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TRANSITION FROM MACH REFLECTION TO REGULAR REFLECTION OVER A CYLINDRICAL CONCAVE SURFACE WITH INITIAL WEDGE ANGLE

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ABSTRACT

Method of prediction for a critical angle of transition from Mach reflection (MR) to regular reflection (RR) over a concave cylindrical surface with initial wedge angle has been developed. In the analysis, relation between the position and the propagating velocity of a reflection point on a concave surface is obtained analytically when reflection is Mach reflection. Based on this relation, new transition criterion from MR to RR over a cylindrical concave surface is developed taking account of the influence of initial angle of incidence. Analytical results of the transition angle obtained by applying this criterion are in better agreement with experimental results than the prediction of Ben-Dor *et al.* (1985) for the concave surface with large initial wedge angle.

INTRODUCTION

When a planar shock wave encounters the concave wedge with initial wedge angle θ_0 (Fig. 1), MR occurs. As the incident shock wave proceeds, effective wedge angle θ increases. As a result, transition from MR to RR occurs. The problems of shock reflection over concave surfaces are less tractable than over ordinary smooth straight surfaces, since the flow field around a triple point for MR is not pseudo-steady and self-similar. On the other hand, due to such unsteadiness, the reflection configuration has a distinctive feature unlike reflection over smooth straight surfaces.

Itoh *et al.* (1980, 1981) analyzed transition angle from MR to RR over a cylindrical concave surface by applying the Whitham's ray shock theory. They also measured transition angles and obtained good agreement with their theory when initial angle of incidence was zero.

Takayama and Sasaki (1983) investigated the effects of initial angle together with radius of curvature on the transition wedge angle experimentally. They showed that the transition angle θ_c from MR to RR decreases as the initial wedge angle increases.

Ben-Dor *et al.* (1985) developed analytical formulae for the transition wedge angle over the concave surface with initial wedge angle. They have derived the transition criterion by assuming that the flow properties

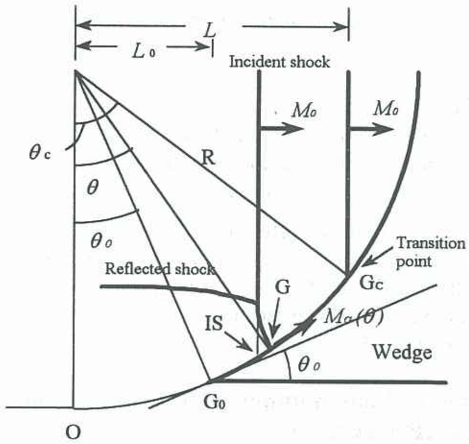

Figure 1: Schematic illustration of a Mach reflection over a cylindrical concave wedge with initial wedge angle θ_0. G_0 and G_c are the tip of the wedge and the transition point from MR to RR, respectively.

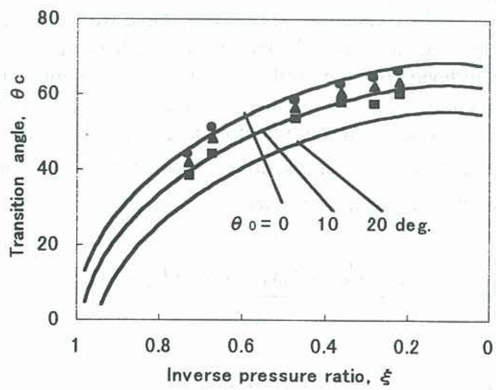

Figure 2: Comparison of the transition lines predicted by Ben-Dor & Takayama (1985) with the experimental results of Itoh & Itaya (1980). Symbols, ●,▲ and ■ indicate experimental data of Itoh & Itaya in the case where $\theta_0 = 0,\ 10^\circ$ and 20°, respectively.

behind the reflected shock wave maintain their values behind the incident shock. Their analytical results, however, are not in good agreement with the experimental ones as the initial wedge angle increases (Fig.2).

The authors, who have studied shock reflection over non-straight surfaces for the past several years, explained the reflection mechanism by use of step-like wedges (Kobayashi *et al.* 1996). As a by-product, analytical formula for transition angle over the cylindrical concave surface with zero initial wedge angle was derived (Suzuki *et al.* 1997). Adachi *et al.* (1998) proposed a new approach to the transition criterion from MR to RR over the cylindrical concave surface, which is valid for the large initial wedge angles. In the present paper we improve this transition criterion in order to hold in a wide range of incident shock strengths and initial wedge angles.

ANALYSIS

Our transition criterion is made of two parts. In the first, the relation between the position θ and the propagating Mach number M_G of a reflection point G on a concave surface is derived analytically when reflection is Mach reflection. Based on this relation, new transition criterion from MR to RR over a cylindrical concave surface is developed taking account of the influence of initial angle of incidence.

Propagation Mach number of reflection point on the concave surface

When incident shock wave propagates over the concave surface and the reflection is Mach type, disturbances issue from the surface behind the Mach stem. These disturbances accumulate to pressurize the region behind the Mach stem. As a result, Mach stem is accelerated as the incident shock proceeds. However, since the Mach stem is nearly normal to the concave surface, these disturbances issuing from the surface just behind the stem could be considered to be weak. Then we can apply the glancing incidence theory to the Mach stem in the neighborhood of the reflection point G, which is intersection of Mach stem and concave surface. That is, the relation between the trajectory angle χ of the weak disturbance and the propagating Mach number M_G of the Mach stem is expressed as

$$\tan\chi = \frac{1}{M_G^2}\sqrt{\frac{\{(\gamma-1)M_G^2+2\}(M_G^2-1)}{(\gamma+1)}}, \qquad (1)$$

where γ is the specific heat ratio.

Let us suppose that reflection point G moves along the chord GG' with small wedge angle $(1/2)d\theta$ instead of actual path, arc GG', as shown in Fig. 3. Then, by considering the flow direction behind the local incident shock wave and approximating the flow field uniform and wave configuration similar, Mach number $M_{G1}(\theta+d\theta)$ at G' can be written as

$$M_{G1}(\theta+d\theta) = M_G(\theta)\frac{\cos\chi}{\cos(\chi+\frac{1}{2}d\theta)}, \qquad (2)$$

where χ is the trajectory angle of a point at which a wave front of disturbance generated by a shallow wedge can catch up with a plane shock.

Another path which approximate circular arc GG' is GAG' composed of two straight line segments. Since lines GA and AG' are tangent to the circular surface at G and G' respectively, wedge angle which accelerate Mach stem is $d\theta$. In this case, Mach number $M_{G2}(\theta+d\theta)$ at G' is expressed as

$$M_{G2}(\theta+d\theta) = M_G(\theta)\frac{\cos\chi}{\cos(\chi+d\theta)}. \qquad (3)$$

Since the actual path, arc GG', lies between two approximated path GG' and GAG' which are composed of straight line, when reflection point moves along the arc GG', Mach number $M_G(\theta+d\theta)$ at G' satisfies the next inequality

$$M_{G1}(\theta+d\theta) < M_G(\theta+d\theta) < M_{G2}(\theta+d\theta). \qquad (4)$$

Consequently we postulate the true Mach number $M_G(\theta+d\theta)$ at the reflection point G' as

$$M_G(\theta+d\theta) = M_G(\theta)\frac{\cos\chi}{\cos(\chi+\alpha d\theta)} \qquad (5)$$

where $1/2 \le \alpha \le 1$ from geometrical consideration, and

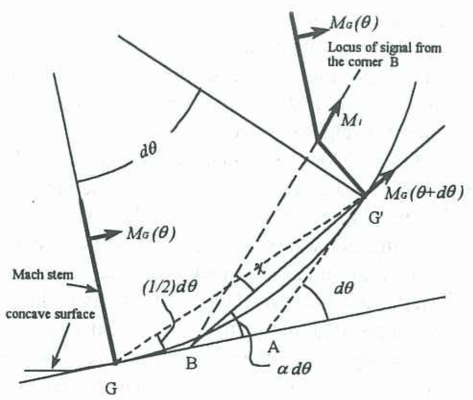

Figure 3: Schematic illustration of a Mach reflection when reflection point moves along a concave corner GBG'. χ is a trajectory angle of a triple point. Other paths GG' and GAG' are also shown for reference.

$\alpha d\theta$ is considered as a equivalent wedge angle if we substitute straight path GBG' for actual path arc GG'.

Since $\alpha d\theta$ in equation (5) is far smaller than unity, the increment of $1/M_G$ corresponding to the small increment of wedge angle $d\theta$ can be expressed as

$$d\left(\frac{1}{M_G}\right) = -\left(\frac{1}{M_G}\right)\cdot\alpha\cdot\tan\chi\cdot d\theta, \qquad (6)$$

Before integrating equation (6), we assume here that α is a function of only inverse pressure ratio, ξ, across the incident shock. From the comparison between analytical and experimental results, we found that lower and upper limits of α correspond to weak and strong incident shock, respectively. Namely, when ξ is unity, propagation path GBG' of reflection point coincides with the straight path GG'. On the other hand, when ξ is zero, path GBG' is coincident with GAG'. Then, if we assume that ξ is a linear function of the length of segment AB, α is expressed as

$$\alpha = \frac{1}{1+\xi}. \qquad (7)$$

Since α is constant for a given value of M_0, integration of (6) can be carried out, to results in

$$\theta - \theta_0 = \frac{1}{\alpha}\sqrt{\frac{\gamma+1}{8}}\left\{f(1/M_{G0}^2) - f(1/M_G^2)\right\}, \qquad (8)$$

where

$$f(u) = \sqrt{\frac{2}{\gamma-1}}\log\left|\frac{(1-\frac{\gamma-1}{2})u + (\gamma-1) - 2\sqrt{g(u)}}{u}\right|,$$

$$g(u) = \frac{\gamma-1}{2}\left(u + \frac{\gamma-1}{2}\right)(1-u),$$

and M_{G0} is the flow Mach number ahead of the Mach stem which is formed just after the incident shock strikes the tip of the wedge G_0. Equation (8) gives the relation between the position and propagation Mach number of the reflection point on the concave surface.

Transition criterion from MR to RR

Transition criterion from MR to RR is deduced from the geometry in Fig. 1. Transition occurs when the intersection IS of the incident shock and the concave surface catches up with the reflection point G. Then next relation holds

$$M_0 a_0 \int_{\theta_0}^{\theta_c} \frac{R d\theta}{M_G a_0} = R(\sin\theta_c - \sin\theta_0), \qquad (9)$$

where M_0 is the incident shock Mach number, a_0 the speed of sound of the flow ahead of the incident shock, R the radius of curvature of the cylindrical concave wedge, and θ_c the transition wedge angle. By use of (6), the equation (9) is rewritten by

$$\int_{M_{G0}}^{M_{Gc}} \frac{1}{\alpha\cdot\tan\chi}\cdot\frac{M_0}{M_G^2} dM_G = \sin\theta_c - \sin\theta_0. \qquad (10)$$

By solving equations (8) and (10) numerically, we can obtain the transition angle θ_c for given initial wedge angle θ_0 and incident shock Mach number M_0. Initial condition M_{G0} for the integral of (10) is the propagating Mach number of Mach stem on the smooth straight wedge which has a wedge angle corresponding to the initial angle of concave wedge. In the present analysis we use the analytical formulae obtained by Henderson (1980) as the initial condition.

RESULTS AND DISCUSSIONS

Numerical calculations are performed in the case of $\gamma = 1.4$. The transition line predicted by the present theory in the case of zero initial angle of incidence is shown in figure 4 as curve A. Curve B is the transition line as obtained by Itoh *et al.* (1981). Solid circles and open circles are experimental results obtained by Ben-Dor & Takayama and us, respectively. The agreement

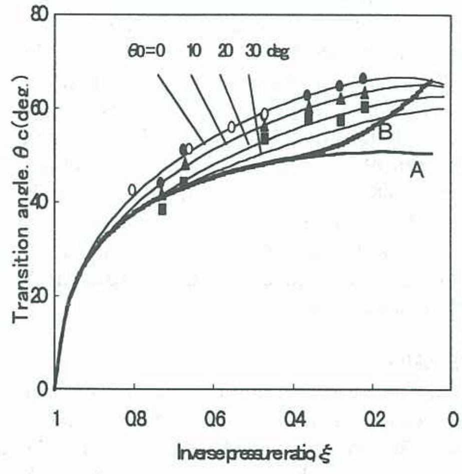

Figure 4: Angle of transition from MR to RR over concave surface with zero initial wedge angle. ξ is the inverse pressure ratio across the incident shock; θ_c is the critical angle of transition; ●, experimental results of Ben-Dor & Takayama (1985); ○, our experimental results; Curve A, the transition angle predicted by the present theory; B, the transition angle predicted by Itoh *et al.* (1981).

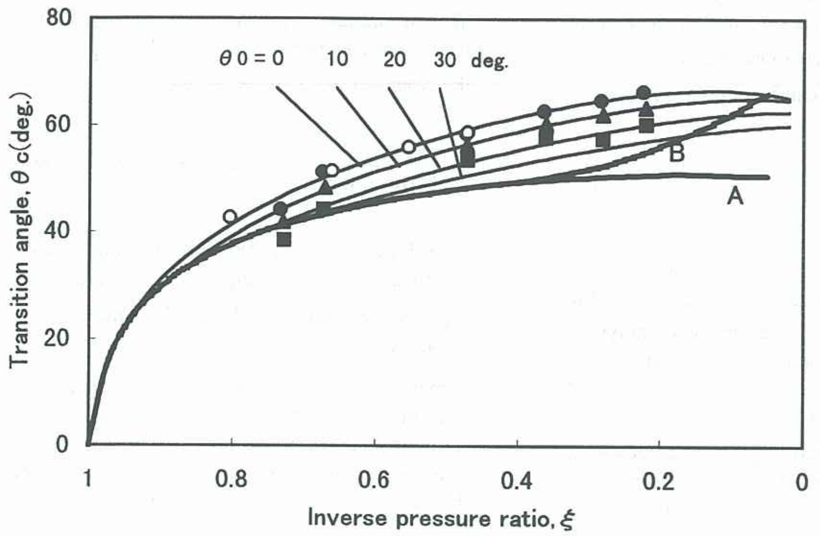

Figure 5: Angle of transition from MR to RR over concave surface with initial wedge angle. Curve A is the detachment criterion; B, the mechanical -equilibrium criterion. For solid symbols and open symbols see caption to figure 2 and 4, respectively.

between the experimental results and the transition line predicted by the present theory is quite good in a wide range of inverse pressure ratio. This means that the assumption for the dependence of equivalent wedge angle on incident shock strength, namely, equation (7) is satisfactory to estimate the angle of transition.

Comparison between transition lines predicted by the present theory and experimental results of Itoh & Itaya (1980) is shown in figure 5. Compared with figure 2, our predicted transition lines are in better agreement with experimental results than those of Ben-Dor & Takayama (19885). This is due to the fact that analytical model of Ben-Dor & Takayama holds only when reflected shock formed just after the incident shock strikes the tip of the wedge is weak, whereas our theory need not put such assumption.

CONCLUSION

New transition criterion from MR to RR over a cylindrical concave surface with initial wedge angle was developed. This criterion includes the effect of the initial angle of incidence as the initial Mach number of Mach stem at the tip of the wedge; so it can accurately predict actual experimental results of the transition angles in a wide range of incident shock strengths and initial wedge angles.

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