AN EMPIRICAL MODEL TO EXPLAIN TURBULENT FLOW IN A CONICAL DIFFUSER

Rakesh K. Singh and Ram S. Azad
Department of Mechanical and Industrial Engineering
University of Manitoba
Winnipeg, Manitoba
Canada

ABSTRACT

A new tentative model has been proposed for the complex turbulent flow in an 8° nominal total divergence angle near-optimum-geometry conical diffuser fed with a fully developed pipe flow. This qualitative model is able to explain the behaviour of initiation and growth of instantaneous backflow, development of inflectional streamwise mean velocity profiles, high relative intensities of wall shear stresses, local static pressure profiles, distribution of Reynolds shear stress and Reynolds normal stresses, and dramatic behaviour of skewness and flatness factors of streamwise velocity fluctuations along the conical diffuser centreline. Further experimental investigations of two point correlations and vorticity field will be useful to characterize the proposed instantaneous toroidal vortices evolution in a conical diffuser flow.

INTRODUCTION

In an axisymmetric diffuser the flow is subjected to sudden multiple perturbations of strong streamwise adverse pressure gradient (APG), concave streamline curvature, and lateral divergence. In boundary layer flows, a sudden application of APG has been found to generate extra strain rates and affect inner-layer velocity-gradient and turbulent stresses immediately, resulting in a reorganization of eddy structure and formation of roll cells [1-4]. However, the combined influence of different perturbations can not be assumed to be a simple summation of the separate effects of above perturbations [2-3]. For example, in Smits et al. [5] axisymmetric cylinder-flare body experiment the flow experienced the combined effects of concave curvature and streamline divergence. Both these effects were destabilizing, yet the combined effect was considerably less than might be expected from each effect acting separately and no longitudinal roll-cells were detected in this study. Also, in the experimental investigation of an axisymmetric swirling turbulent mixing layer (Wood et al. [6]) no evidence of identifiable longitudinal vortices similar to the Taylor-Görtler vortices could be found.

In the initial region of a conical diffuser concave streamwise curvature and appreciable crossflow mean velocities (V) exist. Baskaran et al. [3] pointed out that the stabilizing effect of crossflow opposes the destabilizing influence of concave curvature in three-dimensional boundary layers. Bassom and Hall [7] have also shown conclusively that at sufficiently large values of crossflow there are no unstable Görtler vortices in a boundary layer. However, the results obtained in one geometry can not be extended to another geometry as the resulting interaction of multiple perturbations in various flow configurations may be totally different and facility dependent. Therefore it is essential to obtain sufficient experimental evidence in different flow geometries to get an insight into the physics of turbulence. An empirical model has been developed based on the recent extensive experimental study of a conical diffuser flow [8]. A description of this model is presented in the next section followed by a discussion of the supporting results.

DESCRIPTION OF THE MODEL

The development of unswirled mixing layers have been shown to be influenced by the formation and interaction of large-scale spanwise vortices [9-11]. These spanwise vortex rolls can maintain themselves in the presence of fully-three-dimensional background turbulence [11]. The present study demonstrates some new aspects of the turbulent vortex structure in an instantaneously-reversing conical diffuser flow with free discharge and nonswirling incompressible inlet flow. The recent results of quantitative instantaneous backflow distribution and the mean and turbulent flow-field [12-15] in a conical diffuser revealed the possible existence of instantaneous spanwise-rotating toroidal vortical structures in the wall-layer (Fig. 1). As the flow enters in a conical diffuser, instantaneous toroidal vortical structures may evolve in the wall-layer, as a result of the Kelvin-Helmholz instability. These toroidal vortices remain bounded on the outer side by the conical diffuser wall and grow rapidly in size as the flow proceeds downstream.

Figure 1 shows that the instantaneous toroidal vortices may have elliptical (or similar elongated structure) cross-section with one end of the major axis inclined forward in the direction of the flow and other end in the vicinity of the wall. The angles of inclination may be larger than 45° (from the diffuser wall) initially which increases and approaches 70-80° as the flow moves further downstream and approaches diffuser exit, as will be discussed later. This inclination of major axis is consistent with the measured profiles of instantaneous backflow [12] and local radius at various stations of the present diffuser. In a conical diffuser flow, the spanwise mean vorticity remains concentrated in the wall-layer. As the flow proceeds downstream, the spanwise vorticity profiles also develop a local maxima in the region away from the wall and the centre of the elliptical cross-section toroidal vortices rapidly moves away from the wall, coinciding approximately with the local Reynolds shear stress maxima, outer-region peak spanwise mean vorticity, and the outer extremities of the instantaneous backflow region (Fig. 1). The rapid growth of vorticity thickness and very high values of cross-stream pressure gradient in the latter part of the present diffuser also support the possible existence of spanwise vortical structures.

It is well known that the curvature extra strain rate $\partial V/\partial x$ affects the spanwise mean vorticity [3]. In a recent extensive direct numerical simulation of incompressible turbulent mixing layers, Rogers and Moser [16] pointed out that extra strain rate $\partial V/\partial x$ will be negative where there are rollers (rotation dominant clumps of spanwise vorticity) and will be positive where there are braids (strain dominant thinner regions that have less spanwise vorticity). The present results support the above observation of Rogers and Moser. It may be noted that in the present conical diffuser no instantaneous backflow was detected (at 0.05 mm from the wall) at station 1, $x=0.046$ m (Fig. 1). Also, the strain rate $\partial V/\partial x$ was found to have a positive value in this region of a conical diffuser of similar geometry (Azad and Kassab [17]). These results would suggest that no rollers were present in the wall layer of the initial stages of the diffuser and this is a region of braids. As the flow moves downstream the strain rate $\partial V/\partial x$ becomes negative and instantaneous backflow is detected about 1% of the time (at 0.05 mm from the wall) at station 2, $x=0.156$ m, indicating this latter part of the diffuser flow to be the region of rollers.

Rogers and Moser [16] also concluded that linear growth and self-similarity in experiments are observed by averaging statistics from the passage of many rollers, which are at varying stages in their evolution. In the present diffuser, the rapid growth of the wall layer may be due to passage of many instantaneous toroidal shape elliptical-cross-section roller vortices in different stages of their growth and a random pairing process of adjacent spanwise vortices. Rogers and Moser further indicated that an elliptical vortex with a vertical major axis produces more vertical velocity fluctuations than a vortex with a horizontal major axis. In the present diffuser the radial velocity fluctuations increase as the flow moves downstream, suggesting that the elliptical-cross-section vortices major axis may have higher inclination angle with the diffuser wall in the later stages, as discussed earlier.

SUPPORTING RESULTS

The present model is strictly a qualitative one used to aid interpretation of the flow-field in an incipient-separating conical diffuser. However, at this stage the assumed toroidal shape and elliptic cross-section of the instantaneous vortices has been able to correctly explain the behaviour of various quantities including: the initiation and growth of instantaneous backflow, its profiles at various axial stations, its development along the wall, and the outer extremities of its regions [12]; the development of inflectional streamwise mean velocity profiles at various axial stations [13]; the distribution of Reynolds shear stress; the development of relative intensity, skewness, and flatness factors of streamwise velocity fluctuations, u'/U, S_u, and F_u, respectively, along the diffuser centreline (including their peculiar behaviour in the final stages and the slight reduction of magnitudes of S_u and F_u near diffuser exit in Fig. 2) and at various axial stations [14]; the extremely high relative intensities of wall shear stress [12]; and the local static pressure profiles at various stations in the conical diffuser.

In Fig. 2, the rapid growth in the magnitudes of u'/U, S_u, and F_u along conical diffuser centreline in the latter part of the diffuser can be explained by the proposed toroidal vortices model. This may occur in the downstream region from the location where the wall-layer toroidal structures start influencing the centreline region of the conical diffuser from all radial directions, i.e. the inner diameter of the instantaneous toroidal structures becomes zero and the elliptical cross-section vortices from all radial directions start interacting in the central region of the conical diffuser flow. In this latter stages of the diffuser flow, the centreline region will intermittently receive large amount of low momentum fluid from the wall region (transported by the instantaneous toroidal vortices), resulting in high negative values of skewness factor S_u and high values of flatness factor F_u, as shown in Fig. 2. Figures 1 and 2 also suggest that as the instantaneous vortices propagate downstream, in the regions close to the diffuser exit, the centreline region may not receive increasing large amounts of low momentum fluid from the wall region (due to forward inclination and possible truncation of the vortices structure near diffuser exit), resulting in slightly reduced magnitudes of skewness S_u and flatness F_u near diffuser exit. The results of momentum and turbulence energy balances obtained in this conical diffuser [8] also seem to support the present empirical model.

The instantaneous spanwise-rotating toroidal vortical structures may evolve in a conical diffuser flow as a consequence of the introduction of high-momentum flow from the pressure side beneath the low momentum fluid of the wall-layer. In the final stages of the diffuser, the instantaneous vortex structures may have a cross-section dimension approximately equal to the local radius of the diffuser. In this region, the relative strength of large eddies was found to be nearly constant, indicating the asymptotic nature of turbulent structure in the final one-third length of a conical diffuser of similar geometry [17]. The integral length scale was also found to increase slowly in this latter stages of the conical diffuser flow [13]. Townsend [18] proposed that after a sufficient development distance and at sufficiently high Reynolds number, all mixing layers achieve a self-similar condition. It is important to note that the mean velocity profiles in the present diffuser also achieve an asymptotic velocity defect profile [13] in the final one-third length.

The above results show that the proposed empirical model based on instantaneous spanwise-rotating elliptic-cross-section toroidal vortices in the wall-layer is able to explain various characteristics of instantaneous flow reversals as well as mean and turbulent flow-field in the present conical diffuser. Further investigations such as the measurements of two-point correlations and vorticity-field in near-optimum-geometry conical diffuser flows will be valuable in giving greater insight into the nature of complicated vortex dynamics in severe APG axisymmetric flows with appreciable instantaneous reversals and extremely high turbulence intensities.

ACKNOWLEDGEMENT

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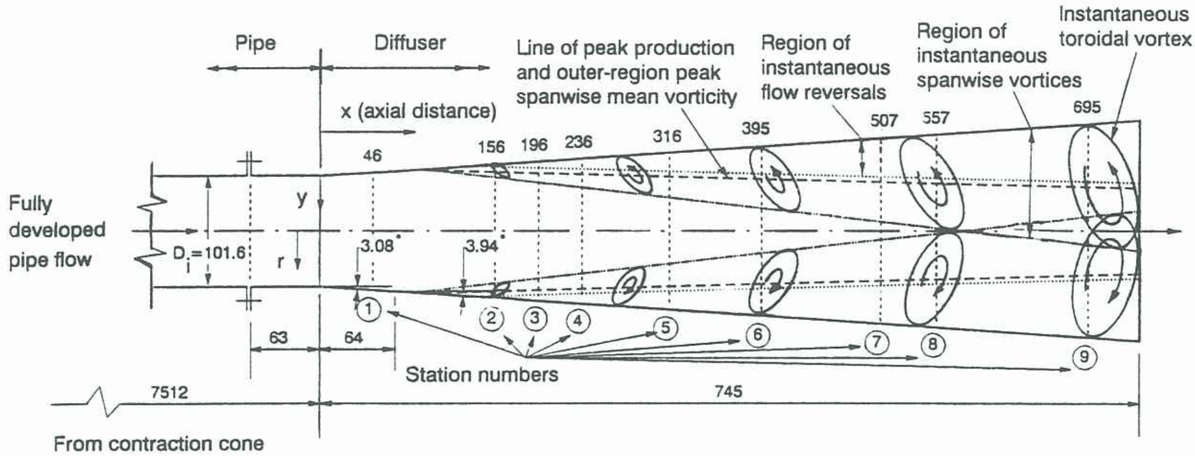

Figure 1. Schematic sketch of the instantaneous elliptical-cross-section toroidal vortices evolution in the conical diffuser flow.

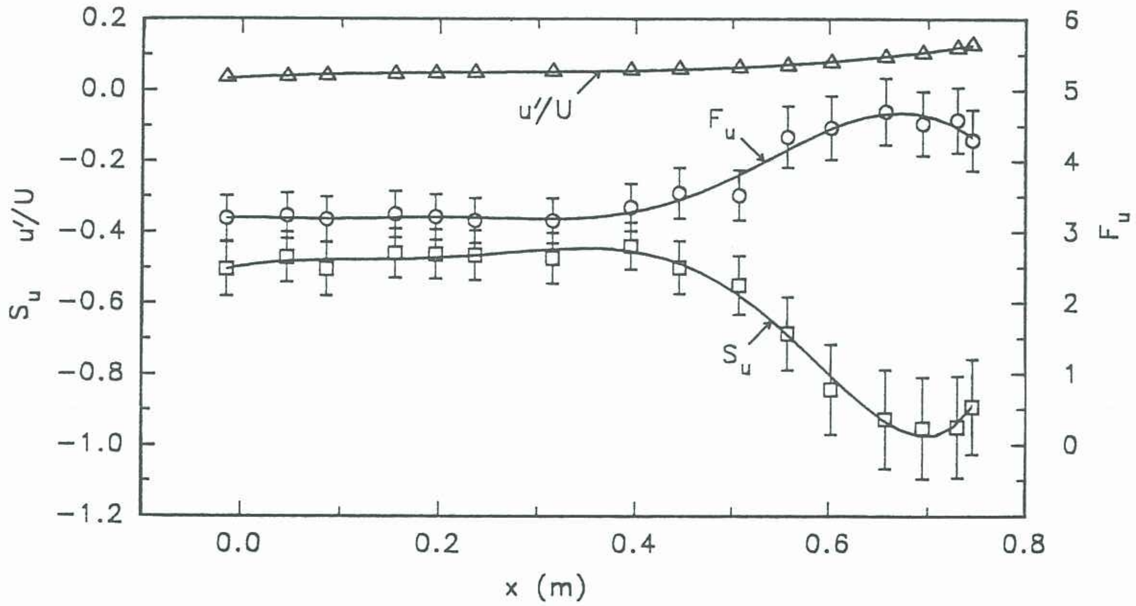

Figure 2. Distribution of relative intensity, skewness factor, and flatness factor of streamwise velocity fluctuations along conical diffuser centreline. Error bars represent band of repeatability. Lines merely indicate the interpreted trend.