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A SMOOTH TUBE SIMULATION OF ROUGHENED WALL PIPE FLOW

D R H Beattie

Australian Nuclear Science and Technology Organisation
Private Mail Bag 1, Menai 2234, AUSTRALIA

ABSTRACT

Rough pipe flow is modelled by smooth pipe flow with an artificially large near-wall viscosity chosen to simulate the effect of roughness elements on near-wall velocity profile. Traditional explanations for behaviour differences for different roughness types are shown to be incorrect. The Colebrook-White friction equation for naturally rough pipes, and the friction equation for hydraulically smooth conditions, are derived. Predicted velocity profiles for these conditions, including those for the near-wall region, agree with published experimental data.

INTRODUCTION

Classical pipe flow theory has two distinct strands: one for smooth pipes and one for rough pipes. Both are based on Nikuradse's (1932, 1933) experimental results.

The velocity profile of Squire (1948),

$$u^+ = y^+ \text{ for } y^+<7.87 \quad (1a)$$
$$= 5.66\{1+\log(y^+-5.41)\} \text{ for } y^+>7.87, \quad (1b)$$

is chosen as a basis of the present analysis instead of earlier classical forms as it is more compatible with experimental near-wall velocity data. It is in effect based on earlier laminar sublayer concepts together with an eddy viscosity of the form

$$\mu_t = 0.407\mu(y^+-7.87) \text{ for } u^+>7.87 \quad (2)$$

and retention of molecular viscosity in the turbulent core.

Averaging the asymptote of the velocity profile

$$u^+ = 5.66(1+\log y^+) \quad (1b')$$

(the earlier classical form of turbulent core profile) over the flow section results in the standard Fanning friction factor expression

$$1/\sqrt{f} = 4\log(Re\sqrt{f})-0.4. \quad (3)$$

For completely rough tubes, Nikuradse's (1933) classic velocity profiles for sand roughness of height ε_s are described in the "fully rough" region by

$$u^+ = 5.66\log(y/\varepsilon_s)+8.5. \quad (4)$$

By averaging this over the flow section,

$$1/\sqrt{f} = -0.4 - 4\log(0.214\,\varepsilon_s/D). \quad (5)$$

This describes Nikuradse's "completely rough" friction factor data. At lower flows, his rough wall friction factor data cover "hydraulically smooth" data compatible with the smooth tube equation and "partially rough" data with friction factors between the completely rough and smooth equations.

White (Colebrook 1939) synthesised the completely rough and smooth wall friction factor equations to produce

$$1/\sqrt{f} = -0.4 - 4\log\{0.214\,\varepsilon_s/D + (Re\sqrt{f})^{-1}\}, \quad (6)$$

where ε_s is now the equivalent sand roughness. Although this agrees with Nikuradse's completely rough data, it does not agree with his partially rough pipe data nor his hydraulically smooth rough pipe data. Nevertheless, more importantly, it <u>does</u> agree with pressure loss data from commercially produced pipes, even in the partially rough regime, and also reverts to the correct form for zero roughness. These outcomes are sufficient to justify White's non-mechanistic synthesis.

The present analysis does not require separate treatments of smooth pipe flow (dependent only on viscosity) and completely rough pipe flow (dependent only on roughness parameters). Moreover, the analysis also covers "partially rough" flows dependent on both relative roughness and Reynolds numbers. These results are achieved by noting that roughness elements of height ε impede the flow in a way that can be simulated by a smooth wall flow with effective near-wall "roughness viscosity" μ_ε chosen to provide the correct local velocity at the outer edge of the roughness layer. The analysis follows classical concepts, with the eddy viscosity form of Squire being chosen over others since, at least for smooth pipes, the Squire model predicts more accurate near-wall velocity profiles.

ANALYSIS

From the definition of viscosity and the standard means of non-dimensionalising parameters,

$$du^+/dy^+ = \mu_{eff}/\mu \quad (7)$$

where, in view of the model outlined above,

$$\mu_{eff} = \mu_\varepsilon,\ y<\varepsilon; \quad (8a)$$
$$= \mu, \text{ from } y^+=\varepsilon^+ \text{ to } u^+=u_o^+, \text{ and} \quad (8b)$$

$$=\mu+\kappa\mu(y^+-y_\delta^+),\ u^+>u_o^+. \tag{8c}$$

In the above, equation (8c) is the form proposed by Squire; u_o^+ is a transition-to-turbulence criterion having a value 7.87 for smooth pipes; and y_δ^+ is an apparent reference position for the onset of turbulence. For smooth pipes, choosing it as the location where the critical velocity for turbulence onset occurs gives it the same value 7.87

Integrating equation (7) provides the velocity profile

$$u^+=(\mu/\mu_\varepsilon)y^+,\ y^+<\varepsilon^+ \tag{9a}$$
$$u^+=y^++(\mu/\mu_\varepsilon-1)\varepsilon^+ \text{ from } y^+=\varepsilon^+ \text{ to } u^+=u_o^+ \tag{9b}$$

(ie, equation (9b) covers the range $\varepsilon^+<y^+< u_o^+ +\varepsilon^+(1-\mu/\mu_\varepsilon)\equiv y_o^+$), and

$$u^+=(u_o^+ +\kappa^{-1}\ln \kappa) +\kappa^{-1}\ln(y^+-y_\delta^+ +\kappa^{-1}) -\Delta \text{ from } y^+>y_o^+ \tag{9c}$$

where

$$\Delta=\kappa^{-1}\ln\{1+\kappa(y_o^+-y_\delta^+)\} \tag{10a}$$
$$\equiv\kappa^{-1}\ln\{1+\kappa(u_o^+-y_\delta^+)+\kappa\, \varepsilon^+(1-\mu/\mu_\varepsilon)\} \tag{10b}$$

is the roughness function.

Averaging the asymptote of equation (9) over the flow section leads to the friction factor expression.

$$1/\sqrt{f}=A+B \log C. \tag{11}$$

where

$$A=u_o^++\kappa^{-1} \ln \kappa -\kappa^{-1} \{1.5+ \ln (2\sqrt{2})\}=-0.4 \tag{12}$$
$$B=(\sqrt{2}\kappa \log e)^{-1}=-4 \tag{13}$$
$$C=\{1+\kappa(u_o^+-y_\delta^+)+\kappa\, \varepsilon^+(1-\mu/\mu_\varepsilon)\}/(Re\sqrt{f}) \tag{14a}$$
$$\equiv\{1+\kappa(y_o^+-y_\delta^+)\}/(Re\sqrt{f}). \tag{14b}$$

The numerical values for A and B are based on values for κ and u_o^+ as 0.407 and 7.87 respectively. These are the values from which the smooth pipe version of Squire's model provides the smooth pipe equations (1) and (3) for velocity profile and friction factor. These values are necessary to ensure that the analysis yields the correct smooth tube equations for zero roughness.

The correct high ε limit, equations (4) and (5), can be ensured by choosing

$$\varepsilon (1-\mu/\mu_\varepsilon)=0.714\, \varepsilon_s . \tag{15}$$

The remaining unknown in the analysis is the effective origin for turbulence y_δ^+ of Squire's turbulence model, equation (8c). Different choices for this quantity permit the present model to simulate the different types of hydraulic behaviour which occur for different roughness types. These are discussed below.

APPLICATION

Pipes of Natural Roughness (Commercial Pipes)

An automatic choice for y_δ^+ is 7.87, the value for which Squire's model leads to the smooth tube friction factor equation, equation (3). It is readily seen that this value converts equation (11) to the Colebrook-White equation. Moreover, the analysis provides a means of predicting the velocity profile.

Robertson et al (1968) measured friction factors and velocity profiles for a naturally rough tube. Their friction data agrees with the Colebrook-White equation with an equivalent sand roughness value $\varepsilon_s/D=0.0015$ (Fig 1).

Robertson et al's velocity data are compared with the present predictions in Figs 2 and 3. The "roughness sublayer" predictions are included for schematic purposes only.

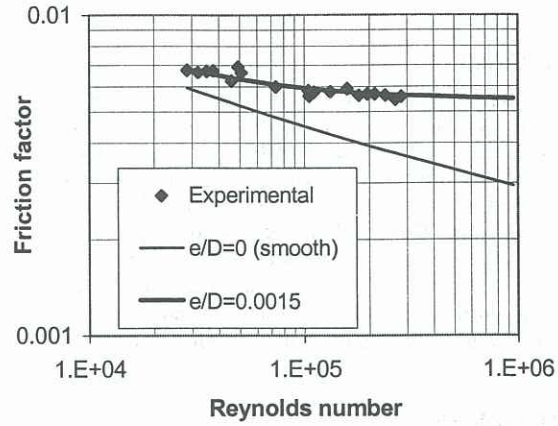

Figure 1 : Comparison of Robertson et al's friction factor data with the Colebrook-White equation for $\varepsilon_s/D=0.0015$.

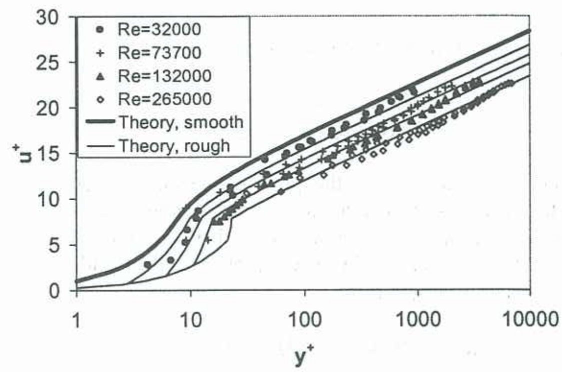

Figure 2 : Comparison of Robertson et al's (1968) velocity data and the present predictions

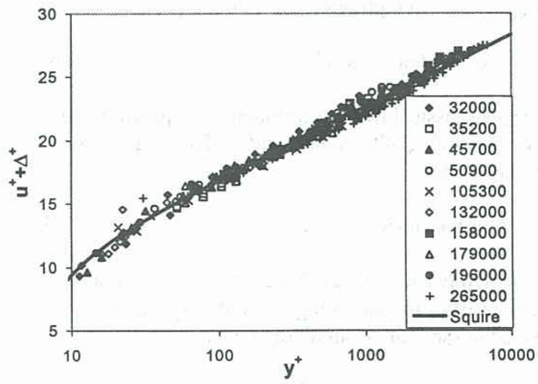

Figure 3 : Comparison of Robertson et al's (1968) shifted velocity data for various Reynolds numbers (shown) and the present predictions.

Prediction of the roughness sublayer velocity profile requires an additional assumption. The predictions in fig. 2 are for the assumption $\varepsilon_s=\varepsilon$. The data of fig 3 only include data in the range beyond the predicted laminar sublayer since only these velocity data when shifted by Δ are predicted to fall on the Squire curve. It can be seen from figs. 2 and 3 that predictions are close to the data, including those data in the predicted laminar sublayer and also those data below $y^+\approx70$ normally regarded as being in the transition-to-turbulence region.

Hydraulically Smooth Rough Pipes

An alternative choice for y_δ^+ to that above (ie $y_\delta^+=7.87$) is the location where, in the Squire model, turbulence starts. The Squire model is here interpreted as having turbulence onset at a critical u^+ value $u_o^+(=7.87)$ rather than at a critical y^+ value. In this case, $y_\delta^+= u_o^++\varepsilon^+(1-\mu/\mu_\varepsilon)$ is the location of turbulence onset, and the predicted turbulent core velocity profile (ie for $u^+>7.87$) becomes

$$u^+ =5.66\{1+\log(y^+-5.41-0.714\,\varepsilon_s^+)\}. \tag{16}$$

This is simply a shifted version of Squire's version of the "universal" profile. Although his version agrees with data better than earlier versions of the "universal" profile, it still has discrepancies in the so-called "buffer" region. The analysis suggests that shifted velocity profiles for hydraulically smooth rough pipe flow will agree with more accurate versions of the "universal" profile.

The asymptote of equation (16) is that for smooth tubes, so, as expected, equation (11) reverts to the normal smooth tube friction equation, equation (3). The rough walls are thus "hydraulically smooth" for such flows.

Figure 4 compares Nikuradse's hydraulically smooth rough pipe velocity data, shifted by d^+ values chosen to optimise agreement with theory, with Squire's equation and Beattie's (1992) version of the "universal" profile. The data include Nikuradse's "y=0" data, neglected by him, indicating a zero error in his traversing mechanism. In line with the above expectations, the data are close to Squire's equation and are in excellent agreement with Beattie's more accurate version of the "universal" profile. (Similarly, Squire's equation appears to slightly overpredict the "buffer" region data of the rough wall velocity data as presented in Figure 3.)

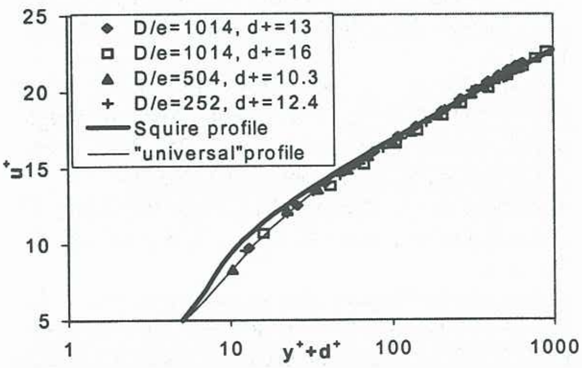

Figure 4 : Comparison of Nikuradse's (1933) shifted hydraulically smooth rough pipe velocity data and the present predictions.

CONCLUDING DISCUSSION

Four observations may be straightforwardly made.

Firstly, a unifying approach to smooth and rough wall flows has been achieved by extending smooth tube concepts to rough pipes. Previously, rough and smooth walls have been presented as having separate, unrelated boundary conditions for the turbulent core.

Secondly, the model provides a simple derivation of the Colebrook-White equation. This equation has not previously had a theoretical basis.

Thirdly, the present model is incompatible with the current widely held view that hydraulic differences for different roughness types arise partly as a result of different extents to which roughness elements extend into the turbulent core. Instead, for the traditional cases examined here, the laminar sublayer is "pushed" to larger y^+ values with increasing ε_s^+, and roughness elements always remain in the laminar sublayer. The dependence on Reynolds number reduces with increasing Reynolds number not because roughness elements extend into the turbulent core, as is normally considered, but because, as demonstrated in Figure 2, the "roughness" sublayer becomes an increasingly dominant component of the total sublayer.

The fourth observation is that the theoretical derivation of velocity profiles has extended closer to the wall than previous similar rough wall analyses. Moreover, as demonstrated, available published experimental velocity data, including those near the wall, are adequately compatible with the present analysis.

A closer examination of Squire's concepts provides more insight into how different roughness types differently affect flow. The apparent reference location for turbulence in Squire's smooth wall model, $y^+=7.87$, is a consequence of overlooking turbulent processes in the so-called laminar sublayer. As the actual reference location for turbulence generation is the y=0 location, the true reference location using Squire's concepts can be regarded as being 7.87 wall units closer to the wall than the apparent reference location built into Squire's model. In this framework, the present analyses for naturally rough pipes and hydraulically smooth rough pipes can be interpreted as having reference locations respectively at the wall and at the edge of the roughness elements. Perhaps, for non-random roughness, the larger eddies which exist at lower Reynolds numbers cannot get through the relatively small spaces between the roughness elements, so the reference location for the origin of turbulence is at the tops of the roughness elements, leading to the actual and predicted hydraulically smooth behaviour. Conversely, the smaller eddies at higher Reynolds numbers can reach the wall, leading to the actual and predicted agreement with the Colebrook-White equation. It follows that the transition between the two could then be a result of a higher fraction of eddies extending to the wall with increasing Reynolds number.

It may be noted that, as shown by Beattie (1996), Squire's empirical form of eddy viscosity (equation 2) is a first-order approximation of the mechanistically-based eddy viscosity of Beattie (1993).

As noted earlier, contrary to the traditional viewpoint, roughness elements, as modelled here, remain in the laminar (in reality, "viscous") sublayer even in the "completely rough" region of friction characteristics. Nevertheless, with increasing Reynolds number, roughness elements will eventually extend into the turbulent core, as can be deduced from the trends of Figure 2. Although the present analysis has not examined this case, a possibility suggested by the model is that, at least if a constant effective near-wall "roughness" viscosity simulates the velocity profile within the roughness region rather than just the local velocity at its edge, then, when the roughness extends to $u^+=7.8$ and beyond, the turbulent core velocity profile will revert to a new universal-like profile insofar as it will be independent of Reynolds number. Consistent with this concept are the experimental friction factor data of Millionshchikov et al (1973) for artificially roughened ducts. After reaching an apparent Reynolds-number independent "completely rough" friction factor asymptote at higher Reynolds numbers, their friction factors eventually revert to a strong dependence on Reynolds numbers at even higher Reynolds numbers. An example is shown in Figure 5.

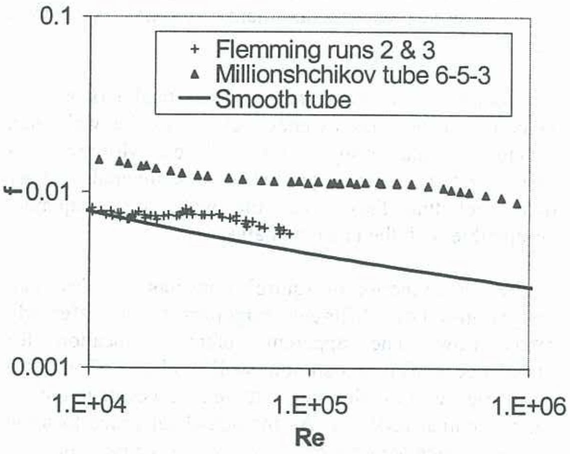

Figure 5 : Rough pipe (hemispherical projections) friction factors reverting to a dependence on Reynolds number at sufficiently large Reynolds numbers (Millionshchikov et al 1973), and hydraulically similar behaviour shown by smooth pipe flow with increased near-wall viscosity (Fleming et al 1972).

The present analysis, in which rough pipe flows are simulated by smooth tube flows with increased near-wall viscosity, implies such smooth tube flows should have friction characteristics similar to those of rough pipes. Available data support this. Fleming et al (1972) measured pressure losses for mercury flowing in copper pipes. The resulting near-wall amalgam layer fluid has a higher viscosity. At moderate Reynolds numbers, friction factors are similar to those for rough pipes, including the trend of constant friction factors, similar to "fully rough" friction factors. At sufficiently high Reynolds numbers, their friction factors reverted to a strong dependence on Reynolds numbers. This is also compatible with the present predictions. An example of their data is given in Figure 5.

Another, more widely encountered flow with increased near wall viscosity is "annular" gas-liquid pipe flow, in which the liquid naturally flows as a film adjacent to the wall. At low pressures, the gas phase Reynolds number is necessarily confined to relatively low values, and the liquid film is confined to the viscous sublayer if the gas phase. In line with the present analysis, such flows are widely modelled by gas flow in a rough pipe of effective roughness determined by the film thickness (Hewitt and Hall-Taylor 1970). At higher pressures, higher gas densities result in higher gas Reynolds numbers for nominally similar annular flows, so the liquid film region extends into the gas turbulent core. The present analysis suggests friction factors for such flows would become independent of film thickness and instead depend on Reynolds number. Friction factors for high pressure "annular" gas-liquid pipe flows do in fact correlate with Reynolds number and not film thickness (Beattie 1973)

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