analysis indicates that the initial, final and spreading heights of the fountain are given by

$$z = f(\sigma) M_o^{3/4} F_o^{-1/2}, \quad (1)$$

where the dimensionless parameter, σ, is defined by $\sigma = M_o^2 N^2 / F_o^2$, $\rho_i M_o = Q_o^2/(\pi r_e^2)$ is the momentum flux at the source, $\rho_i F_o = \rho_i \Delta_o Q_o$ is the buoyancy flux at the source and $N^2 = -(g/\rho_o)(d\rho/dz)$ is the square of the buoyancy frequency. In these expressions, Q_o is the volume flux at the source, $\Delta_o = g(\rho_i - \rho_o)/\rho_o$, g is the gravitational acceleration, z is the height above the source and ρ is the ambient fluid density, with ρ_o the density at the base of the tank and ρ_i the density of the input fluid. Our experimental determination of the functions $f(\sigma)$ are given in figure 2.

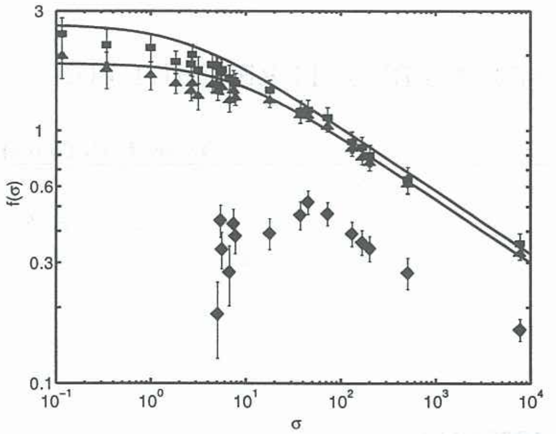

Figure 2: Dimensionless initial fountain height (■), final fountain height (▲) and spreading height (♦) as a function of σ. Equations (5) and (6) are also shown.

In the limits of small and large σ, the appropriate $f(\sigma)$ for the initial, final and spreading heights, respectively, is given by:

$$f_i(\sigma) = \begin{cases} 2.65 & \sigma < 0.1 \\ 3.25\sigma^{-1/4} & \sigma > 40, \end{cases} \quad (2)$$

$$f_f(\sigma) = \begin{cases} 1.85 & \sigma < 0.1 \\ 3.00\sigma^{-1/4} & \sigma > 40, \end{cases} \quad (3)$$

and

$$f_s(\sigma) = \begin{cases} 0 & \sigma < 5 \\ 1.53\sigma^{-1/4} & \sigma > 40. \end{cases} \quad (4)$$

For all values of σ, the simple functions

$$f_i(\sigma) = \left(2.65^{-4} + 3.25^{-4}\sigma\right)^{-1/4} \quad (5)$$

and

$$f_f(\sigma) = \left(1.85^{-4} + 3.0^{-4}\sigma\right)^{-1/4} \quad (6)$$

are a good fit to the experimental results.

MODEL OF THE FRONTS AND FOUNTAIN

Descending front

The motion of the descending front is found by writing the equation for the conservation of volume flux in the region below the front. Hence, if the cross-sectional area of the tank, A, is much greater than that of the fountain,

$$A \frac{dz_d}{dt} = -Q_e, \quad (7)$$

where $z_d(t)$ is the height of the front above the virtual point source and Q_e is the volume flux of fluid entrained into the fountain from below z_d. Since the upflow in the fountain is almost identical to that in a jet, the entrained volume flux between z_v and z_d is

$$Q_e = 2\alpha Q_o \frac{z_d - z_v}{r_e}, \qquad (8)$$

where $\alpha = 0.076 \pm 0.004$ is the jet entrainment coefficient. To simplify this and subsequent expressions, we introduce the dimensionless heights $\tilde{z}$ and times $\tilde{t}$ defined by

$$\tilde{z} = \frac{z}{r_e} \text{ and } \tilde{t} = \frac{Q_o t}{A r_e}. \qquad (9)$$

With the use of (8) and (9), (7) is integrated to give a solution for the height of the descending front above the base of the tank:

$$(\tilde{z}_d - \tilde{z}_v) = (\tilde{z}_s - \tilde{z}_v) e^{-2\alpha\tilde{t}}. \qquad (10)$$

This exponential decrease in the height of the descending front contrasts with the algebraic decrease in the height of the descending fronts formed by an axisymmetric plume in either a homogeneous (Baines and Turner, 1969) or a stratified fluid (Cardoso and Woods, 1993).

Ascending front

The motion of the ascending front, $z_a(t)$, is determined by writing the expression for the conservation of volume flux at the level of the front. Thus

$$A\frac{\mathrm{d}z_a}{\mathrm{d}t} = Q_o + Q_e, \qquad (11)$$

where Q_e is the volume flux of ambient fluid entrained into the downflow from above the front. Baines *et al.* (1990) found that, in a homogeneous fluid, the entrained volume flux per unit height into the downflow of the fountain is constant and is given by

$$\frac{\mathrm{d}Q_e}{\mathrm{d}z} = B\frac{Q_o}{r_e}, \qquad (12)$$

where B was found experimentally to be $B \approx 0.25$. Baines *et al.* (1990) also explained that the observation of constant entrainment per unit height can be understood by viewing the downflow as a line plume which encircles the upflow. In a linearly stratified environment, the behaviour of a plume is little different to that in a uniform fluid until close to the spreading height (Cardoso and Woods, 1993). As a result, (12) also accurately predicts the entrainment into the downflow of a fountain in a stratified fluid, and the total volume flux entrained between $\tilde{z}_a$ and $\tilde{z}_f$ is

$$Q_e = \frac{BQ_o}{r_e}(z_f - z_a). \qquad (13)$$

Using (9), and introducing (13) into (11), leads to

$$\frac{\mathrm{d}\tilde{z}_a}{\mathrm{d}\tilde{t}} = 1 + B(\tilde{z}_f - \tilde{z}_a). \qquad (14)$$

After the ascending front has reached the top of the fountain at a time t^*, the position of the front increases at the same rate as which the free surface rises due to the addition of fluid to the tank, so that $\mathrm{d}\tilde{z}_a/\mathrm{d}\tilde{t} = 1$. To integrate (14) for times $t < t^*$, we need an expression for the change in the fountain height with time.

Fountain height

In developing an expression for the fountain height, we must consider both that the environment evolves with time from being stratified to being homogeneous, and that the addition of dense source fluid increases the average ambient density with time. We therefore derive two expressions for the fountain height: $z_{fs}(t)$, which gives the fountain height in a stratified environment with decreasing stratification, and $z_{fh}(t)$, which gives the fountain height in a homogeneous fluid of increasing density.

To quantify z_{fs}, the average ambient density gradient over the height of the fountain is approximated by

$$\frac{\mathrm{d}\rho}{\mathrm{d}z}(t) = \frac{\rho_{z_f}(t) - \rho_o}{z_{fs} - z_v}, \qquad (15)$$

where $\rho_{z_f}(t)$ is the ambient density at the level of the top of the fountain, which is at a height $z_{fs} - z_v$ above the base of the tank. At small times, a good estimate for ρ_{z_f} is obtained by assuming that all ambient density levels above the ascending front rise at the same rate as the free surface. The position of a thin layer which is initially at a height z_o is therefore given by $z(t) = z_o + Q_o t/A$. When this layer reaches z_{fs}, $\rho_{z_f} = \rho_o + \left.\frac{\mathrm{d}\rho}{\mathrm{d}z}\right|_o (z_o - z_v)$, giving

$$\rho_{z_f} = \rho_o + \left.\frac{\mathrm{d}\rho}{\mathrm{d}z}\right|_o (z_{fs} - z_v - Q_o t/A), \qquad (16)$$

where $\left.\frac{\mathrm{d}\rho}{\mathrm{d}z}\right|_o$ is the initial density gradient. Combining (15) and (16), we obtain an expression for $N^2(t)$:

$$N^2(t) = N_o^2 \frac{z_{fs} - z_v - Q_o t/A}{z_{fs} - z_v}, \qquad (17)$$

where N_o is the initial buoyancy frequency. Introducing (17) into the definition of σ, and combining (1), (6) and (9), leads to

$$\frac{z_{fs}}{M_o^{3/4} F_o^{-1/2}} = \left(\frac{1}{1.85^4} + \frac{\sigma_o}{3.0^4}\frac{\tilde{z}_{fs} - \tilde{z}_v - \tilde{t}}{\tilde{z}_{fs} - \tilde{z}_v}\right)^{-1/4}, \qquad (18)$$

where $\sigma_o = M_o^2 N_o^2 / F_o^2$. Rearranging this expression gives the result that

$$\frac{\tilde{z}_{fs}}{\tilde{z}_{fs}(0)} = \left(1 - \frac{3.0^{-4}\sigma_o}{1.85^{-4} + 3.0^{-4}\sigma_o}\frac{\tilde{t}}{\tilde{z}_{fs} - \tilde{z}_v}\right)^{-1/4}. \qquad (19)$$

At small times when $\tilde{t} \ll \tilde{z}_{fs}$, we can replace $\tilde{z}_{fs}$ by $\tilde{z}_{fs}(0)$ on the right hand side of (19), so that

$$\tilde{z}_{fs} \approx \tilde{z}_{fs}(0) + \frac{3.0^{-4}\sigma_o}{1.85^{-4} + 3.0^{-4}\sigma_o}\frac{\tilde{z}_{rs}}{4}\tilde{t}, \qquad (20)$$

where $\tilde{z}_{rs} = \tilde{z}_{fs}(0)/(\tilde{z}_{fs}(0) - \tilde{z}_v)$.

An expression for z_{fh} is obtained by assuming that the ambient fluid below this height is homogeneously mixed, and then using (1) combined with (3) to find the height to which the fountain would rise in a fluid with this density. If $\bar{\rho}_o$ is the average environmental density at $t = 0$, and the fountain reaches a height $z_{fh}(0)$ in this homogeneous fluid, then

$$\bar{\rho}_o = \rho_o + \left.\frac{\mathrm{d}\rho}{\mathrm{d}z}\right|_o \frac{(z_{fh}(0) - z_v)}{2}. \qquad (21)$$

The initial buoyant acceleration of the source fluid is

$$\bar{\Delta}_o = \frac{g(\rho_i - \bar{\rho}_o)}{\bar{\rho}_o} = \frac{\Delta_o + \frac{N_o^2}{2}(z_{fh}(0) - z_v)}{1 - \frac{N_o^2}{2g}(z_{fh}(0) - z_v)}. \qquad (22)$$

Using (1) and (3), we obtain an equation which can be solved numerically for $\tilde{z}_{fh}(0)$:

$$\tilde{z}_{fh}(0) = \frac{1.85 M_o^{1/2}}{\pi^{1/4} r_e^{3/2}} \left(\frac{1 - \frac{N_o^2 r_e}{2g}(\tilde{z}_{fh}(0) - \tilde{z}_v)}{\Delta_o + \frac{N_o^2 r_e}{2}(\tilde{z}_{fh}(0) - \tilde{z}_v)} \right)^{1/2}. \qquad (23)$$

In a homogeneous fluid, Baines *et al.* (1990) have shown both experimentally and theoretically that the fountain height rises at close to half the rate at which the free surface rises due to the inflow. If the same theoretical arguments are applied to estimate $\tilde{z}_{fh}$, the equivalent result in terms of dimensionless parameters is

$$\tilde{z}_{fh} = \tilde{z}_{fh}(0) + \frac{1}{2}\tilde{z}_{rh}\tilde{t}, \qquad (24)$$

where $\tilde{z}_{rh} = \tilde{z}_{fh}(0)/(\tilde{z}_{fh}(0) - \tilde{z}_v)$.

To quantify the fountain height at all times, we combine (20) and (24) into a single expression for the fountain height which characterizes the transition from $\tilde{z}_{fs}$ at small times to $\tilde{z}_{fh}$ as $\tilde{t} \to \tilde{t}^*$. A suitable expression for $\tilde{z}_f(\tilde{t})$ is therefore

$$\tilde{z}_f(\tilde{t}) = (1 - w(\tilde{t}))\tilde{z}_{fs} + w(\tilde{t})\tilde{z}_{fh}, \qquad (25)$$

where the weighting function $w(\tilde{t})$ is chosen to be

$$w = \frac{\tilde{z}_a - \tilde{z}_d}{\tilde{z}_f - \tilde{z}_v}. \qquad (26)$$

Experimental results

The position of the fountain height and the fronts were measured in a series of experiments performed for a range of values of σ. The data from one of these experiments is shown in figure 3 along with the result of integrating (14) for the ascending front, the predicted position of the fountain height and the expression for the descending front (10). The good agreement between theory and experiment for the fountain height and ascending front indicates that the assumptions made, and the simple weighting function used, describe the actual fountain behaviour well. The experimental results indicate that the descending front falls slightly faster than predicted by (10). This faster descent is almost certainly due to the additional entrainment into the overshooting fluid below the front, which is not included in our model.

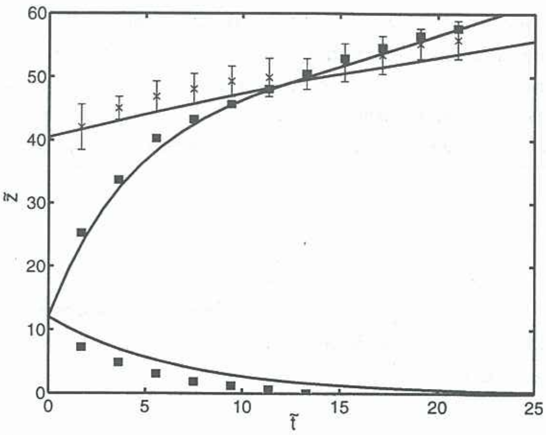

Figure 3: Experimental measurements of the fountain height (×) and the fronts (■) along with the theoretical predictions of our model. In this experiment, $\sigma = 14$ and $f_s(14) = 0.4$ (see figure 2).

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