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MIXING IN THE WAVY VORTEX REGIME OF TAYLOR-COUETTE FLOW

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ABSTRACT

The flow regimes found in a narrow-gap Taylor-Couette vessel with the inner cylinder rotating are simulated using a second-order numerical method. The flow conditions span the range from wavy vortex flow (with only one azimuthal frequency), through modulated wavy vortex flow, to flow that can be considered weakly turbulent. The chaotic advection of massless marker particles is calculated, and statistical evidence is presented that suggests that the axial mixing found in all of these flow regimes can be modelled as a diffusion process. Additionally, the effective axial diffusion coefficient appears to be linearly proportional to the Reynolds number based on the inner cylinder rotation velocity and the gap width between cylinders. The motion of dense particles is also considered and it is found that the mean settling velocity of an ensemble of dense particles is of the same order as their free settling velocity – a surprising result that is not expected from results obtained from axisymmetric vortex flow.

INTRODUCTION

Taylor-Couette flow is the flow that occurs between two concentric cylinders, either or both of which are allowed to rotate. The flow has been studied extensively since the seminal work of Taylor (1923), and new flow regimes are regularly discovered. Andereck *et al.* (1986) classify the possible regimes in their vessel into eighteen primary regimes, with some coexisting. When only the inner cylinder rotates, there are 5 primary flow regimes:

- cylindrical Couette flow,
- axisymmetric Taylor vortex flow,
- wavy vortex flow,
- modulated wavy vortex flow
- and turbulent vortex flow.

In the axisymmetric Taylor vortex regime, fluid particles are constrained to lie upon a toroidal surface, and the only global mixing possible results from molecular diffusion. Little work has previously appeared in the literature examining particle trajectories or mixing in wavy and modulated wavy vortex flow. Broomhead and Ryrie (1988) performed a theoretical analysis on a simple model of a wavy vortex flow and showed that the presence of even a small wave amplitude on top of an axisymmetric vortex flow would be sufficient to introduce chaos into the system. This would have a significant impact on particle paths and the mixing in wavy vortex flow. Recent experimental results reported by Moore and Cooney (1995) suggest that axial dispersion in Taylor-Couette vessels varies almost linearly with the rotational Reynolds number for a wide range of aspect ratios, through-flow Reynolds numbers and rotational rates – even for rotation rates below the onset of waviness. This result is somewhat surprising and suggests that either the non-axisymmetric inflow and outflow arrangements used in their vessel or the presence of the throughflow (or both) are having a significant effect on the dispersion in their experiments.

The study reported here originates in an industrial application in which a through-flow Taylor-Couette vessel was used to shear a suspension of particles. For certain operating conditions it was observed that particles appeared to have a much shorter residence time than the the nominal liquid residence time. Numerical simulations of the flow regimes found in the industrial study were therefore undertaken in order to understand why the phenomenon was observed and to estimate the solids' residence time in the vessel. As a first step, the small (axisymmetric) throughflow applicable to the industrial vessel was neglected.

A MEASURE OF MIXING

Once Taylor-Couette vortex flow ceases to be axisymmetric, fluid particles are no longer constrained to lie upon a toroidal surface (and hence within one vortex) and are transported in the axial direction, moving from vortex to vortex. From a statistical point of view, particles appear to diffuse in the axial direction, although the process that causes this movement is chaotic advection, not diffusion. Ottino (1989) states that 'particle paths, streamlines and (to a lesser extent) streaklines are not sufficient to give a good picture of mixing'. Determining chaotic particle paths is problematic because small numerical errors will result in large errors in individual particle positions after long times. However, Metcalfe (1995) has suggested that although individual particle paths may be in considerable error, the ensemble of a large number of particle paths can give reliable statistical information, and that the numerical results obtained from particle tracking can be statistically robust.

Following Broomhead and Ryrie (1988), a measure of mixing is defined using an *effective* particle diffusion coefficient which is estimated on the basis of particle paths. Because particles are constrained to lie between the inner and outer radii of the vessel, and because they are advected by a mean flow in the azimuthal direction, only an axial (z) diffusion coefficient is considered here.

For a purely random process the *r.m.s.* displacement for an ensemble of particles can be related to a diffusion coefficient. Defining

$$D'_z(t) = \frac{<(z(t) - z_(0))^2>}{2t},$$

where $< Q >$ denotes the *r.m.s.* value of Q, an effective axial diffusion coefficient is defined by

$$D_z = \lim_{t\to\infty} D'_z(t) = \lim_{t\to\infty} \frac{<(z(t) - z(0)^2>}{2t}, \qquad (1)$$

In axisymmetric vortex flow the effective axial diffusion coefficient (eqn. 1) tends to zero as time becomes very large, which states there is no global fluid mixing. Once the flow ceases to be axisymmetric, fluid particles move between vortices and the possibility of efficient global mixing arises.

NUMERICAL METHOD

The numerical method is a three-dimensional finite difference code based on the Marker and Cell (MAC) method and utilises cylindrical coordinates. A nominally third order Flux-Corrected Transport (FCT) methodology is used which ensures stability of the scheme. Time-stepping is by way of an improved Euler algorithm which is second-order in time. Particle trajectories are calculated using bi-quadratic interpolation and second-order time integration. Because the flow is in general unsteady (even when referred to a rotating coordinate system) the fluid equations must be solved at each time-step before updating the particle positions. The full solution must be followed for tens of thousands of time-steps to provide meaningful statistical information.

Visual inspection of the industrial vessel suggested that the azimuthal wavelength of the wavy vortex flow was approximately one sixteenth of the circumference of the shear vessel and that the axial extent of a vortex pair was approximately 2.5 times the gap width between cylinders. Hence, a computational domain of one gap width in the radial direction, 2.5 gap widths in the axial direction and one sixteenth of the circumference in the azimuthal direction is used for all simulations here. Although it is well known that the azimuthal and axial wavelengths of wavy vortex flow depend on the Reynolds number, and further that for a fixed Reynolds number there is no unique combination of wavelengths (Coles 1965), a fixed domain size and aspect ratio was used here for convenience. Fixing the domain size and aspect ratio will determine the exact flow state predicted by the simulations. The effect this will have on the mixing statistics is unknown and must be invetsigated further.

The ratio of gap width to inner cylinder radius is 1:20, which corresponds to a radius ratio $\eta = 0.952$. Boundary conditions are no-slip on the inner and outer cylinder walls and periodic in both the axial and azimuthal directions. The computational domain is thus repeated infinitely in both θ and z. The mesh size is $32 \times 80 \times 80$ or approximately 200 000 nodal points. Initially, 20 000 particles (5 000 fluid particles and 5 000 each of three dense particle types) are scattered randomly in the computational domain and are followed using the particle tracking algorithm discussed in Rudman et al. (1994). The initial flow conditions used for particle tracking are determined by running the simulation until a statistically steady flow pattern has been obtained.

RESULTS

Four values of Re_I $(= V_I d/\nu)$ are considered, $Re_I =$ 524, 1047, 2094 and 4188 which correspond to operating conditions in the experiment of 12.5, 25, 50 and 100 r.p.m. for a liquor viscosity equal to that of water. A value of $Re_I = 524$, corresponds to wavy vortex flow with only one azimuthal frequency, $Re_I = 1047$ and 2094 correspond to modulated wavy vortex flow and $Re_I = 4188$ corresponds to weakly turbulent flow. Although the numerical resolution was probably insufficient to accurately resolve all scales of motion in the case of $Re_I = 4188$, the results are included in subsequent discussion. The effective diffusion coefficient calculated in this case must be treated with caution, although it is more likely to be underpredicted than overpredicted by the simulation.

Fluid Particles

Plots of the initial fluid particle positions (projected onto an axial plane) and subsequent particle positions for $Re_I = 1047$ and 2094 after two rotations of the inner cylinder (or approximately 6-8 vortex turnover times) are shown in figure 1. It is clearly seen that fluid particles disperse widely in the axial direction in a very short time. Circumferential mixing has also been considerable, and after two rotations of the inner cylinder, fluid particles that started in a region one sixteenth of the total circumference of the vessel are dispersed around the entire circumference (not shown).

The normalised number density of particles per unit length is shown in figure 2. The initial particle distribution is sharply peaked about zero, and the other curves for particle number density after 2 revolutions of the inner cylinder are also shown. Although the curves are not identical, they are similar and their Gaussian-like shape suggests that particle dispersion in these flow regimes is behaving like a diffusion process.

A plot of the non-dimensional $D'_z(t)$ for the four values of Re_I is shown in figure 3, (plotted as a function of number of rotations of the inner cylinder). Although the results shown do not prove that $\lim_{t\to\infty} D'_z(t)$ is non-zero, they do suggest that it is approaching an asymptotic value of approximately 0.003. However, the simulations must be continued for a considerable time before a value could be stated with confidence.

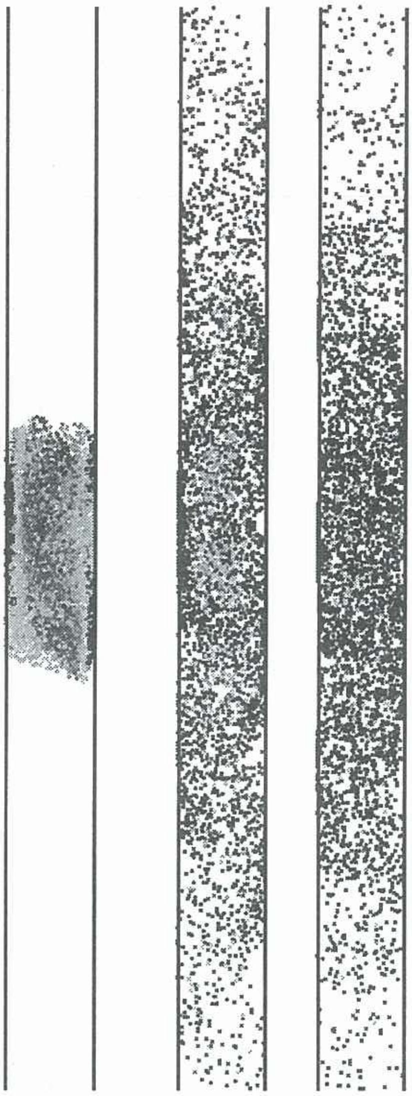

Figure 1: PARTICLE POSITIONS (PROJECTED ONTO AN AXIAL PLANE) FOR FLUID PARTICLES. FROM LEFT TO RIGHT: INITIAL PARTICLE POSITIONS AND POSITIONS AFTER 2 ROTATIONS OF THE INNER CYLINDER FOR $Re_I = 1047$ AND 2094. THIS FIGURE SHOWS THE CONSIDERABLE AXIAL MIXING THAT RAPIDLY OCCURS IN THESE FLOW REGIMES.

More interesting is that the non-dimensional $D_z(t)$ behaves almost identically for the values of Re_I considered here. This spans the flow regimes from pure wavy vortex flow, $Re_I = 524$, to weakly turbulent vortex flow, $Re_I = 4188$. The results also suggest that D_z appears to be independent of Re_I.

To obtain a dimensional value of the effective diffusion coefficient, D_z must be scaled by dU, (where U is the velocity of the inner cylinder and d the gap width.) The results therefore suggest that the dimensional diffusion coefficient scales approximately linearly with Re_I. This means that chaotic advection of fluid particles depends purely on rotation rate of the vessel – the important result here is that increasing the rotation rate by a given factor reduces the time taken to achieve the same degree of axial mixing by a similar factor.

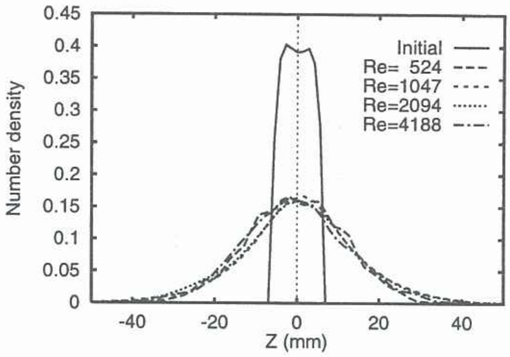

Figure 2: NORMALISED NUMBER DENSITY (PARTICLES/UNIT z-LENGTH) VS. z FOR FLUID PARTICLES. THE SOLID LINE IS THE INITIAL NUMBER DENSITY AND THE DASHED LINES ARE FOR $Re_I =$ 524, 1047, 2094 AND 4188 AFTER 2 ROTATIONS OF THE INNER CYLINDER. THE NUMBER DENSITY IS NORMALISED TO GIVE UNIT AREA UNDER THE GRAPH.

In an experimental study of diffusion in turbulent Taylor-Couette flow, Tam and Swinney (1987) find that the effective axial diffusion coefficient scales like Re_I^{β} where β depends primarily on the the radius ratio, η. Tam and Swinney only consider the range $0.494 < \eta < 0.875$ and find that as η increases so does β. For their smallest radius ratio ($\eta = 0.494$) they find $\beta \approx 0.7$ and for their largest radius ratio ($\eta = 0.875$) they find $\beta \approx 0.85$. It may be that in the limit as $\eta \rightarrow 1$, $\beta \rightarrow 1$, although this is purely conjecture at this time. However, the result of the simulations presented here (that $\beta = 1$ for $\eta = 0.952$) is not inconsistent with Tam and Swinney's results. It also must be kept in mind that Tam and Swinney's experiments were truly turbulent vortex flow, whereas the simulations here are primarily for laminar wavy vortex flows. In contrast to Tam and Swinney's results are those of Moore and Cooney (1995) who suggest that for a wide range of parameters (including η), D_z should scale like $Re_I^{1.05}$. These latter results are for a through-flow vessel with non-axisymmetric inlet and outlet, and once again are not directly comparable to the results presented here. Nevertheless, the current results are not inconsistent with Moore and Cooney's.

Heavy Particles

The transport of heavy particles in wavy vortex flow regimes instigated this study, and particle trajectories are also presented for three typical dense particle types. The particle free-settling velocities are chosen to be 40, 10 and 2.5 m/hr or approximately 1.11, 0.28 and 0.07 cm/sec. A velocity of 40 m/hr (1.11 cm/sec) corresponds to 0.03 times the rotational velocity of the inner cylinder for $Re_I = 524$ and 0.00375 times the inner cylinder velocity for $Re_I = 4188$.

The mean settling velocity of the three different ensembles of particles after two rotations of the inner cylinder is presented in table 1. Although there is considerable axial dispersion in the distribution of dense particles of all types (not shown), the surprising result is that the mean particle-settling velocity is of

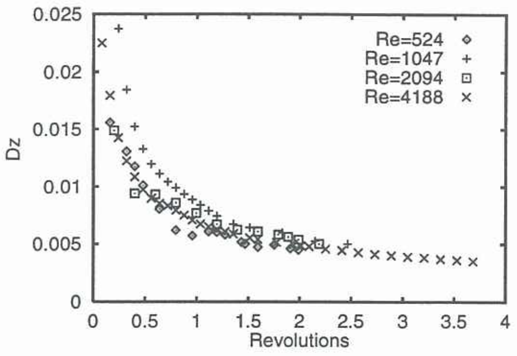

Figure 3: $D'_z(t)$ VERSUS NUMBER OF REVOLUTIONS OF THE INNER CYLINDER.

Table 1: Mean settling velocity (in m/hr) based on mean distance settled after two rotations of the inner cylinder. V_S is the nominal free settling velocity (40, 10 and 2.5 m/hr).

Re_I	524	1047	2094	4188
$V_S = 40.0$	39.9	46.8	49.3	48.4
$V_S = 10.0$	10.0	10.1	7.9	12.0
$V_S = 2.5$	2.6	2.2	2.0	3.6

a similar magnitude to (and sometimes greater than) the particles' nominal free-settling velocity. The same settling effect was inferred from results obtained from the experimental vessel.

It has been shown (Marsh and Maxey 1990, Rudman et al. 1994) that in 2-D recirculating cellular flows (which include axisymmetric Taylor-vortex flow) some global settling of dense particles can occur for any ratio of settling to cellular velocity. However when the settling velocity is significantly less than the vertical component of the cellular velocity, the proportion of particles that are able to settle from the flow is very small. In this case, the majority of particles are trapped inside retention zones.

In Taylor-Couette flow, the axial component of the vortex velocity increases almost linearly as Re_I increases. If the flow were to remain axisymmetric, as Re_I increased the proportion of retained heavy particles would also increase and the average settling velocity of an ensemble of dense particles would tend to zero. After the onset of waviness and the consequent mass transfer of fluid between vortices, this result is unlikely to hold, with dense particles able to settle with respect to the fluid at the same time as being chaotically transported by the flow. The net result of gravitational settling, centrifugal settling and chaotic transport is not possible to predict *a priori*. It is seen here that their combination results in the vortex recirculation having little adverse affect on a dense particle's ability to settle in the mean. The estimates of mean settling velocity obtained here allow an estimate to be made of the mean residence time of different sized particles in the experiment.

CONCLUSIONS

The presence of modulated wavy vortices gives rise to efficient mixing in narrow-gap Taylor-Couette vessels. Even though the flow is laminar, the unsteady three-dimensional nature of wavy vortices results in chaotic advection of fluid elements. This is the primary cause of efficient mixing. The results suggest that the effective axial diffusion coefficient is a linear function of Re_I. This observation has consequences for the way in which Taylor-Couette flow is modelled in process applications. For flow that is known to be axisymmetric, the assumption of a vortex pair being a 'well-mixed tank' is in error because a fluid particle is constrained to lie upon a torus and does not mix with neighbouring tori except via molecular diffusion. For flow occurring after the onset of waviness, chaotic advection ensures that at least some, if not all, fluid in a vortex pair communicates with all other vortex pairs – in this case the entire vessel may be considered to be a well-mixed tank.

The flow also allows dense particles to settle through the vessel, and the general observation is that the mean settling velocity of an ensemble of dense particles is of a similar magnitude (and sometimes slightly faster) than the particles' free settling velocity. This has important consequences for estimating residence times of dense particles in Taylor-Couette flow. It suggests that the mean residence time is not strongly dependent on rotation rate, and is a function of the free settling velocity of the particle.

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