THE STUDY OF VORTICITY-STRAIN FIELD ALIGNMENT CHARACTERISTICS USING DIRECT NUMERICAL SIMULATION OF HOMOGENEOUS TURBULENT FLOWS

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ABSTRACT

Data obtained from direct numerical simulation of the Navier-Stokes equations have shown that there is a strong tendency for the vorticity vector to align in the direction of the intermediate rate of strain direction. In this paper a study of the alignment of vorticity with the principal strain directions in different topological regions of homogeneous isotropic turbulence is presented. The effect of Reynolds number in aligning the vorticity vector in the direction of the intermediate rate of strain of the Taylor-Green vortex has also been investigated and is discussed. It is shown here that for the Taylor-Green vortex, tendency for the vorticity vector to align itself in the intermediate rate of strain direction is Reynolds number dependant.

INTRODUCTION

One of the most striking features of the structure of turbulence observed in many recent direct numerical simulations of turbulent flows is the emergence of high vorticity magnitude regions concentrated in tube-like structures. Jiminez *et al.* (1993) and Vincent & Meneguzzi (1991) have investigated the properties of these tube-like structures for a statistically stationary homogeneous flow field. Brachet *et al.* (1983) have also concluded in their studies that tube-like structures exists in the Taylor-Green vortex. Analysis of data for forced homogeneous isotropic turbulence and homogeneous sheared turbulence by Ashurst *et al.* (1987) found that vorticity has a tendency to align itself with the intermediate rate of strain. An experimental study by Tsinober *et al.* (1992) showed a similar trend for grid turbulence. Probability histograms obtained by evaluating the alignment of vorticity with the three principal strain directions throughout the flow field were used to reach this conclusion.

Chong *et al.* (1990) proposed that topological methodology be used to study these fine scale structures. Studies using topological methods have been carried out by Ooi *et al.* (1994) and Soria & Chong (1993) on the evolution of the tube-like structures in a direct numerical simulation of a three-dimensional plane wake. Soria *et al.* (1994) have also used these methods to analyse data from the direct numerical simulation of a plane mixing layer. In this paper, probability histograms showing the alignment property of vorticity with the three principal rate of strain directions in different topological regions is presented. This paper also shows that in the case of the Taylor-Green vortex, the tendency for the vorticity vector to align itself with the intermediate rate of strain direction is dependant on the Reynolds number.

LOCAL TOPOLOGY

For incompressible flow, the characteristic equation

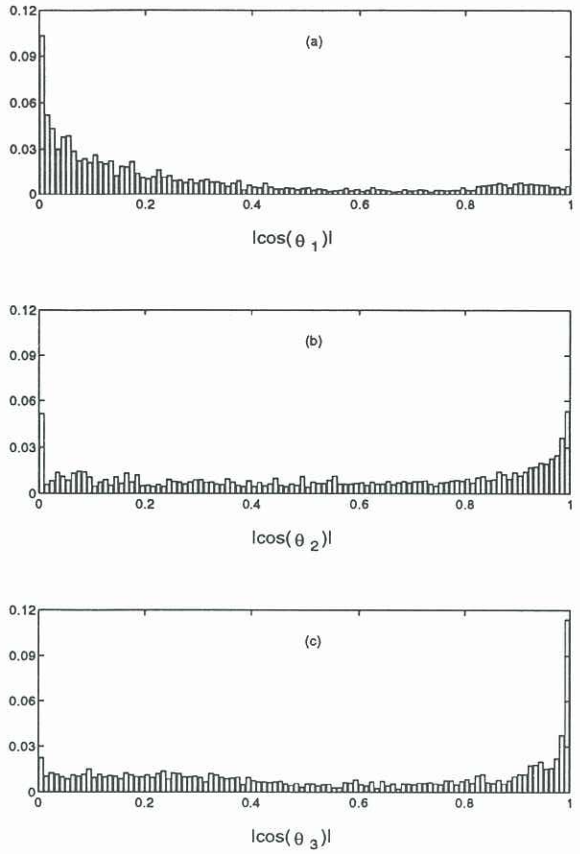

Figure 1: Histograms of alignment of vorticity with the 3 principal rate of strain weighted by Q_w. (a) probability histogram for $|\cos(\theta_1)|$, (b) probability histogram for $|\cos(\theta_2)|$, (c) probability histogram for $|\cos(\theta_3)|$. Histograms created from data of the Taylor-Green vortex with $\mathbf{R}$=100 at t $\simeq$4.0.

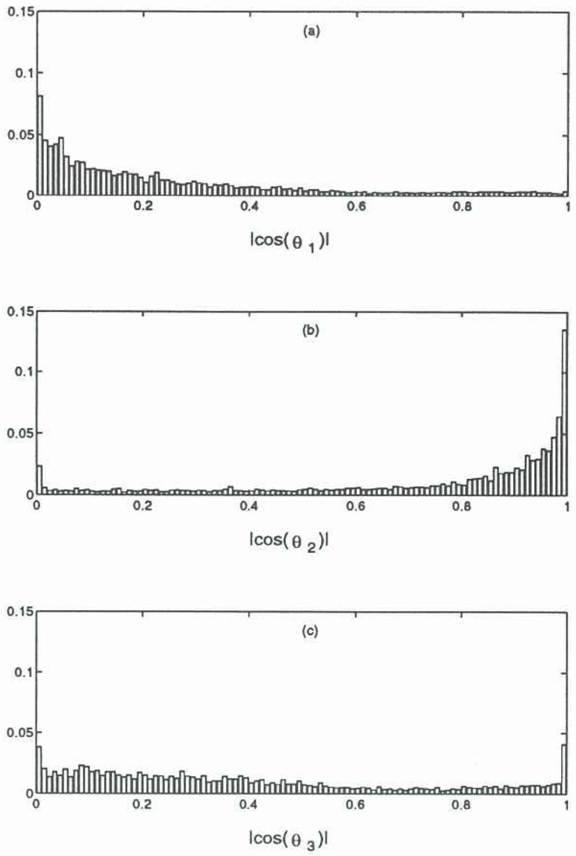

Figure 2: Histograms of alignment of vorticity with the 3 principal rate of strain weighted by Q_w. (a) probability histogram for $|\cos(\theta_1)|$, (b) probability histogram for $|\cos(\theta_2)|$, (c) probability histogram for $|\cos(\theta_3)|$. Histograms created from data of the Taylor-Green vortex with $\mathbf{R}$=800 at t$\simeq$4.0.

for the velocity gradient tensor, A_{ij}, is

$$\lambda^3 + Q\lambda + R = 0, \quad (1)$$

Where Q and R are the second and third invariant of the velocity gradient tensor respectively. Using the topological classification scheme proposed by Chong *et al.* (1990), every point in an incompressible flow can be classified as either stable-focus stretching (SF/S), unstable-focus contracting (UF/C), stable-node saddle-saddle (SN/S/S) and unstable-node saddle-saddle (UN/S/S). These regions are completely determined by Q and R and are summarised in the table below.

CONDITION	TOPOLOGY
$27R^2/4 + Q^3 > 0$ and $R < 0$	SF/S
$27R^2/4 + Q^3 > 0$ and $R > 0$	UF/C
$27R^2/4 + Q^3 < 0$ and $R > 0$	UN/S/S
$27R^2/4 + Q^3 < 0$ and $R < 0$	SN/S/S

A_{ij} can be written in terms of a symmetric part S_{ij} and an antisymmetric part W_{ij}. S_{ij} contains information about the local strain field and its real eigenvalues α_1, α_2 and α_3 are the three principal strain rates. The corresponding eigenvectors $\mathbf{e_1}$, $\mathbf{e_2}$ and $\mathbf{e_3}$ are the three principal strain directions. From continuity $\alpha_1 + \alpha_2 + \alpha_3 = 0$, and using the convention in this paper that $\alpha_1 \leq \alpha_2 \leq \alpha_3$, α_1 must be always negative, α_3 always positive and α_2 can either be positive or negative. θ_i is the angle between $\mathbf{e_i}$ and the vorticity vector. All histograms presented are weighted by Q_w, which is the second invariant of W_{ij}. It can be shown that Q_w is directly proportional to the enstrophy density.

DATA

The data for this investigation was obtained by solving the Navier-Stokes equations using a pseudospectral technique with periodic boundary conditions. Details of this numerical method are found in Orszag (1971). The time-stepping scheme used in this simulation is similar to Vincent & Meneguzzi (1991). A second order Adams-Bashforth scheme was used for the convective terms of the Navier-Stokes equations and the viscous terms were integrated exactly using an integrating factor. The Taylor-Green vortex for Reynolds number, $\mathbf{R}$, of 100 and 800 was simulated using 64^3 and 96^3 grid points respectively. Data for homogeneous isotropic turbulence with Taylor Reynolds number, R_λ, of approximately 20 was computed using a 64^3 grid.

DISCUSSION OF RESULTS

The effect of $\mathbf{R}$ on the alignment of vorticity with the three principal strain directions is investigated by studying the evolution of the Taylor-Green vortex (Taylor & Green (1937)). Data obtained from the evolution of the Taylor-Green vortex shows that there is a clear alignment of vorticity with the direction of intermediate strain, but only when $\mathbf{R}$ is large. This is clearly illustrated in figures 1 and 2 which shows probability histograms for the alignment of vorticity with the three principal directions for two different $\mathbf{R}$. For the Taylor-Green vortex, $\mathbf{R}= 1/\nu$ where ν is the kinematic viscosity and t is time in non-dimensional form as introduced by Taylor & Green (1937). For $\mathbf{R}$=100, figure 1(a) shows that vorticity does not align with the eigenvector associated with α_1. Figures 1 (b) and (c) shows no obvious tendency for vorticity to align with either $\mathbf{e_2}$ or $\mathbf{e_3}$. However, figure 2 (b) clearly shows that for $\mathbf{R}$=800 at the same point in time, the vorticity vector has a strong tendency to align with $\mathbf{e_2}$.

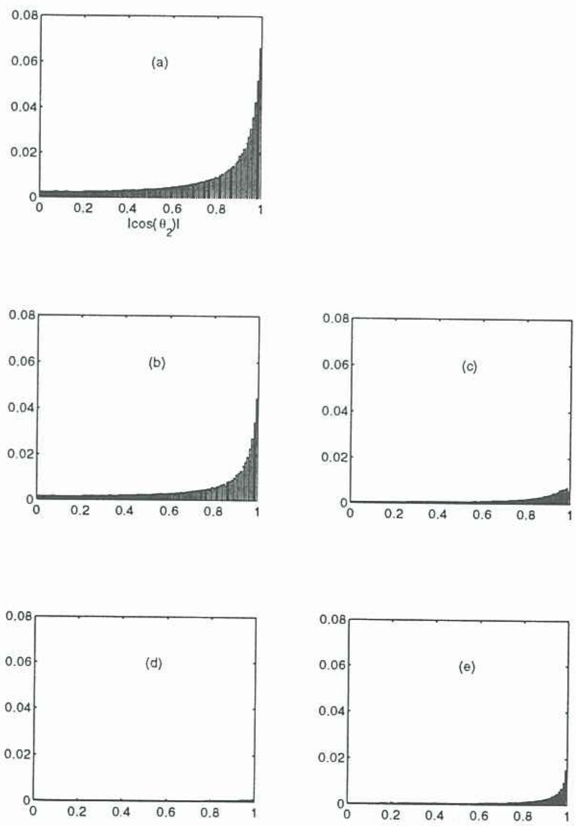

Figure 3: Histograms showing the alignment of vorticity with the intermediate rate of strain with $\alpha_2 > 0$ weighted by Q_w. (a) all topologies (b) SF/S topology, (c) UF/C topology, (d) SN/S/S topology,(e) UN/S/S topology.

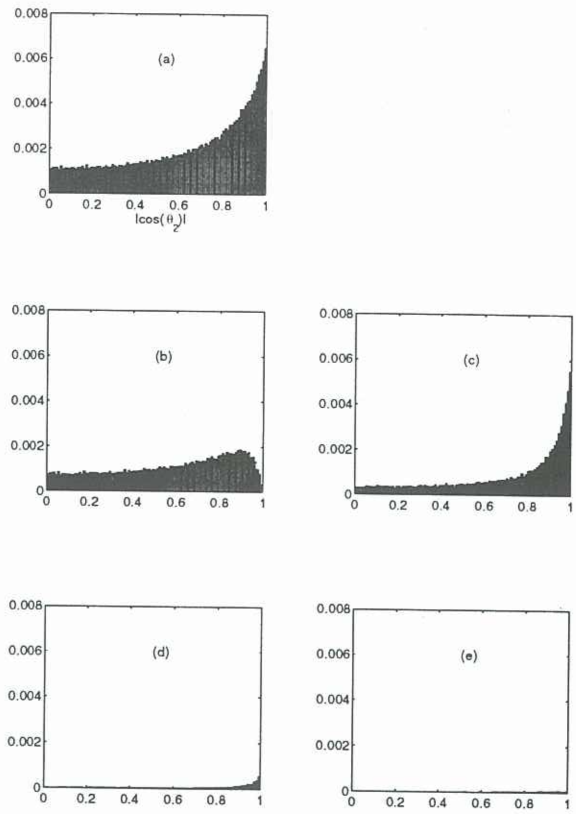

Figure 4: Histograms showing the alignment of vorticity with the intermediate rate of strain with $\alpha_2 < 0$ weighted by Q_w. (a) all topologies (b) SF/S topology, (c) UF/C topology, (d) SN/S/S topology,(e) UN/S/S topology.

Figures 3 and 4 shows probability histograms of the alignment of vorticity vector in the direction of the intermediate rate of strain weighted by Q_w from direct numerical simulation of decaying homogeneous isotropic turbulence. Data used to generate figures 4 and 5 were taken when $R_\lambda \simeq 20$. Only data points where $\alpha_2 > 0$ were used to construct the histograms in figure 3. Figures 4 and 5 was constructed from data points of regions with $\alpha_2 < 0$. Figure 3 shows that if $\alpha_2 > 0$ the alignment of vorticity with the intermediate rate of strain is more prominent in regions which have local topology SF/S and UN/S/S. However if $\alpha_2 < 0$ the alignment of vorticity is more obvious in regions which have local topology UF/C and SN/S/S as shown in figure 4. Figure 5 shows that if $\alpha_2 < 0$, the vorticity has a tendency to align itself with $\mathbf{e_3}$ in regions where the local topology is SF/S. Simulations of forced isotropic turbulence with $R_\lambda \simeq 70$ show similar trends.

CONCLUSIONS

From this study, it can be concluded that for decaying homogeneous isotropic turbulence with $R_\lambda \simeq 20$, vorticity has a tendency to align itself with the eigenvector associated with the intermediate eigenvalue of the rate of strain tensor. For $\alpha_2 > 0$, this align-

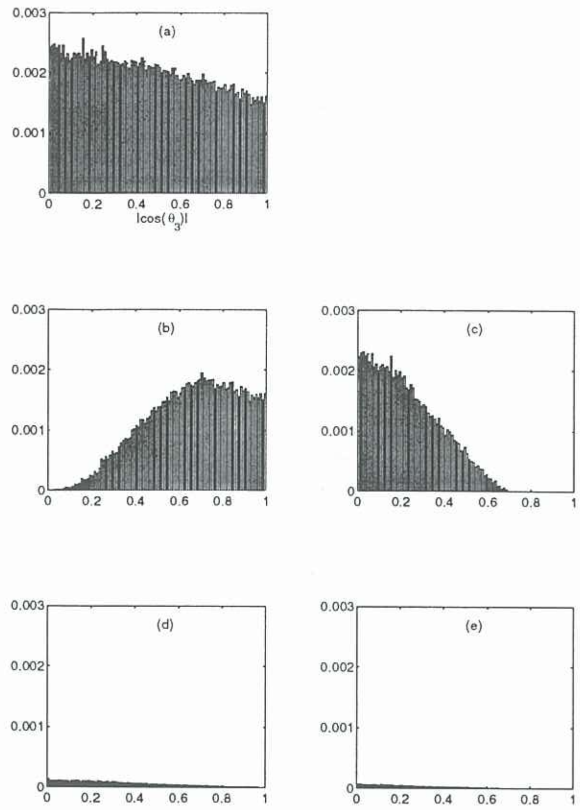

Figure 5: Histograms showing the alignment of vorticity with the largest rate of strain with $\alpha_2 < 0$ weighted by Q_w. (a) all topologies (b) SF/S topology, (c) UF/C topology, (d) SN/S/S topology,(e) UN/S/S topology.

ment is more prominent in regions with local topology SF/S and UN/S/S. Vorticity will align itself with $\mathbf{e_2}$ in regions with local topology UF/C and SN/S/S when $\alpha_2 < 0$. It was also found that if $\alpha_2 < 0$ the vorticity vector will align itself with $\mathbf{e_3}$ in regions with local topology SF/S. Analysis of data from the Taylor-Green vortex shows that the tendency for the vorticity vector to align with $\mathbf{e_2}$ is enhanced for larger Reynolds number. These conclusions are consistent with the alignment of vorticity vector with the three principal strain directions in different topological regions of Burgers-type vortices.

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