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MODELLING STEADY STATE FREE SURFACE FLOWS USING FASTFLO

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ABSTRACT

This paper describes in outline algorithms for solving steady free surface flow problems in two dimensions. The fluid is assumed to be incompressible with constant viscosity and constant surface tension coefficient. For steady problems two principle algorithms are outlined — the **kinematic iteration** and the **normal stress update**. These algorithms are incorporated into a *Fastflo* module. Both algorithms employ an elastic deformation of a fixed topology mesh. Results for a double roll coating flow problem are presented.

1. INTRODUCTION

The main feature of a free surface problem is that the position of the free surface must be found together with the flow variables (velocity and pressure), either in a steady state configuration (steady state problem) or else tracked through time (transient problem). The fluid is assumed to be incompressible with constant viscosity (μ) and constant surface tension coefficient (γ). An important measure of the flow is the capillary number, defined as $\mathrm{Ca} = \mu U/\gamma$, where U is a characteristic velocity; Ca measures the relative strength of surface tension to viscous forces.

This paper describes in outline methods used to solve steady free surface flow problems in two dimensions using the finite element package *Fastflo*. These methods employ an elastic deformation of a fixed topology mesh. Two principle methods are used — the **kinematic iteration** and the **normal stress update**. In both methods the solution for the flow field is segregated from the free surface motion. The kinematic iteration works well for Ca greater than about 0.1, the normal stress update works for the complementary range.

Numerical results were obtained using the *Fastflo* package. This is an advanced and flexible finite element package that has been developed at CSIRO Division of Mathematics & Statistics. The package can solve problems in two and three dimensions. It also has a flexible problem definition and command language called *Fasttalk*.

The presence of surface tension has a major impact on the computational procedures required to solve the problem: this is because the curvature of the free surface must be taken into account. Particular care is required to enforce slope conditions on the free surface. Both the kinematic iteration and the normal stress update use special finite element assembly and solution procedures restricted to the free arc. These procedures are supported by facilities available in the associated *Fasttalk* language and both methods are incorporated into a *Fastflo* free surface module. Results for a double roll coater flow are given.

2. FLOW EQUATIONS

The solution domain is, in general, a time-varying domain, labelled $\Omega(t)$, and is assumed to be generated by a continuous coordinate (mesh) deformation process

$$\mathrm{r}(t) = \mathrm{r}(0) + \boldsymbol{\xi}(t): \qquad \Omega(0) \rightarrow \Omega(t).$$

$\boldsymbol{\xi}$ is chosen as an elastic deformation of the domain Ω so as to minimise local mesh deformation.

To handle the surface tension it is necessary to formulate the flow equations with stress boundary conditions. Using suitably defined finite element spaces, we seek a solution $\mathrm{u} \in \mathcal{V}_g$ and $p \in \mathcal{H}$ such that $\forall \mathrm{v} \in \mathcal{V}_0,\ \forall q \in \mathcal{H}$

$$\int_\Omega \rho \frac{\partial \mathrm{u}}{\partial t} \cdot \mathrm{v}\, d\mathrm{x} + \int_\Omega \mu \nabla \mathrm{u} \cdot \nabla \mathrm{v}\, d\,\mathrm{x} + \int_\Omega \mu \sum_{i=1}^{2} \frac{\partial \mathrm{u}}{\partial x_i} \cdot \nabla v_i\, d\mathrm{x} + \int_\Omega \rho\, (\mathrm{u} \cdot \nabla)\mathrm{u} \cdot \mathrm{v}\, d\mathrm{x} -$$

$$- \int_\Omega \mathrm{f} \cdot \mathrm{v}\, d\mathrm{x} - \int_\Omega p\, \nabla \cdot \mathrm{v}\, d\mathrm{x} = \int_{\partial\Omega_f} \boldsymbol{\sigma} \cdot \mathrm{v}\, ds, \tag{1}$$

$$\int_\Omega \nabla \cdot \mathrm{u}\, q\, d\mathrm{x} = 0.$$

Standard notation is used for physical quantities; $\partial\Omega_f$ is the free boundary. The flow field must also satisfy the kinematic condition

$$\mathrm{u} \cdot \mathrm{n} = \frac{\partial h}{\partial t} + \mathrm{u} \cdot \mathrm{t}\, \frac{\partial h}{\partial s} \qquad \text{on} \qquad \partial\Omega_f, \tag{2}$$

where h is the normal displacement of the free surface.

Surface tension at the free surface is given by $\boldsymbol{\sigma} = \gamma\, dt/ds$, noting that dt/ds is the arc curvature. In assembling the momentum equation (1) the surface stress integrals are converted to an alternative form by integration by parts:

$$\int_{\partial\Omega_f} \boldsymbol{\sigma} \cdot \mathrm{v}\, ds = \int_{\partial\Omega_f} \gamma\, \frac{d\mathrm{t}}{ds} \cdot \mathrm{v}\, ds = [\gamma\, \mathrm{t} \cdot \mathrm{v}]_0^L - \int_{\partial\Omega_f} \gamma\, \mathrm{t} \cdot \frac{d\mathrm{v}}{ds}\, ds.$$

The alternative form can be evaluated with much greater accuracy. The end point residue terms can be used to impose slope conditions on the free arc.

3. KINEMATIC ITERATION

The kinematic iteration is a natural segregated solution method for transient free surface problems: from some initial flow field and domain the mesh is advanced according to the flow across the free surface, a new flow field is calculated and the mesh advanced again, and so on. At each step the mesh is adjusted so as to comply with the kinematic condition (2). Thus, a sequence of velocity/pressure fields $\{\mathrm{u}^n, p^n\}$ is generated, together with a corresponding sequence of meshes $\{\Omega^n\}$ approximating the changing flow domain. An initial flow field u^0, p^0 and domain Ω^0 must be provided. The kinematic iteration can be used to solve a full transient problem or just to solve a steady state problem. The basic form of the algorithm is formally first order accurate in the time step. In the *Fastflo* module the flow fields at each step are calculated using an augmented Lagrangian method. For details of the algorithm see [2].

4. NORMAL STRESS UPDATE

In contrast to the kinematic iteration the normal stress update is essentially a steady state algorithm. Again, a sequence of flow fields and meshes is calculated from an initial flow field and domain until convergence to a steady state is achieved. The key difference is that a *steady* kinematic condition (zero normal flow) is imposed at the outset in place of the (locally) normal momentum equation; the tangential momentum equation is retained. In the *Fastflo* free surface module the surface tension coefficient is constant and so the tangential momentum equation operates with zero imposed surface stress. The successive flow fields so calculated do not in general match the the surface tension generated by the curvature of the free surface. The unbalanced normal stress is calculated and the surface is given a normal displacement h which adjusts the curvature to rebalance the normal stress. This is done with a pseudo-time stepping process (relaxation) to prevent excessive mesh deformation in any one step. Once h has been calculated an elastic mesh deformation $\boldsymbol{\xi}$ is calculated as before and the mesh is updated. For details of the algorithm see [2].

5. DOUBLE ROLL COATER PROBLEM

The double roll coater problem is selected as an example: it is based on example 24 of the FIDAP Examples Manual [1]. In this problem the flow between two adjacent rollers counter-rotating at different speeds is modelled. The main interest is the free surface formed by the coating fluid as the rollers rotate. The shape of the free surface is determined by the roller speeds, their radii, the gap thickness and the physical properties of the liquid.

The problem geometry is indicated by the initial mesh in Fig. 1(a). Each roller subtend an angle of 28 degrees at its centre and hence has slopes of $\pm$ 28 degrees to the horizontal at the outlets. A simple shear flow ($u_x = 1 - y/4$, $u_y = 0$) is imposed at the inlet gap. Tangential velocities of 1, 0.5 are imposed on the upper and lower roller surfaces respectively, in the same sense as the inlet flow. At the two outlet sections the flow moves freely under a zero stress boundary condition. Slope conditions are imposed on the free surface: at both ends the free surface must be parallel to the roller surfaces, with slopes of $\pm$ 28 degrees to the horizontal.

The physical properties of the fluid are taken as $\rho = 1$, $\mu = 1$, $\gamma = 10$: Re $\sim$ 1, Ca $\sim$ 0.1. The problem is suitable for the kinematic iteration, but near the lower limit of Ca. A mesh of 6-node triangular elements with 400 corner nodes is used, distributed uniformly over the initial domain (Fig. 1(a)).

The final mesh at the steady state is shown in Fig. 1(b). The steady state mesh is strongly stretched near the central part of the free surface; the mesh deformation process has managed quite well. The steady pressure contours and streamlines are shown in Figs. 1(c)–1(d). The minimum distance from the inlet to the free arc is 29.15 units, consistent with [1], Fig. 24.2. This flow problem has also been solved using the normal stress update: the capillary number is in the overlap range.

Other FIDAP examples have been run successfully using the Fastflo free surface module: these are the die-swell problem (example 6), the slot coater problem (example 22) and the fountain flow problem (example 23). For details of these simulations see [2].

6. CONCLUSIONS

The *Fastflo* free surface module can solve a variety of two-dimensional free surface flow problems. The methods adopted can clearly cope with substantial deformations from the starting mesh.

The kinematic iteration can also be used for transient free surface problems. Work is currently in progress using this algorithm for two- and three-dimensional flow problems.

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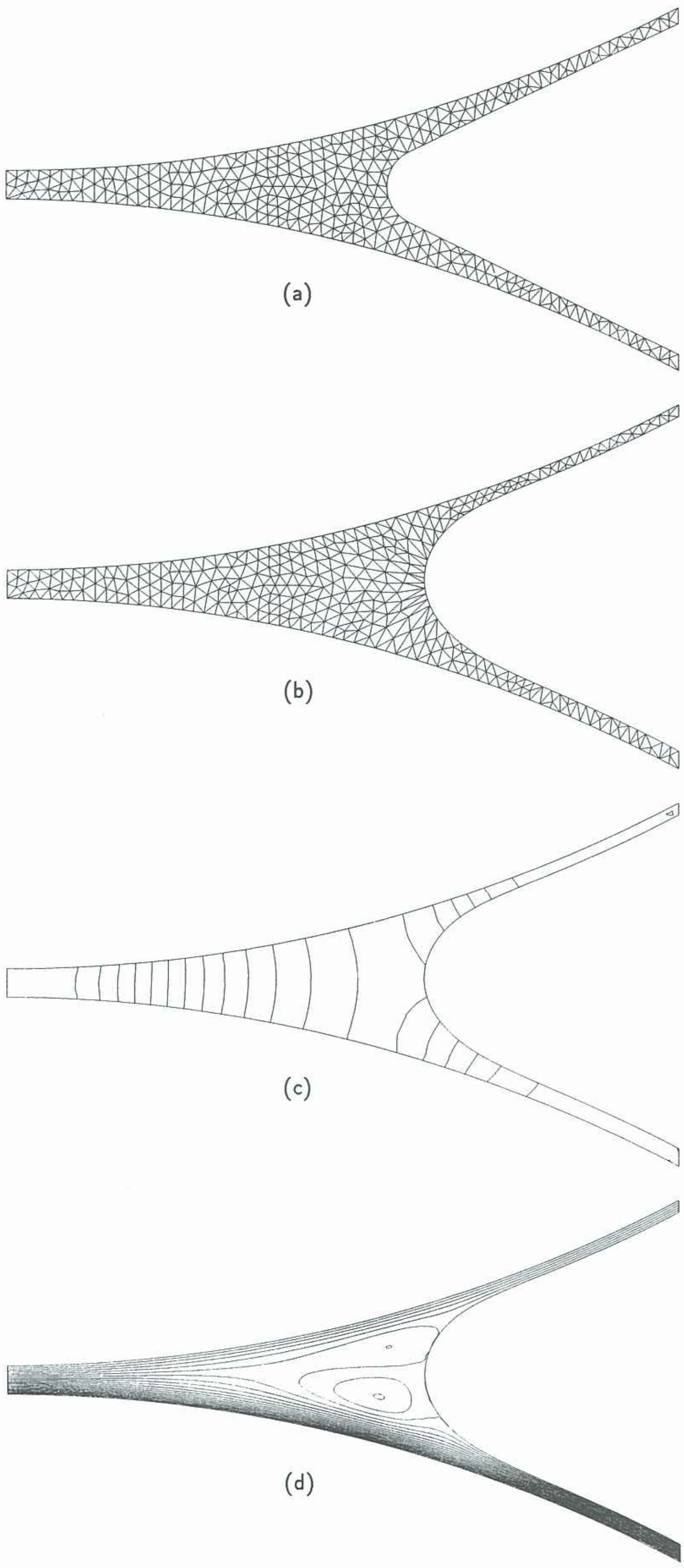

FIGURE 1. DOUBLE ROLL COATER FLOW: (a) INITIAL MESH;
(b)–(d) MESH, PRESSURE CONTOURS AND STREAMLINES FOR STEADY STATE.