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REAL 3-DIMENSIONAL SIMULATION OF NEAR-FIELD

L. Yu
Dept. of Hydraulics and Sanitary Engineering
School of Engineering at São Carlos, USP
São Carlos, SP
Brazil

A. M. Righetto
Dept. of Hydraulics and Sanitary Engineering
School of Engineering at São Carlos, USP
São Carlos, SP
Brazil

Y. Zhu
Dept. of Mechanical Engineerin
University of Newcastle
Newcastle, NSW
Australia

ABSTRACT
In the paper, a refined hybrid grid system adequate for illustrating and establishing the full three-dimensional model for flow and mass transport in the near-field of natural waters (the area of neighbourhood of the structures of hydraulic, environmental and coastal engineering) is presented. The governing equations of the hybrid grid algorithm in the conservative form are established. A tentative computational example modelled the flow fields in a reach of 90^0 curved natural river is reported. The computational results have shown that the hybrid grid has great potential to establish a powerful numerical analysis tool for refinedly modelling natural waters.

1. HYBRID GRID SYSTEM
The refined grid generation and numerical model of full three-dimensional modelling in the near-field of natural waters is an undeveloped problem in the case of both finite element and finite difference. The main difficulty which places restrictions on the development of more general algorithm and versatile computational programs for natural waters is the domain discretization, as the boundaries of natural waters do not match any frequently used orthogonal co-ordinate system. One of the urgent tasks for engineering computations is to search for a suitable grid system, which can most efficiently make use of geographical data such as the banks (shore lines) and the bottom topography, and can obtain expected computational results at lower cost. The frequently used full three-dimensional mathematical models for engineering computations usually adopt some cell-blockage techniques in all three spatial dimensions simultaneously, such a treatment obviously is only suitable in case of slight blockage. It is clear that having a reasonable co-ordinate system which matches the main boundary of natural waters is imperative. The paper presented a model adopted a refined, special designed grid system appropriate for illustrating three-dimensional natural waters, which includes part orthogonal body-fitted curvilinear co-ordinates in horizontal directions (Yu,L.,1989), and part Cartesians co-ordinate in the vertical direction. In the horizontal plane, the orthogonal body-fitted co-ordinate (BFC) system exactly fits shore lines; in the vertical direction, the blockage coefficients in the Cartesian co-ordinate system approximately fit the bottom topography (Yu,L. & Chen,D.,1992). This newly developed grid system correctly fits the accuracy of the data provided by engineering departments, at the same time, can achieve higher quality numerical results at lower cost. According to the value of water-depth corresponding to the solved co-ordinate location of each grid node, the blockage coefficients of each control volume can be determined easily. Two examples of the generated hybrid grid system (Yu,L.,1995) are shown in the paper. Figure 1 and Figure 2 show the generated three-dimensional perspective drawings of a 90^0 and a 180^0 curved river reaches under the hybrid grid system on physical space respectively.

2. GOVERNING EQUATIONS
In order to derive and establish governing equations under this grid system, it is convenient for us to assume the static water surface of natural waters is the physical plane (*xoy*), also the meshes of every horizontal levers are the same and independent of the water depth direction *z*. The time-independent transformation from the physical domain (x, y, z) to the transformed unit cube (ξ, η, ς) is then described by following functions:

$$\tau = t \qquad \xi = \xi(x,\ y) \qquad \eta = \eta(x,\ y) \qquad \varsigma = z / H \tag{1}$$

where the characteristic length H in the vertical direction z is no less than the maximum static water depth.

The fundamental governing equations of incompressible Newtonian fluid under the hybrid grid system, i.e. the continuity equation, the momentum equations in ξ, η and ς directions and the transport equation can then be expressed respectively. The momentum equation in ξ direction can be expressed as follows

$$\begin{aligned}
&\left(\frac{\rho u}{J}\right)_\tau + (\rho U u)_\xi - (C_1 u_\xi)_\xi + (\rho V u)_\eta - (C_3 u_\eta)_\eta + (\rho W u)_\varsigma - (C_5 u_\varsigma)_\varsigma = \left[\frac{\mu_{eff}}{Re}(u_\xi y_\eta - u_\eta y_\xi) y_\eta J - \frac{\mu_{eff}}{Re}(v_\xi y_\eta - v_\eta y_\xi) x_\eta J\right]_\xi \\
&\quad + \left[\frac{\mu_{eff}}{Re}(v_\xi y_\eta - v_\eta y_\xi) x_\xi J - \frac{\mu_{eff}}{Re}(u_\xi y_\eta - u_\eta y_\xi) y_\xi J\right]_\eta + \left[\frac{\mu_{eff}}{Re}\frac{1}{H}(w_\xi y_\eta - w_\eta y_\xi)\right]_\varsigma - [p_\xi y_\eta - p_\eta y_\xi] \\
&\quad + \rho g^x (1 - \beta \Delta T) / J
\end{aligned} \tag{2}$$

where u, v and w are velocity components in x, y, z directions; Re and J stand for Reynolds number and Jacobian; ρ, p and T denote the density, pressure and temperature; g^x, g^y and g^z are components of the acceleration due to gravity in x, y and z direction respectively; the effective eddy viscosity μ_{eff} is equal to $\mu + \mu_t$ with μ being the fluid kinetic viscosity and μ_t being the turbulent eddy viscosity; β is thermal expansion coefficient for temperature transport; the velocity components in ξ, η and ς directions and the coefficients, C_1, C_2 and C_3 are defined as following two equations

$$U = u y_\eta - v x_\eta \qquad V = v x_\xi - u y_\xi \qquad W = w / JH \qquad C_1 = \frac{\mu_{eff}}{Re} J\left(x_\eta^2 + y_\eta^2\right) \qquad C_3 = \frac{\mu_{eff}}{Re} J\left(x_\xi^2 + y_\xi^2\right) \qquad C_5 = \frac{\mu_{eff}}{Re}\frac{1}{JH^2}$$

All governing equations are discretized in the conformed computational grid (ξ, η, ς) by utilization of finite volume approach and pressure-velocity correction algorithm to solve iteratively the unknown variables u, v, w and p. An example, the flow field of a 90^0 curved river reach under the hybrid grid has been computed tentatively, in which the mean width and slope of the water surface within a 290 m long river reach are about 25 m and 0.00382 respectively, the flowrate is equal to 6.0 m^3/s, the longitudinal velocity is no more than 0.8 m/s. The nodal number of the computational grid adopted actually in conformed space has to be reduced from $183 \times 30 \times 10$ used for the grid generation to $92 \times 31 \times 10$ (Yu, L.,1995), in order to diminish the computer's storage and CPU time. The simple constant turbulence eddy viscosity was used in the computation to avoid the possible effect of the mediles of various turbulence models at current research stage. The value of eddy viscosity was evaluated by the authors according to the fluvial dynamics parameters.

3. NUMERICAL DETAILS

All governing equations were discretized in the conformed computational grid (ξ, η, ς) by utilization of finite volume approach to solve the unknown variables u, v, w, p and (or) T. The key for solving the discretized algebraic equations is how to solve pressure fields, or say, how to improve a solved pressure field. The widely adopted pressure-velocity coupling technique is one of the algorithms for improving obtained pressure fields (Patanker,S.V.,1980) The basic idea of the pressure-velocity correction algorithm is that the computed velocity components u, v and w, corresponding to a pressure field initially guessed or determined by last iteration cycle, usually do not exactly satisfy the mass conservation equation, and further improvement for the computed pressure field is necessary. By introducing the pressure-velocity coupling relation determined by the discretization formations of momentum equations into the discretized continuous equation, the so called pressure-correction equation can be derived (Yu,L.,1995). Using the corrected pressure values computed by the pressure-correction equation to correct the current velocity fields, the relative solution is satisfied with the continuous equation at the present iteration cycle. Taking the new velocity to make further improvement to the coefficients of discretized momentum equations, then the next iteration cycle starts and repeats itself until a pre-specified convergence is achieved.

4. COMPUTATIONAL RESULTS AND DISCUSSION

The computational results of the velocity fields by using uniform velocity distribution both in inlet and downstream section have been presented in Figure 3 and Figure 4 respectively. In this computation, the variation of bottom topography is approximately linearized within *10* grid lines in η direction from the inlet to outlet. In Figure 3, the velocity vector distribution on three different water depths are drawn respectively. In Figure 4, a three-dimensional perspective velocity field i.e. a cut-open view of the 90^0 curved river reach in physical space is presented, which was cut away along with the longitudinal axis of the river reach. Generally speaking, the configuration of the computed velocity distribution is reasonable. The sizes of the velocity vectors near banks and bottom are less than ones in the centre part; the directions of the velocity vectors are changed gradually along with the variation of the solid boundary. The velocity distributions both in the vertical and horizontal directions are coincident well with the logarithmic law, which agrees with the regulation of fluvial dynamics. Though it is clear that more numerical work is needed to set up a perfect full three-dimensional model under the hybrid grid system, however, a bright prospect of the hybrid grid method for modelling refinedly the real flows and transport phenomena of the near-field has appeared. The successive numerical tasks, in the opinion of the authors, should mainly include to investigate the suitable boundary conditions at inlet and outlet sections for full three-dimensional computations, to improve and develop the advanced algorithm(s) for accelerating the iteration convergence (single-block correction, double-block correction techniques, and so on) in the very mixture grid system and to understand the interaction of grid transformation and cell-blockage technique.

At present, the complex and irregular boundaries of computational domains in natural waters compel most mathematical models to keep to Cartesian or cylindrical coordinates in all three dimensions for numerical simulations with smaller scales, or to keep Cartesian or polar coordinates in horizontal directions and to use some simplified methods, such as rather coarse σ-coordinate (relative height method), even more coarse plat-bottomed assumption in the vertical direction for numerical simulations in the natural waters with larger scales. Generally speaking, frequently used full three-dimensional mathematical models for engineering computations adopted some cell-blockage techniques in all three spatial dimensions simultaneously. It is obvious that such a treatment is only suitable for the computations in case of slight blockage. In this paper, the specialized refined hybrid grid system suitable for fully simulating three-dimensional natural waters has been established. This mixture curvilinear grid system can strictly satisfy orthogonal and conformal relations simultaneously and can make the non-simplified governing equations of fluid flow and mass transport as well as their corresponding boundary conditions in computational domains relatively simpler. Such a mathematical model has great potential to be developed further as an essential numerical model to make use of advanced higher order turbulence closure models. In the hybrid grid system, the blockage coefficient technique is merely employed in the vertical dimension, and the advanced orthogonal BFC technique, which can prearrange the grid nodes on a pair of adjacent boundaries without constructing any special functions, has been utilized to transform irregular domains of natural waters into a regular square on horizontal plane. It is no doubt that the hybrid grid system will reduce the computational storage and cost greatly, especially for full three-dimensional unsteady tidal flow computations.

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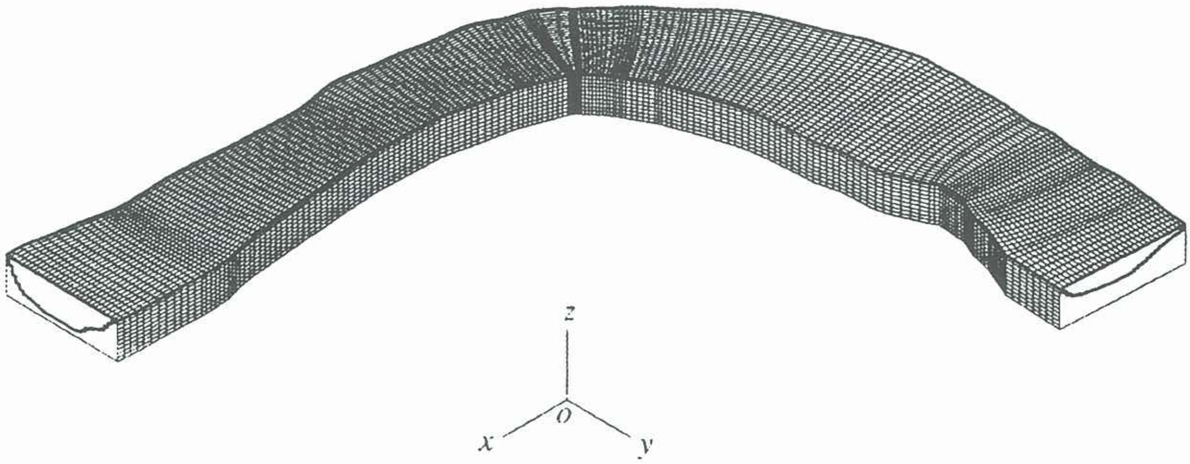

FIGURE 1 *90⁰* CURVED REACH UNDER HYBRID GRID SYSTEM

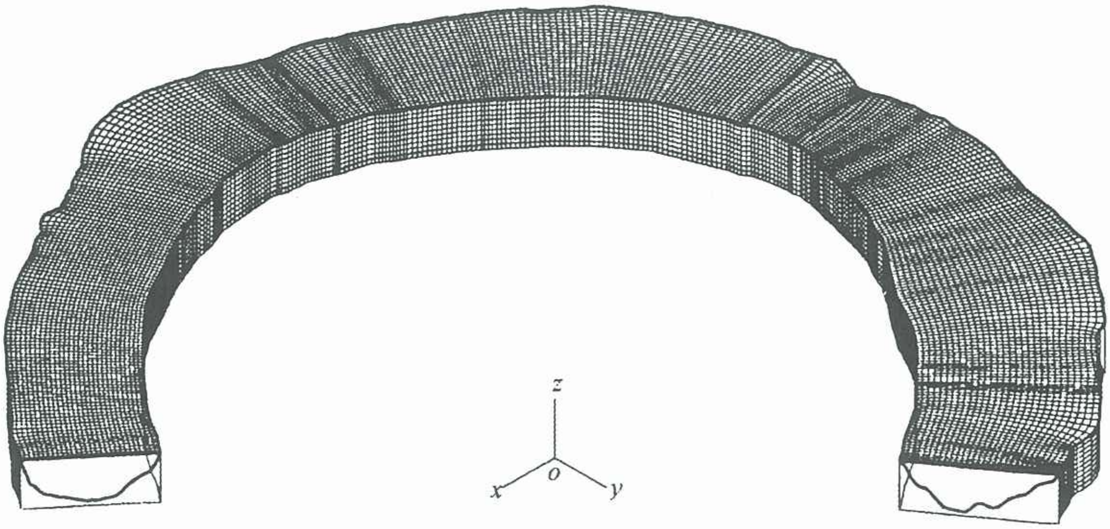

FIGURE 2 *180⁰* CURVED REACH UNDER HYBRID GRID SYSTEM

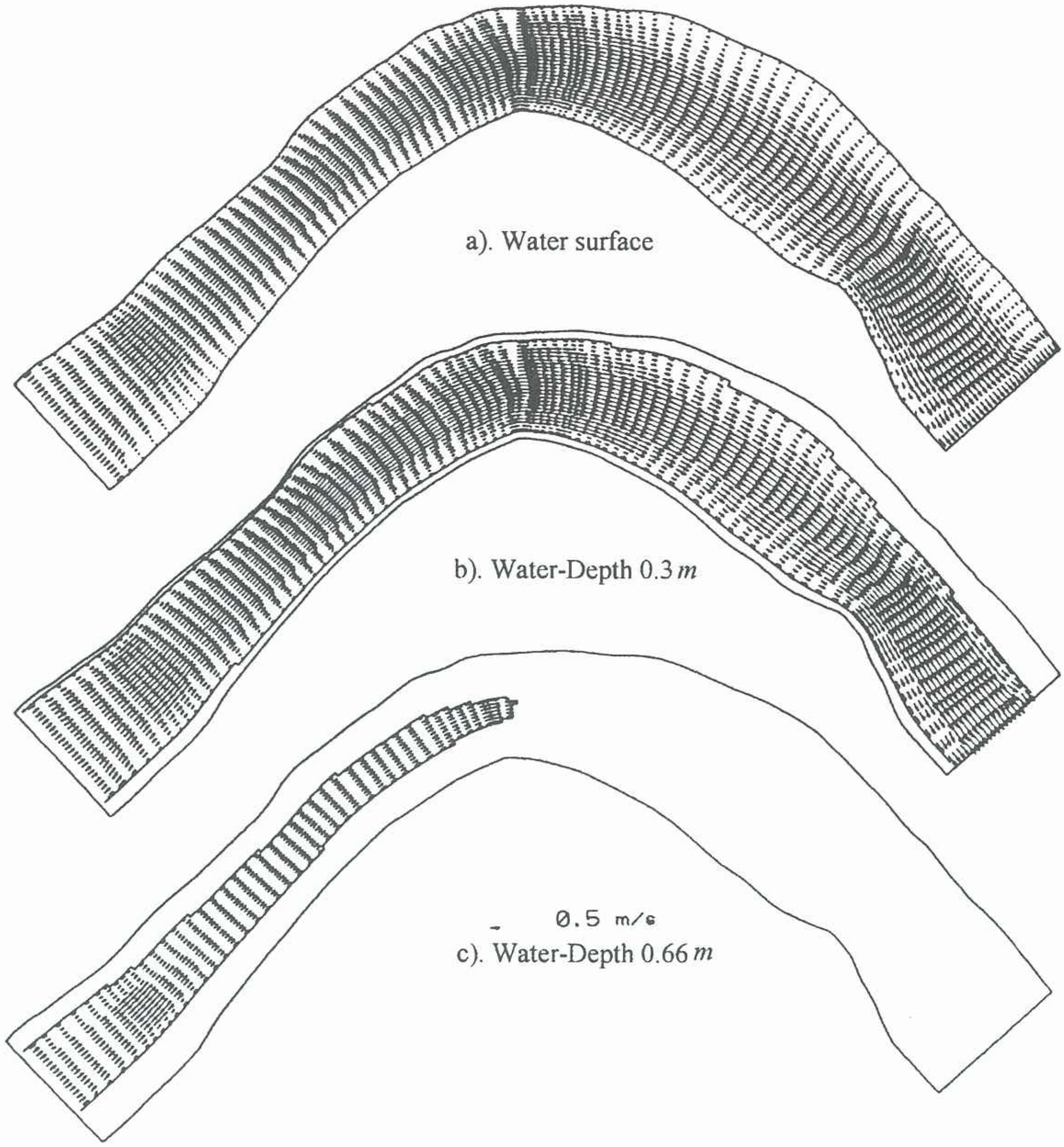

FIGURE 3 VELOCITY FIELDS IN DIFFERENT WATER-DEPTHS CALCULATED BY HYBRID GRID ALGORITHM

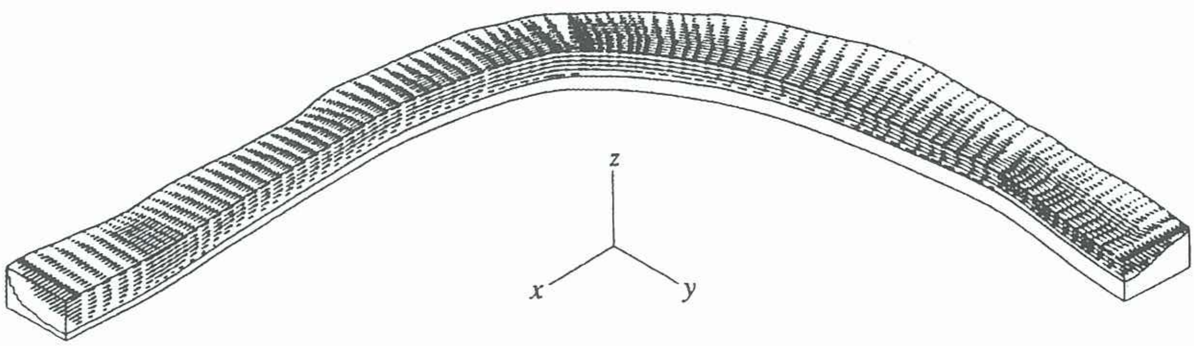

FIGURE 4 PERSPECTIVE DRAWING OF VELOCITY FIELD IN PHYSICAL SPACEof