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Experiments on turbulence and dispersion in a rotating stratified fluid

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ABSTRACT

Experiments on turbulence in a rotating, stratified are conducted to determine the behaviour of geophysical flows. In the nonrotating case, the inhibition of vertical motion by the stratification leads to an inverse energy cascade similar to that found in two-dimensional turbulence. This upscale energy transfer occurs by the coalescence of eddies of like sign. The effect of rotation is to cause the large structures to be unstable to baroclinic instability, leading to eddies on the scale of the Rossby deformation radius. Relative dispersion is measured and the rates are found to be very different inside the coherent structures compared with regions outside.

INTRODUCTION

The study of turbulence in constrained systems leads to significant changes in the qualitative behaviour of the flow compared with fully three-dimensional turbulent flow. For example, in two-dimensional turbulence energy cascades to large scales rather than to small scales. This upscale transfer of energy results from the fact that vortex lines can not be stretched or twisted in a two-dimensional flow. Vortex structures merge with neighbours of like sign to give large coherent structure, and the flow self organises. In geophysical flows, geometry, stratification and the rotation of the Earth produce constraints on the motion. The atmosphere and the oceans are very thin layers of fluid and so the motion within them is primarily horizontal. Stratification inhibits vertical motion, and rotation reduces vertical shear (by the Taylor-Proudman theorem). We observe that atmospheric and oceanic flows contain very energetic coherent vortices. To what extent are these a consequence of the constraints placed on the flow?

We describe laboratory experiments which investigate the dynamics of turbulence in a stratified rotating fluid. The aim of this work is to show how the motion reacts to the constraints of stratification and rotation, and to examine the mechanisms operating in the flow. In particular, we are concerned with the energy transfers between different scales of motion, and the consequences for dissipation and dispersion. The experiments are described in section 2, and the results for stratified flow are given in section 3. The effects of rotation are described in section 4, and the dispersion in the flow is discussed in section 5. The conclusions are given in section 6.

THE EXPERIMENTS

The experiments were carried out in a tank 610mm square and 400mm deep, filled with a stratified fluid, with either a constant buoyancy frequency N or a region of strong stratification between two uniform layers. The flow was forced by an arrangement of sources and sinks placed around a horizontal ring of diameter 563mm. The sources and sinks were directed horizontally, and connected to each other by a peristaltic pump, so that fluid was sucked from the tank and re-injected at a source at the same density level. This arrangement was designed to produce horizontal motion at the forcing level with the minimum of vertical mixing. The number of sources could be varied up to a maximum of forty, and in every case they were arranged symmetrically around the ring so that (within experimental error) no net angular momentum was imparted to the flow. For the rotating experiments, the system was mounted on a rotating table and spun about a vertical axis with Coriolis parameter f. The flow was visualised by placing small particles in the flow which were neutrally buoyant at the forcing level and tracking them using an image processing package DigImage (Dalziel, 1993). Further details of the experiments can be found in Linden, Boubnov & Dalziel (1995) – hereafter LBD.

The behaviour of the flow is characterised by the following parameters: the strength of the forcing, $F=U/Nl$, where U is the velocity of the flow from the sources and l is the spacing between them and the ratio N/f of the buoyancy and Coriolis frequencies. The flow

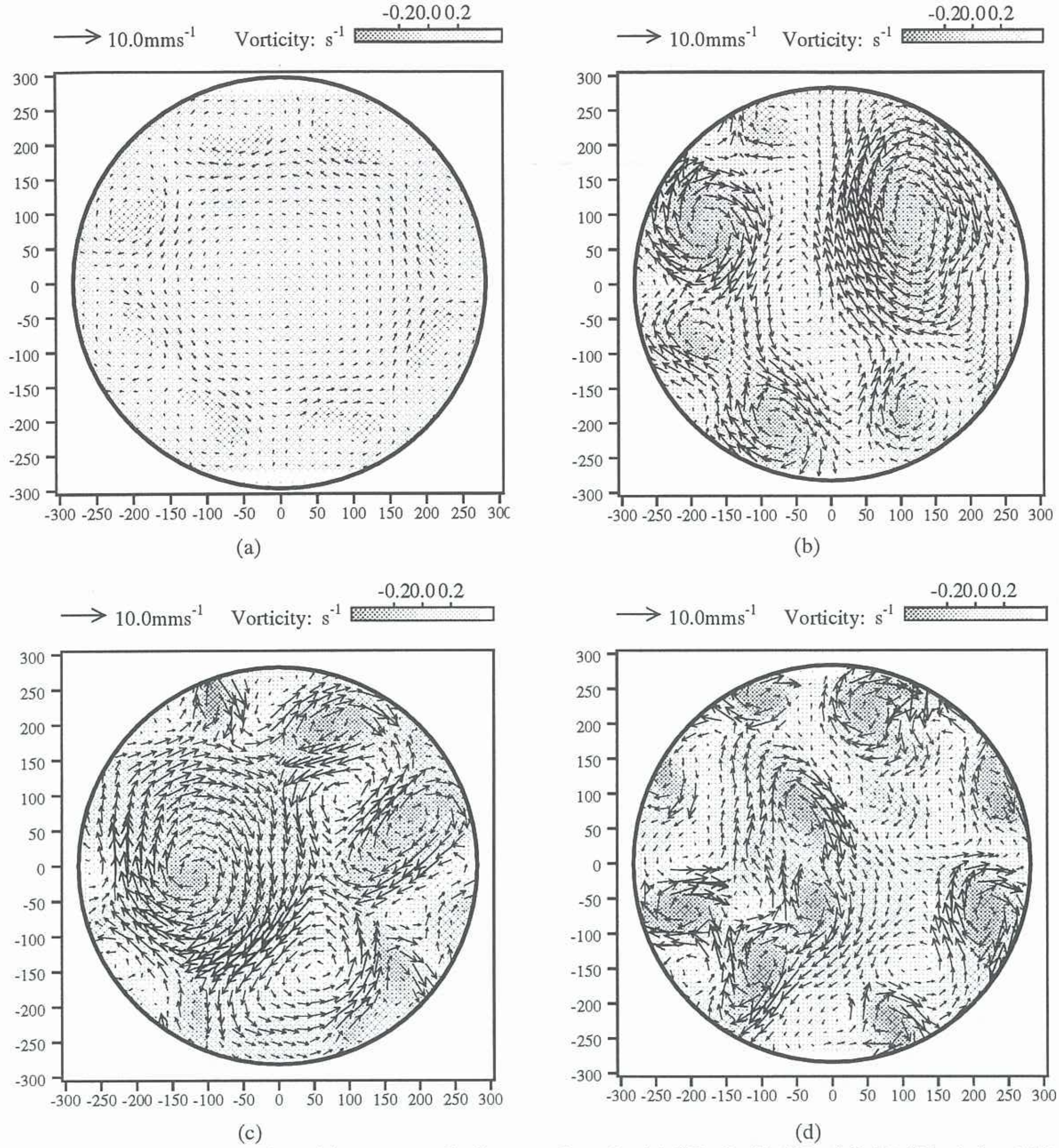


Figure 1: Velocity vectors and vorticity contours for 8 source flows for (a) $f/N = 0$, (b) $f/N = 0.8$, (c) $f/N = 1.6$ and (d) $f/N = 4.0$. The flows shown are representative of the ultimate states of these systems.

also depends weakly on the Reynolds number and Prandtl number, but we will not consider these further here. The forcing parameter F has the form of a Froude number and measures the strength of the forcing compared with the stratification. At large values of F the flow from the sources produces vertical mixing, and in the region of weak stratification that is formed the turbulence is three-dimensional. This case is discussed in Boubnov, Dalziel & Linden (1994) - hereafter BDL, and is not considered further.

STRATIFIED, NONROTATING FLOW

At low values of the forcing parameter F flow develops into a large scale circulation. Initially a number of vortices are produced by the flow from the sources, and these penetrate throughout the flow domain. As time progresses these vortices merge with neighbours of like sign to produce structures of increasing scale until eventually a circulating flow on the scale of the tank persists. An example of this flow when there are 8 sources is shown in figure 1a. Although vertical velocities are suppressed by the stratification, in the early stages of the flow development there is considerable vertical shear of the horizontal velocities. This shear twists vertical vortex lines towards the horizontal and energy is transferred to vertical motions which can generate internal waves. They can also generate small scale vertical mixing and three-dimensional motions. Both of these processes are resisted by the stratification, and as the motion spreads vertically (by a combination of viscous diffusion over short scales and internal wave radiation and dissipation over larger scales) from the forcing plane the vertical shear decreases. Below a threshold value the vertical shear is no longer capable of twisting vortex lines and the two-dimensional inverse cascade operates. In BDL where experiments were carried out in a smaller tank the onset of the circulation took about ten times longer than in the present experiments where the larger flow domain reduced the vertical shear.

Once established the large scale circulation persisted as long as the forcing was maintained, and the direction of the motion remained unchanged. This circulation excludes the continuing flow from the sources from penetrating into the interior of the tank. The vorticity within the circulation is relatively small and uniform, so that the fluid is in approximate solid-body rotation in the centre of the flow domain. This circulation is maintained against dissipative processes such as viscosity and internal waves by the advection of vorticity from the sources into the interior. The details of this process are described in BDL and it is found that this vorticity advection occurs in an unsteady manner with the occasional penetration of energetic vortices. The question then arises as to the mechanism whereby the direction of the circulation is constant given the presence of vortices of both signs being produced by the sources.

When a vortex encounters a shear flow it undergoes radically different behaviour depending on whether the vorticity in the vortex and the shear are of like or opposite signs. Vortices of like sign tend to merge and so the vortex will be incorporated into the shear flow and help maintain it. If the vortex if of opposite sign the strain field of the circulation will distort the vortex causing it to be stretched out into long filaments. These filaments lead to large gradients of vorticity which are then dissipated by viscosity - the 'enstrophy cascade'.

Thus the stratified flow exhibits the behaviour characteristic of two-dimensional turbulence. However, geophysical flows do not have dominant circulations on the scale of the domain. The reason for this is the rotation of the Earth as will be discussed in the next section.

STRATIFIED ROTATING FLOW

The influence of rotation is characterised by the nondimensional frequency ratio f/N. Figure 1b,c and d show the final states of the flow for the values of f/N = 0.8, 1.6 and 4.0, respectively. As the rotation rate is increased the flow changes qualitatively in the following ways:

- a large scale motion is not observed
- the flow consists of a collection of vortices
- the vortices become isolated at high rotation rates
- there is a bias towards anticyclonic vortices.

At the lowest rotation rate shown in figure 1b, the flow has the same basic features as the nonrotating case and a large scale circulation is still observed. In this case the circulation is weak and the vortices around the boundary are more energetic. It is also noticeable that the vortices are predominantly anticyclonic. As the rotation rate increases the large scale circulation is no longer established and the ultimate state of the flow consists of a number of coherent vortices. The horizontal; scale of these vortices decreases with increasing rotation rate, and they become isolated from each other. Figures 1c and 1d show that the predominance of anticyclones continues to be a consistent feature.

Video film of the vortex motions show that there a range of complex interactions which contribute to the evolution of the flow. When the forcing is turned on vortices are produced at the sources and propagate into the centre of the tank. Vortices of like sign merge to produce larger vortices but, in contrast to the nonrotating case, once the vortices reach a certain scale they are observed to fission into two smaller ones. Interactions between anticyclones produce coherent structures but merging of cyclones was observed to produce more diffuse regions of cyclonic vorticity. Stronger anticyclones were also formed at the sources, and these two effects led to the observed predominance of anticyclones in the final state.

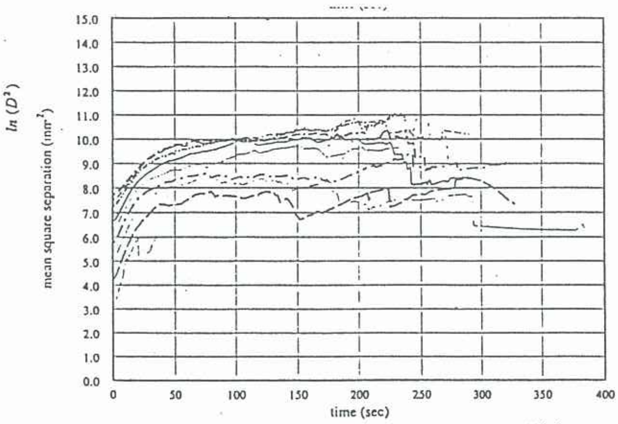
The scale of the vortices is comparable with the Rossby deformation radius $R=NH/f$, based on the full depth H of the tank. For the case shown in figure 1b, R=600mm, and so rotation is unable to inhibit the formation of the large scale circulation. In comparison the flows in figures 1c and 1d have R=150mm and R=75mm, receptively. As these deformation radii are significantly smaller than the flow domain, rotation plays an important role in the dynamics. The obvious mechanism whereby large vortices are split is by baroclinic instability. This is a long-wave instability which produces vortices on the scale of the deformation radius (Griffiths & Linden, 1981). Baroclinic instability is essentially three-dimensional in character and so its presence in these experiments relies on departures from strict two-dimensional flow. An effect of rotation is to reduce the vertical shear compared to the nonrotating case (by the Taylor-Proudman theorem) making the flow more two-dimensional. However, the nature of the forcing which is concentrated in one horizontal plane introduces sufficient three-dimensionality to enable baroclinic effects to operate.

In all cases the vortices are quasi-geostrophic, with Rossby numbers ω/f about 0.2. With these low values of the relative vorticity it is not clear why there is such a difference between cyclones and anticyclones. Yamagata (1982) suggests that differences in the density structure may be responsible. Anticyclones produce a doming of isopycnal surfaces and are somewhat self-enclosed, while cyclones cause a divergence of isopycnals at their edges which may lead to less coherent structures when they merge. The details of these differences remain to be resolved.

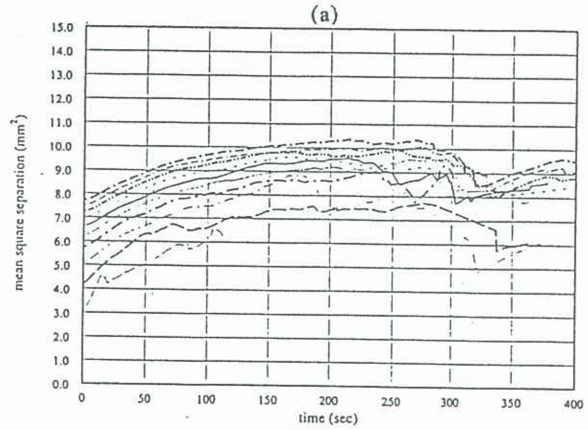
DISPERSION

The dispersion of passive scalars in flows where coherent structures dominate the motion are of particular relevance to the deep ocean. Recently full scale tests have been carried out on the dispersion of SF_6 in the Atlantic which show that the tracer is pulled out into long streaks and then diffused into a more regular patch. In the present configuration we have calculated dispersion rates by measuring the separation of particle pairs in stratified, nonrotating flows. In these experiments a two-layer, nonrotating stratification was used with the forcing level within the density interface. The flow exhibited the same qualitative features as the linearly stratified flow described in section 2. Lagrangian particle tracks were used to determine the two-particle separation. The measurements were restricted to the time before the large scale circulation was established.

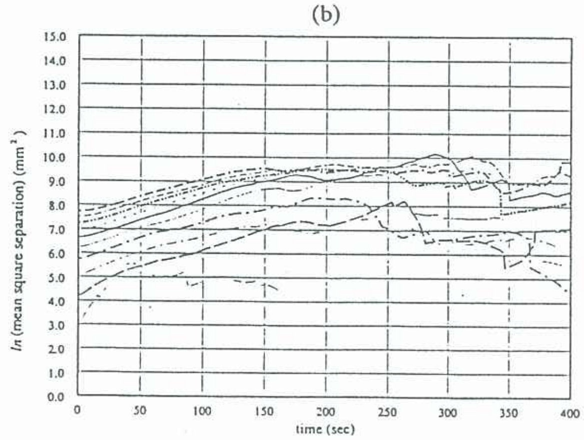
The possible trapping of particles in eddies raises the question of the rôle of coherent structures in dispersing passive tracers. Different rates of dispersion of patches of tracer are expected for releases in different parts of the



(a)



(b)



(c)

Figure 2: Dispersion in the different regions of the flow. Particles release in regions of (a) high shear, (b) streaming flow and (c) inside vortices.

flow because of their position relative to the vortices. In order to address this question the relative dispersion was calculated in three types of flow regions. The regions are defined as eddy regions, streaming regions and high shear regions. The regions were classified by the magnitudes of the horizontal shear

$$\gamma = \left| \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right|$$

and the speed $q = (u^2 + v^2)^{1/2}$, were calculated at points throughout the flow. High shear regions were classified as regions where $\gamma > \gamma_*$; streaming regions where $\gamma < \gamma_*$ and $q > q_*$, and eddy regions (regions of low shear and low speed) where $\gamma < \gamma_*$ and $q < q_*$. Threshold values were set such that these regions corresponded with the structures observed in the flow.

Figure 2 shows the rates of dispersion for the three regions, and they are observed to be dramatically different in different regions of the flow. Relative dispersion is fastest for pairs of particles released initially in regions of high shear (figure 2a) and slowest for those released in vortices (figure 2c).

An unexpected result is that there seems to be an exponential increase in D^2 for pairs of particles in the vortices. Inside the vortices other coherent structures can not separate the particles unless they destroy the vortex. Small scales may exist within the vortices which are continually interacting with one another. The dispersion in the vortices may then be occurring by the formation of progressively larger scales, as in the classical theory.

CONCLUSIONS

These experiments have shown that turbulence in stratified flow exhibits the inverse energy cascade of two-dimensional turbulence. In order for this cascade to occur the vertical shear must be sufficiently small. The addition of rotation reduces the vertical shear further, but introduces a long-wave instability which leads to the formation of vortex structures on the scale of the Rossby deformation radius. These structures are predominantly anticyclonic and have long lifetimes. Dispersion in these flows depends on where a tracer is released relative to the flow structures. Consequently, overall dispersion rates depend on the distribution of vortex structures within the flow.

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