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DNS OF TURBULENT HEAT TRANSFER IN CHANNEL FLOW: NEAR-WALL TURBULENCE QUANTITIES

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ABSTRACT

A direct numerical simulation (DNS) of turbulent heat transfer in channel flow was carried out for $Re_\tau = 180$ and 395 with various molecular Prandtl numbers. Turbulence statistics such as mean temperature profile, temperature variance, turbulent heat fluxes and turbulent Prandtl number were calculated and the near-wall behaviors were investigated.

INTRODUCTION

The recent DNS has enabled the near-wall turbulence quantities to be obtained with a sufficient accuracy. The DNS of turbulent channel flow was performed firstly by Kim et al. (1987) for $Re_\tau = 180$, where Re_τ is the Reynolds number based on the friction velocity u_τ and the channel half width δ. The same group (Kim-Moin, 1989) extended their DNS to include the scalar transport for the Prandtl number of 0.1, 0.71 and 2.0. Later, Kasagi and his co-workers (Kasagi et al., 1992; Kasagi-Ohtsubo, 1993) made the DNS for a slightly lower Re_τ of 150 with a low Prandtl number of 0.025.

Antonia-Kim (1991) analyzed the DNS data by Kim et al. (1989) and obtained various turbulence quantities in the near-wall region. Among them, they found that the turbulent Prandtl number Pr_t tends to a constant level irrespectively of the molecular Prandtl number as the wall is approached.

The present authors' group (Kawamura et al., 1997, 1998a) also performed the DNS for the same configuration with $Re_\tau = 180$ for various Prandtl numbers up to $Pr = 5$. More recently they (Kawamura et al., 1998b) carried out the DNS of the turbulent heat transfer in the channel flow with a higher Reynolds number of $Re_\tau = 395$, which was the highest Reynolds number ever calculated in conjunction with the scalar transport.

In this paper, The effect of the Reynolds and Prandtl number on the turbulent scalar transport is discussed based on the DNS.

NUMERICAL PROCEDURE

The fully developed turbulent channel flow was assumed. The flow was heated with a uniform heat flux from both walls (see Fig. 1).

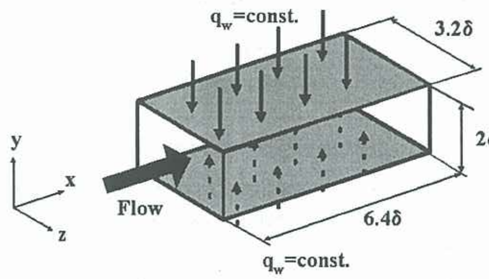

Figure 1: Calculation domain

The instantaneous temperature is converted to θ^+ by the following relation.

$$T^+(x^*,y^*,z^*) = \frac{d\left\langle \overline{T}_m^+ \right\rangle}{dx^*} x^* - \theta^+(x^*,y^*,z^*), \quad (1)$$

where θ^+ satisfies the uniform wall boundary condition of $\theta^+ = 0$. Wall value of the temperature fluctuation θ'^+ is also assumed to be zero.

The simulation was made with use of the finite difference method in which a special attention was paid to the consistency between the analytical and numerical differential operations (Kawamura -Kondoh, 1996). To perform a DNS with a higher Reynolds number, a larger grid size of 256 × 128 × 256 was adopted. The calculation was carried out with use of a parallel computer called as Numerical Wind Tunnel (NWT) located at National Aerospace Laboratory.

The computational condition is given in Table 1.

RESULTS

The obtained mean temperature profile is shown in Fig. 2. The logarithmic and the wake regions are

Table 1: Computational condition

Grid		Staggered Grid
Coupling Algorithm		Fractional Step Method
Time Advancement	Viscous Terms	2nd-order Crank-Nicolson Method (y-direction)
	Others	2nd-order Adams-Bashforth Method
Discretization Scheme	Nonlinear Terms	2nd-order Central Scheme (Consistent)
	Viscous Terms	2nd-order Central Scheme
Boundary Condition		Periodic(x, z direction), Non-slip (y-direction)
Grid Number		$128 \times 66 \times 128$, $256 \times 128 \times 256$
Computational Volume		$6.4\delta \times 2\delta \times 3.2\delta$
Reynolds Number		$Re_\tau = 180, 395$
Prandtl Number		$Pr = 0.025, 0.1, 0.2, 0.4, 0.71, 5.0$ $(Re_\tau = 180)$
		$Pr = 0.025, 0.2, 0.71$ $(Re_\tau = 395)$

better distinguished when Re_τ is 395 with $Pr = 0.2$ and 0.71. But, in case of $Pr = 0.025$, no logarithmic region seems to exist irrespectively of the Reynolds number because the conductive sublayer is thickened.

Taylor series expansion of the mean temperature $\overline{\theta}^+$ away from $y^+ = 0$ is (Antonia-Kim, 1991)

$$\overline{\theta}^+ = Pr \cdot y^+ - \frac{Pr}{6\langle \overline{u}^+ \rangle Re_\tau} y^{+3} + \cdots, \quad (2)$$

where $\langle \overline{u}^+ \rangle$ is the mean streamwise velocity averaged over the channel cross section. Figure 3 shows $\overline{\theta}^+/Pr$ versus y^+. The plot indeed tends to a straight line independently of the Prandtl number as the wall is approached. Thus, it can be confirmed that the above relation is satisfied.

The rms of the temperature variance θ'^+_{rms} is given in Fig. 4. Its peak value becomes less dependent upon Re_τ with increase of Re_τ when the Prandtl number is large as $Pr = 0.71$. On the other hand, for a smaller Prandtl number, the peak increases with the increasing Reynolds number.

The temperature fluctuation can be expanded in terms of y^+ as

$$\theta'^+ = b_\theta^* \cdot y^+ + \cdots. \quad (3)$$

No definite conclusion has been obtained on the dependence of $b_\theta{}^*$ on Pr. The present author (Kawamura et al, 1998) indicated that b_θ^* should be proportional to Pr. Thus, it becomes

$$\theta'^+ = Pr \cdot b_\theta \cdot y^+ + \cdots. \quad (4)$$

To confirm the above relation, θ'^+_{rms}/Pry^+ is plotted in Fig. 5. In case of $Re_\tau = 180$, the coefficient b_θ still depends upon Pr; but it indeed becomes almost independent of the Reynolds number when $Re_\tau = 395$. Thus, the expansion of Eq. (4)

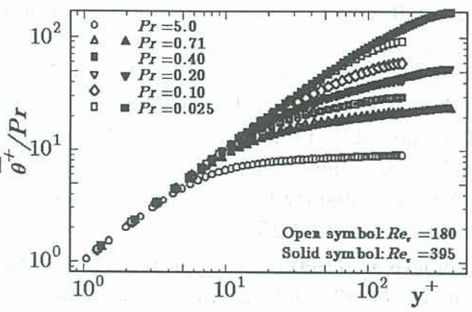

Figure 3: $\overline{\theta}^+/Pr$ profile

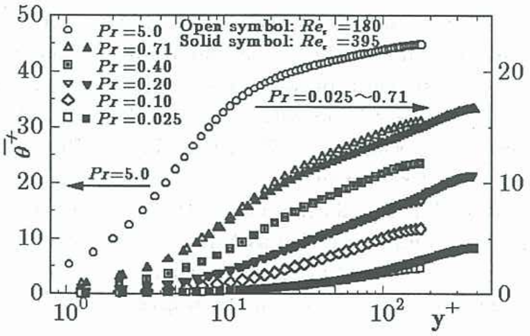

Figure 2: Mean temperature profile

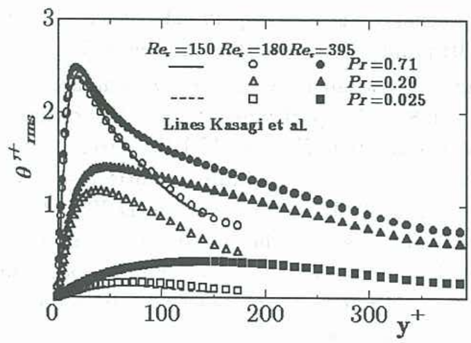

Figure 4: Rms of temperature variance

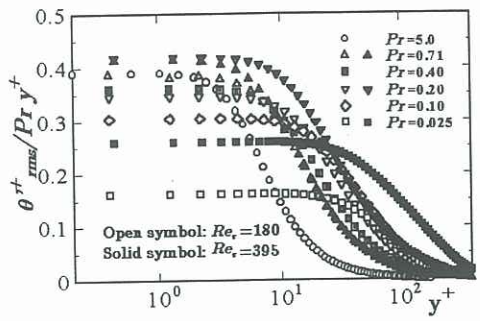

Figure 5: $\theta'^+{}_{rms}/Pry^+$ profile

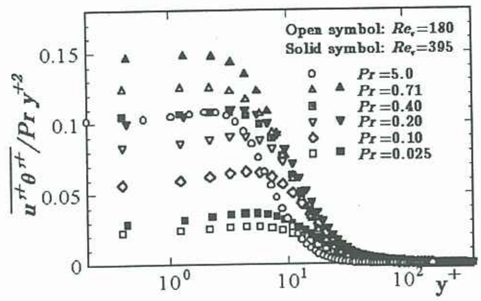

Figure 6: $\overline{u'^+\theta'^+}/Pry^{+2}$ profile

can be confirmed if the Reynolds and Prandtl numbers are higher than a certain limit. The present DNS indicates $b_\theta = 0.42$ for $Re_\tau = 395$.

The fluctuations of the streamwise and the wall-normal velocities are expanded as

$$u'^+ = b_1 \cdot y^+ + c_1 \cdot y^{+2} + \cdots \quad (5)$$

$$v'^+ = c_2 \cdot y^{+2} + \cdots, \quad (6)$$

where b_2 does not appear because of the continuity condition. Combination of Eqs. (4), (5) and (6) results in the following expansions.

$$\overline{u'^+\theta'^+} = Pr \cdot \overline{b_1 b_\theta} \cdot y^{+2} + \cdots \quad (7)$$

$$\overline{v'^+\theta'^+} = Pr \cdot \overline{c_2 b_\theta} \cdot y^{+3} + \cdots \quad (8)$$

Figure 6 shows the ratio of $\overline{u'^+\theta'^+}/Pry^{+2}$ versus y^+. If $Pr < 1.0$, the correlation coefficient $\overline{b_1 b_\theta}$ increases with increasing Re_τ and Pr. On the other hand, if $Pr > 1.0$, $\overline{b_1 b_\theta}$ decreases because the similarity between the velocity and the temperature field is lost with the increase of the Prandtl number. When the Prandtl number is $Pr = 0.71$, the estimated value of $\overline{b_1 b_\theta}$ is 0.12 and 0.15 for $Re_\tau = 180$ and 395, respectively.

A similar plot is given in Fig. 7 for the wall-normal turbulent heat flux $\overline{v'^+\theta'^+}$. The correlation coefficient $\overline{c_2 b_\theta}$ tends to a constant value as the wall is approached but not so exactly as the other correlations. This is because v'^+ tends to y^{+2} only very close to the wall ($y^+ < 1.0$) and the mesh resolution of the present calculation is not enough in this region.

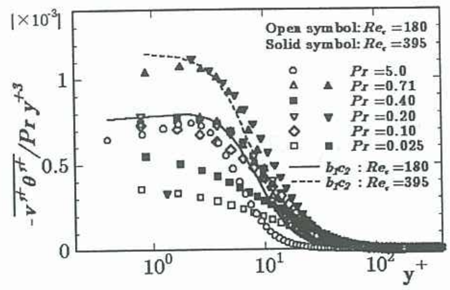

Figure 7: $-\overline{v'^+\theta'^+}/Pry^{+3}$ profile

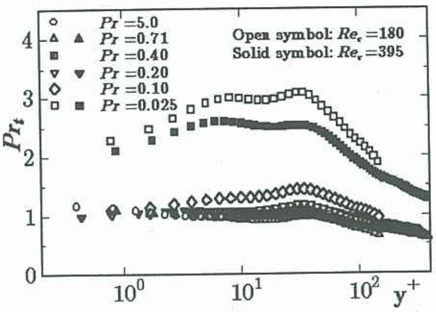

Figure 8: Distribution of turbulent Prandtl number

Nevertheless, it can be seen that the wall value of the coefficient $\overline{c_2 b_\theta}$ tends to a constant of about 0.0007 ± 0.0001 for $Re_\tau = 180$ and 0.0011 ± 0.0001 for $Re_\tau = 395$.

The turbulent Prandtl number is the quantity often used in the engineering heat transfer calculations. It is defined as

$$Pr_t = \frac{\nu_t}{a_t} = \frac{\overline{u'^+v'^+}}{\overline{v'^+\theta'^+}} \frac{\left(d\overline{\theta}^+/dy^+\right)}{\left(d\overline{u_1}^+/dy^+\right)}. \quad (9)$$

Since $d\overline{\theta}^+/dy^+ \to Pr$ and $d\overline{u}^+/dy^+ \to 1$ as the wall is approached, the wall value of the turbulent Prandtl number becomes

$$Pr_t = \frac{\overline{b_1 c_2} y^{+3} Pr}{Pr \overline{c_2 b_\theta} y^{+3}} = \frac{\overline{b_1 c_2}}{\overline{c_2 b_\theta}}. \quad (10)$$

Thus, it is independent of y^+ and Pr, as found by Antonia-Kim (1991) from their analysis of DNS data. The turbulent Prandtl number obtained from the present DNS is shown in Fig. 8. As discussed above, Pr_t tends to about 1.1 independently of Re_τ and Pr if Pr is large enough. This result is in good agreement with the previous numerical simulations by Kim-Moin(1989), Antonia-Kim(1990) and Kasagi et al.(1992). It is interesting to note that even in the case of low Prandtl number of $Pr = 0.025$, Pr_t becomes closer to 1.0 with increase of Re_τ.

The time scale ratio is the quantity defined as

$$R = \frac{\overline{\theta'^{+2}}\varepsilon}{2k\varepsilon_\theta} \quad (11)$$

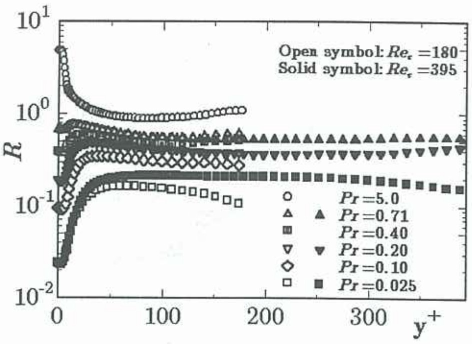

Figure 9: Distribution of time scale ratio

and is plotted in Fig. 9. This quantity is often used to estimate the dissipation rate of the temperature variance ε_θ.

The wall value of R is exactly equal to the molecular Prandtl number, which can be confirmed in Fig. 9. In the central region of the channel, R stays at a constant value and increases with increase of the Prandtl number.

CONCLUDING REMARKS

The DNS of turbulent heat transfer in channel flow was carried out for $Re_\tau = 180$ and 395 with various Prandtl numbers. Turbulence statistics were calculated to investigate the near-wall behaviors.

The conclusions are derived as follows:

1 The effect of the Reynolds and Prandtl number on the near-wall turbulence quantities was examined.

2 The near-wall expansions of the turbulence quantities were examined and their correlation coefficients were obtained for the Reynolds and Prandtl number calculated.

3 Turbulent Prandtl number was calculated and found that Pr_t tended to be about 1.1 independently of Re_τ and Pr for a larger Prandtl number. Moreover, in case of a low Prandtl number of $Pr = 0.025$, the effect of Re_τ on Pr_t is more enhanced.

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NOMENCLATURE

b_i, c_i, d_i	coefficient of expansion
Pr	molecular Prandtl number
Pr_t	turbulent Prandtl number
R	time constant ratio
Re_τ	Reynolds number$=u_\tau \delta/\nu$
T	temperature
T_m	bulk mean temperature
u_i, u, v, w	velocity component
u_τ	friction velocity$=\sqrt{\tau_w/\rho}$
x_1, x	streamwise direction
x_2, y	wall-normal direction
x_3, z	spanwise direction
Greek	
δ	channel half width
ε	dissipation of turbulent energy
ε_θ	dissipation of temperature variance
θ	transformed temperature
ρ	density
τ_w	wall shear stress
Superscripts	
$()^*$	normalized by δ
$()^+$	normalized by u_τ and ν
$()'$	fluctuation component
$\overline{()}$	statistically averaged
$\langle\rangle$	averaged over channel section

REFERENCES

ANTONIA, R., KIM, J., "Turbulent Prandtl number in the near-wall region of a turbulent channel flow" ,*Int. J. Heat Mass Transfer,* **34,** 1905-1908, 1991.

KASAGI, N., TOMITA, Y., KURODA, A., "Direct numerical simulation of passive scalar field in a turbulent channel flow" , *Transactions of the ASME,* **114,** 598-606, 1992.

KASAGI, N., OHTSUBO, Y., "Direct numerical simulation of low Prandtl number thermal field in a turbulent channel flow" , in: *Turbulent Shear Flows 8*, 97-119, Springer-Verlag, Berlin, 1993.

KAWAMURA H., KONDOH Y., "Application of consistent finite difference scheme to DNS of turbulent heat transfer in channel flow", *Proc. of the 3rd KSME-ASME Thermal Eng. Conf.*, Kyongju, Korea, **1**, 53-58, 1996.

KAWAMURA, H., OHSAKA, K., YAMAMOTO, K., "DNS of turbulent heat transfer in channel flow with low to medium-high Prandtl number fluid" , *Proc. 11th Symp. Turbulent Shear Flows*, Grenoble, **1** , 8.7-8.12, 1997.

KAWAMURA, H., OHSAKA, K., ABE, H., YAMAMOTO, K., "DNS of turbulent heat transfer in channel flow with low to medium-high Prandtl number" , *International Journal of Heat and Fluid Flow,* to be published , 1998.

KAWAMURA, H., ABE, H., MATSUO, Y., YAMAMOTO, K., "DNS of turbulent heat transfer in channel flow with respect to Reynolds-number effect" , *Turbulent Heat Transfer 2,* Manchester, **1,** 1.15-1.22, 1998.

KIM, J., MOIN, P., MOSER, R., "Turbulence statistics in fully developed turbulent channel flow at low Reynolds number" , *Journal of Fluid Mechanics,* **177** , 133-166, 1987.

KIM,J., MOIN,.P., "Transport of passive scalars in a turbulent channel flow" , in: *Turbulent Shear Flows* 6, 85-96, Springer-Verlag, Berlin, 1989.