

SIMILARITY LAWS AND ATTACHED EDDY SHAPES IN TURBULENT BOUNDARY LAYERS

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ABSTRACT

This paper presents a similarity formulation for the streamwise component of the fluctuating velocity u_1 in a turbulent boundary layer based on the attached eddy model of wall turbulence being developed at the University of Melbourne by Perry and co-workers. The consequences of this formulation for increasing Reynolds numbers is tested against recent high Reynolds number data. The model is based on the assumption that there exist eddies of different length scales in a turbulent boundary layer and the probability density function (p.d.f.) of the eddy length scale distribution follows a -1 power law for eddies in the turbulent wall region. Such a distribution is necessary to obtain the log-law of the mean velocity: the spectral scaling laws provide indirect evidence for this. In this paper the results from a template matching technique will be presented which gives further support for the proposition of a -1 power law p.d.f. of attached eddy length scales.

Using space time correlation coefficients further details can be obtained regarding eddy shape. The simple Π -shaped representative eddy needs to be modified to give reasonable correlation coefficients.

INTRODUCTION

Based on the Townsend (1976) attached eddy hypothesis and the flow visualization experiments of Head & Bandyopadhyay (1981), Perry & Chong (1982) postulated that wall turbulence consists of a *forest* of hairpin, horseshoe or ' Λ ' vortices. These vortices originate from the viscous buffer zone and are inclined at approximately 45° with the downstream direction. Perry & Chong (1982) found that in order to obtain a logarithmic mean velocity distribution, a region of constant Reynolds shear stress and cor-

rect u_1 spectral behaviour in the turbulent wall region, it is necessary to assume that a range of scales of geometrically similar representative eddies exist. The simplest assumption is that all the representative eddies have the same characteristic velocity scale ($\sim U_\tau$, wall shear velocity) and their length scale δ (eg. eddy height) varies from the smallest scale δ_1 ($\sim \nu/U_\tau$) to the the largest scale equal to the boundary layer thickness δ_c . The probability density function (p.d.f.) of eddy length scale δ follows a -1 power law distribution if a continuous distribution is assumed. Perry *et al.* (1986) extended this model to the whole turbulent boundary layer beyond the viscous region. For simplicity here it will be assumed that the average spacing between eddies scales with the eddy length scale and that all eddies have the same characteristic velocity scale. This later assumption appears to apply reasonably well for zero pressure gradient flows but not for flows with strong pressure gradients - see Marusic & Perry (1995).

P.D.F. MEASUREMENTS

The experiments were carried out in a wind tunnel with a working section of 940×388 mm. The boundary layer was tripped at a distance 3160 mm from the measuring station with a 1 mm diameter trip rod giving Reynolds numbers R_θ of 4140 and 11928 at nominal free-stream velocities of 10 m/s and 30 m/s respectively. Here $R_\theta = U_1\theta/\nu$ where U_1 is the freestream velocity, θ is the momentum thickness and ν is the kinematic viscosity. All the measurements were made with a dynamically-calibrated normal wire.

The estimates for the p.d.f. of the eddy length scale distributions were made using a template matching technique where the velocity template for an eddy of a particular length scale was generated assuming

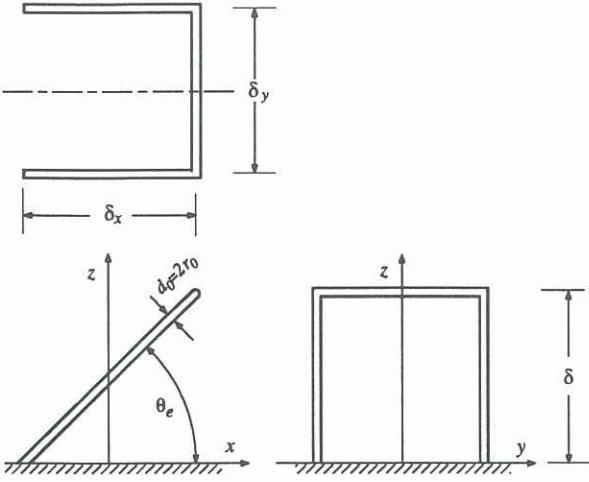


Figure 1: Schematic diagram of a Π -shaped eddy.

a Π -shaped eddy at an angle of 45° with the flow direction as shown in figure 1. This assumed shape is somewhat arbitrary and has been adopted for its simplicity for this preliminary work. The other relevant geometric properties of the assumed eddy shape are: a core-diameter $d_0=0.1\delta$, an aspect ratio of 1 (for the ratio of eddy height to spanwise width) and a Gaussian distribution of vorticity within the core and d_0 is twice the standard deviation. The velocity field of the eddy was calculated from Biot-Savart law calculations and the convection velocity of an eddy with a length scale δ was assumed to be equal to the local mean velocity at $z=0.5\delta$, where z is the distance normal to the wall. It was found that variations to the convection velocity assumptions have only a minor effect on the conclusions for δ/δ_c small but an appreciable effect for δ/δ_c large and needs to be clarified. The Biot-Savart velocity signature was correlated with the measured temporal velocity signal and a match, representing an eddy, is expected to be characterized by a peak in the template-signal correlation. A threshold for this peak needed to be chosen but conclusions are insensitive to this choice.

The p.d.f.'s of the eddy length scale distribution as obtained by the template matching technique for various probe positions are shown in figure 2. It can be seen that the expected p.d.f. is obtained. Full details of the template matching technique are given by Uddin (1994).

STREAMWISE VELOCITY FLUCTUATIONS

The attached eddy model of wall turbulence suggests that, like the mean velocity (Coles 1956), there is a log-law region for $\overline{u_1^2}/U_\tau^2$ for $z_+ \gg U_\tau/\nu$ and $z \ll \delta_c$ - see Perry *et al.* (1986) and Perry & Li (1990). Furthermore, experimental data of $\overline{u_1^2}/U_\tau^2$ from various sources suggests that $\overline{u_1^2}/U_\tau^2$ exhibits two deviations from the log-line, similar to that observed for mean flow profiles, one near the wall and

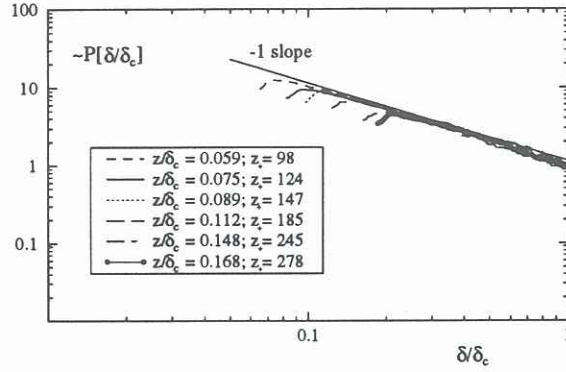


Figure 2: Probability density function (p.d.f.) of eddy length scale distribution in a turbulent boundary layer as obtained by template-matching technique. Authors' data - see Uddin (1994). The z/δ_c and z_+ values refer to the position of the probe; $z_+ = zU_\tau/\nu$.

the other in the outer layer. This leads us to propose that

$$\frac{\overline{u_1^2}}{U_\tau^2} = B_1 - A_1 \ln\left[\frac{z}{\delta_c}\right] - V_{g1}[z_+] - W_{g1}\left[\frac{z}{\delta_c}\right] \quad (1)$$

where the deviation $V_{g1}[z_+]$ is termed as the *viscous deviation function* due to finite Reynolds number and $W_{g1}[z/\delta_c]$ is termed as the *wake deviation function*, which according to Townsend's Reynolds number hypothesis should be a function of z/δ_c alone. In (1) A_1 is a universal constant and B_1 is a characteristic constant which is related to the Coles wake factor which is assumed fixed for high Reynolds number zero-pressure-gradient turbulent boundary layers. The function $V_{g1}[z_+]$ is probably controlled by many effects. One due to the Kolmogoroff cut-off at high wavenumbers (which is an isotropic effect) and another is due in part to the cut-off of geometrically similar attached eddies, i.e. $\delta < \delta_1$. There is no doubt that other additional anisotropic viscous effects are entering as we approach the buffer zone. The viscous deviation function $V_{g1}[z_+]$ is such that

$$V_{g1}[z_+] = C z^{-\frac{1}{2}} \quad \text{as } z_+ \rightarrow \infty$$

from the theory of Kolmogoroff, e.g. see Perry & Li (1990).

The universality of the functions $V_{g1}[z_+]$ and $W_{g1}[z/\delta_c]$ for $z_+ > 100$ imply that if these are deduced by a curve-fitting procedure from one set of data, then they must be applicable for other sets of data. These functions were determined with the experimental data from the authors' data shown in figure 3(a) and its validity was checked by comparing the experimental data of Smith (1994) and Petrie *et al.* (1990) as in shown in figure 3(b). These are the highest Reynolds numbers so far recorded for such smooth wall turbulence measurements. Note that values of $A_1=1.1$ and $B_1=2.0$ were used and are only slightly

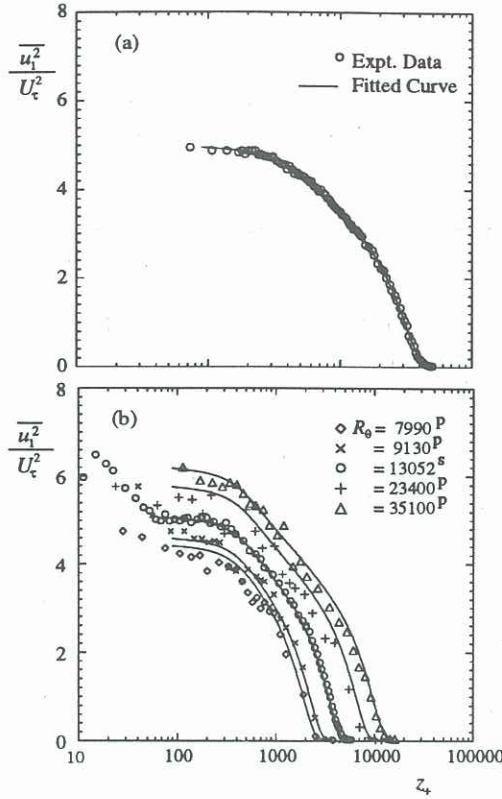


Figure 3: Validity of the similarity formulation of u_1 . (a) Determination of the functional form for $V_{g1}[z_+]$ and $W_{g1}[z/\delta_c]$ by curve-fitting the authors' data at $R_\theta = 11928$. (b) Comparison of similarity predictions and the experimental data of Smith (1994) (marked with s) and Petrie *et al.* (1990) (marked with p).

different to the values 1.03 and 2.39 used by Perry & Li (1990). The observed good agreement between the experimental data of Smith (1994) and Petrie *et al.* (1990) seem to support our conjecture. As seen in the figure, the $\frac{u_1^2}{U_c^2}$ versus z_+ profile has an approximate plateau at $z_+ = O(100)$ which continues to rise with Reynolds number. We need to go to higher and higher Reynolds numbers to see how far this scaling remains valid. According to this formulation the plateau will rise indefinitely with Reynolds number. From these results, it can clearly be seen that conventional wall functions for $\frac{u_1^2}{U_c^2}$ cannot work at least in the turbulent wall region.

SPACE TIME CORRELATIONS FOR EXPLORING EDDY SHAPES

The aspects outlined above are somewhat insensitive to the assumed eddy shapes. The two point space time correlation coefficient depends more on eddy shape, particularly the angle at which the eddy leans in the streamwise direction. One could formulate a mean effective angle θ_i which is defined as

$$\theta_i = \arctan\left(\frac{\Delta z^*}{\xi_m}\right) \quad (2)$$

where $\xi = \Delta x^* - \tau U_c / \delta_c$ where $\Delta x^* = \Delta x / \delta_c$;

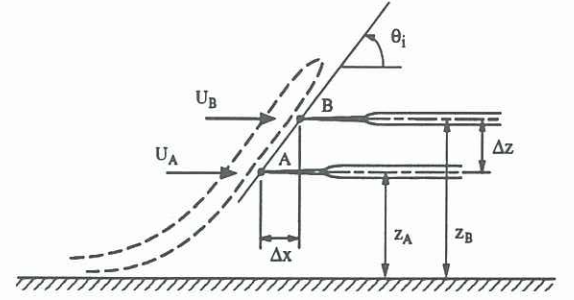


Figure 4: Definition of terms.

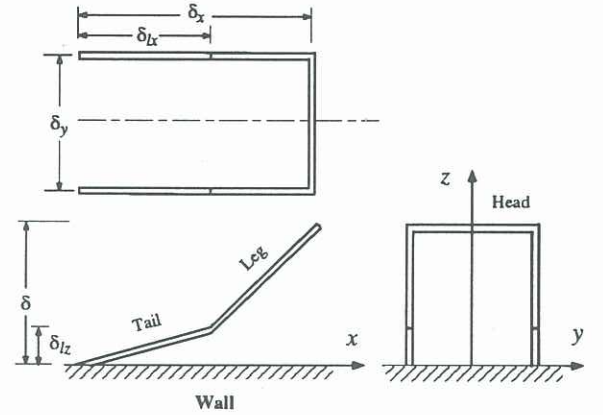


Figure 5: Sketch of a Π -shaped eddy with a one segmented tail.

$\Delta z^* = \Delta z / \delta_c$ as defined in figure 4 and U_c is the eddy convection velocity (typically $0.8U_1$) and τ is the time lag of the U_B velocity signal relative to U_A and ξ_m is the value of ξ at which the correlation $\frac{U_A U_B}{(\sqrt{U_A^2} \sqrt{U_B^2})}$ is a maximum. Because there is a range of scales this simple interpretation becomes obscure but nevertheless it forms a very useful variable for comparing experiment with theory.

Figure 6 shows a comparison between attached eddy model and experimental results for inferred average structure angles in a zero pressure gradient flow. For the attached eddy model, a range of scales of one type of representative eddy are being used, distributed with a -1 power law p.d.f. of length scales. Comparing figures 6(a) and (c) shows that using 45° inclined Π -eddies does not give a reasonable result in the near wall part of the flow. Figure 6(b) shows that using an approximation to a "bent" representative eddy as shown in figure 5 gives better qualitative and quantitative agreement with the experimental results but the model could be improved. The "tail" section of this eddy is tapered with r_o^* decreasing to approximately zero towards the end of the eddy. Organised structures similar in appearance to figure 5 have also been proposed by Jovic (1993) from two-point correlation measurements in a recovering turbulent boundary layer. In reality, the representative eddy is probably curved and figure 5 is only a crude approximation at this stage.

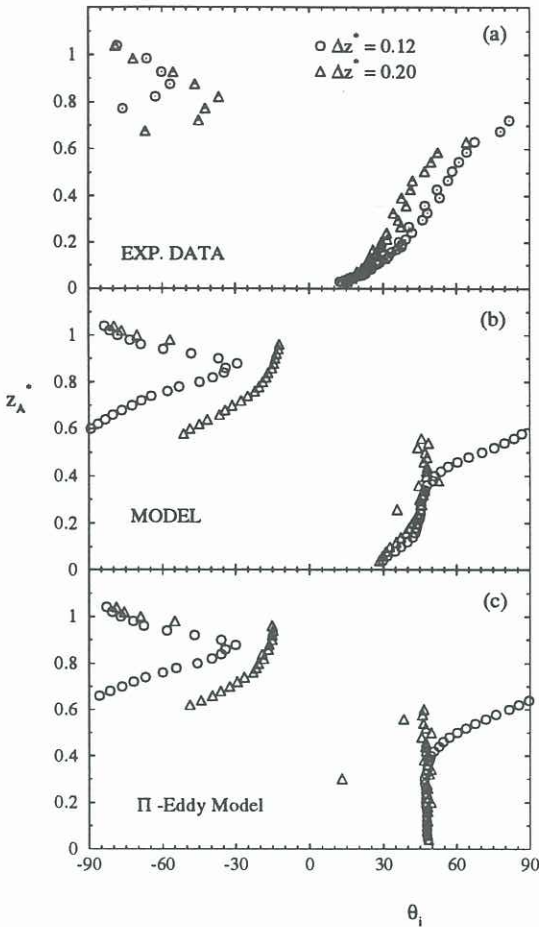


Figure 6: Inferred average structure angle. $z_A^* = z_A/\delta_c$. (a). Experimental data. Flow G1, Uddin(1994). (b). Attached eddy model results using Π -shaped eddy with a one segmented tail as shown in figure 5. $r_0^* = 0.1$, $\delta_y/\delta = 1$, $\delta_{lz}/\delta = 0.5$, $\delta_{lx}/\delta = 1.5$. (c). Model using Π -shaped eddy shown in figure 1. $r_0^* = 0.1$, $\theta_e = 45^\circ$, $\delta_x/\delta = 1$, $\delta_y/\delta = 1$.

CONCLUSIONS

The p.d.f.'s of attached eddy length scales in a zero pressure gradient turbulent boundary layer have been measured experimentally using a template matching technique. The results seem to agree with an expected -1 power law in the turbulent wall region of the flow. Turbulence intensity scaling which is based on -1 power law p.d.f.'s predict that the turbulence intensity at the outer edge of the buffer zone increases without limit with increasing Reynolds number. Two point space time correlation coefficients have also been measured and have been used to infer attached eddy shapes. Comparisons with an attached eddy model indicate that the representative eddy shapes are most likely curved near the wall and not inclined at 45° to the wall as is usually assumed.

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