Twelfth Australasian Fluid Mechanics Conference
The University of Sydney, Australia 1995

MIXING EFFICIENCIES IN THERMALLY STRATIFIED TURBULENT FLOWS

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ABSTRACT

Detailed field experiments conducted in two freshwater lakes were used to investigate mixing efficiencies in thermally stratified water columns. The efficiencies were derived from data collected with profiling instruments capable of measuring vertical and horizontal velocities, temperature and conductivity on microscales. These new findings are compared to previously published experimental data, collected within salt stratified water tunnels, thermally stratified wind tunnels, and numerical results. The quality of the mixing efficiency estimates were observed to be strongly dependent on the number of individual events sampled. It was found that the averaged measured fluxes followed a dependence on the turbulent Froude number which was first established with data from laboratory grid experiments.

INTRODUCTION

To be able to successfully parameterise, model and predict the vertical transfer rate of heat, passive tracers and stratifying species in stably stratified fluids subjected to turbulent overturn events we require an understanding of the energetics of the turbulent mixing process. Specifically, we need a simple tool for predicting the efficiency of the mixing processes in a wide range of environments.

Over recent years an increasing number of laboratory, numerical and field experiments were performed by numerous investigators in an attempt to quantify the vertical buoyancy flux and mixing efficiency in a stable density stratified fluid under various forcing conditions. Mixing efficiencies are defined here as the ratio of the flux of buoyancy, b, to the rate of change of available mechanical energy, m, and are equal to the flux Richardson number,

$$\mathrm{Ri_f} = \frac{b}{m} = \frac{1}{1+(\varepsilon/b)} \quad , \tag{1}$$

where ε is the rate of dissipation of turbulent kinetic energy. Using a range of data sources Ivey and Imberger (1991) found that the mixing efficiencies were a weak function of the Prandtl number,

$$\mathrm{Pr} = \frac{\upsilon}{k} \quad , \tag{2}$$

where υ is the kinematic fluid viscosity and k is the diffusivity of the dominant stratifying species, and a strong function of a turbulent Froude number,

$$\mathrm{Fr_t} = \frac{u'}{NL_c} \sim \left(\frac{\varepsilon^2}{g'^3 L_c}\right)^{1/6} \quad , \tag{3}$$

where u' is the rms turbulent velocity scale of the event, L_c is a statistical average measure of the vertical length scale of the energy bearing eddies within the event (Ivey and Imberger, 1991) and N *is the* Brunt-Väisälä frequency. The modified gravitational acceleration, g', is obtained by firstly monotonising the recorded density profile and then subtracting the monotonised profile from the original. The difference in density at each point represents a measure of the out of balance mass, ρ', which when multiplied by the gravitational constant and divided by the mean density yields g' at each point. The length scale, L_c, is obtained by firstly determining the distance a fluid element has been displaced from its level of neutral buoyancy in a monotonised profile (this distance is referred to as the Thorpe displacement scale, L_d). Following this, the displacement estimates, L_d, are moved vertically by one half of their value so that they are placed at the centre of the overturn event. The absolute values of all the estimates at a particular depth are then summed and averaged at their new position; giving a measure of the eddy displacement, L_c, which is now positioned at the event centre. Note that all these quantities are readily measured with vertically transecting probes capable of measuring temperature and conductivities on a microscale.

When $\mathrm{Fr_t} < 1$ the turbulent Reynolds number, Re_t, was also important in determining $\mathrm{Ri_f}$. It is defined as:

$$\mathrm{Re}_t = \frac{u' L_c}{\nu} \sim \left(\frac{\varepsilon L_c^4}{\nu^3}\right)^{1/3} , \quad (4)$$

As noted by Gibson (1980) a third possible ratio exists, which is the small scale Froude number and is given by $Fr_g = (Fr_t Re_t^{1/2})$

The data used by Ivey and Imberger (1991) were derived from thermally stratified air (Pr=0.7) and salt stratified water bodies (Pr=700). Mixing efficiencies were found to increase the closer the Fr_t was to unity, and to decrease the smaller the Pr value. From the studies they derived simple empirical relations capable of predicting the mixing efficiencies based on Fr_t (which is significantly easier to measure than the mixing efficiency itself).

The question remained as to what are the mixing efficiencies in the highly important thermally stratified water bodies, such as lakes and portions of the ocean, where the mechanisms for mixing vary widely and where Pr $\approx$ 7. This was the question addressed in this study.

EXPERIMENTAL METHOD

Data collected during two detailed field experiments conducted in two separate freshwater lakes were used to investigate mixing efficiencies in thermally stratified water columns The data presented here from were restricted to the turbulent benthic boundary layer, which Lemckert and Imberger (1995a) found to be actively mixing.

The first experiment was performed in Lake Biwa, Japan, from the 25th August to the 16th September, 1993, inclusive. Lake Biwa is the largest natural lake in Japan, and is located on the eastern side of Honshu Island. It has a maximum depth of 104 m and a surface area of 636 km^2 when full. During this study the turbulent benthic boundary layer was being generated by internal seiches generated by passing typhoons.

The second experiment was performed in Lake Kinneret, Israel, from the 17th to the 21st July, 1994, inclusive. This lake is located in the northern part of Israel and has a maximum depth was 50 m and a corresponding surface area of 165 km^2 when full. In this instance, the turbulent benthic boundary layer was being driven by shoaling internal waves initiated by a diurnal wind pattern.

Mixing efficiencies were derived from data collected with profiling instruments capable of measuring vertical and horizontal velocities, temperature and conductivity on microscales (Lemckert and Imberger, 1995a, 1995b). The data collected by the profilers represented a snapshot of the turbulence field.

When determining the nature of turbulent events sensed within a microstructure cast only statistically stationary segments were considered. These were determined by applying the stationary test proposed by Imberger and Ivey (1991) to the measured temperature gradient signals.

From the recorded temperature and conductivity data, profiles of ρ, ρ', L_d and finally L_c were derived. By subtracting the profiling instrument's rise velocity from the recorded vertical velocity, w' estimates were obtained. These were then filtered to remove high frequency instrument noise. The derived quantities, ρ' and w' were then multiplied together at each sample point in a profile to give an estimate of the vertical density fluxes, on a point by point basis. To determine the buoyancy flux ($b = -(g/\rho_o)\{w'\rho'\}$), the vertical density fluxe estimates were then averaged over a stationary segment, multiplied by g and divided by a mean segment density. The flux Richardson number [1] was then determined using a dissipation estimate calculated by a Batchelor curve fitting technique applied to the gradient temperature signal data.

Note that data within the stationary segment was used only if it was free of any large noise spikes induced by underwater objects and satisfied the condition $L_s/L_c > 2$ where L_s is the segment length.

RESULTS

From various profiles performed at different locations within the turbulent benthic boundary layers 52 suitable data segments were obtained. Figure 1 shows the distribution of the of all the data points on the Fr_t vs Re_t activity diagram. From this plot it can be seen that the majority of the data were collected in regions of active mixing where inertia dominated over viscosity and buoyancy.

A plot of the derived Ri_f vs Fr_t estimates for the salt and thermally stratified waters are presented in Figure 2.a, while those for the wind tunnel and numerical studies are presented in Figure 2.b. The wind tunnel data shows the rapid rise in Ri_f to a peak value of 0.19 as Fr_t increases from 1 to 1.5. Beyond this, Ri_f decreases slowly with increasing Fr_t. In these experiments data were collected over periods of approximately 100 s, which gave a typical record length (L_l = mean velocity x sample time) to overturn scale ratio of $L_l/L_c \approx 2400$. Given that the turbulence was apparently homogeneous the data records were constructed from a large number of overturning events. The numerical results of Ivey et al (1995) show a significant difference in absolute values when compared to the wind tunnel data. A similar trend was also shown in Ivey and Imberger (1991) when they compared water tunnel and some wind tunnel data, noting that Fr_t and Re_t values for both cases were in similar ranges. These results indicate that mixing efficiencies do indeed depend on Pr and Fr_t. Any dependence on Re_t has yet to be quantified.

When examining the Ri_f vs Fr_t results for the water tunnel experiments (Figure 2.a.) we observe that they follow the same trend as found for the air and numerical studies, but with increased scatter. A possible cause of this was the reduced averaging performed. As a result of instrument stability the water tunnel data records were derived from time series data collected for periods of up to 16 s, which gives $L_l/L_c < 260$. Therefore, the data records were constructed from relatively fewer passing turbulent overturn events. This result illustrates that the sampling length, or averaging extent, is important in determining the true mean estimate.

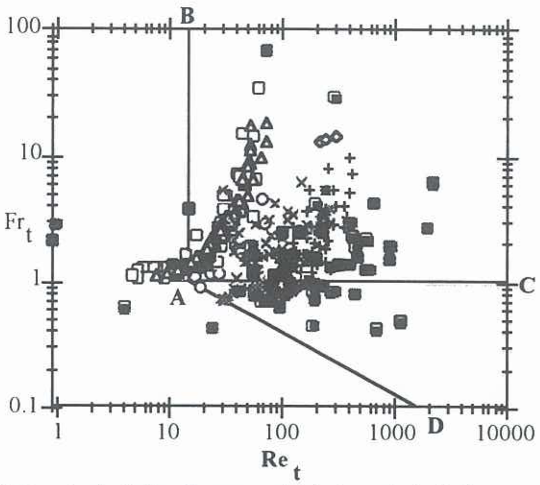

Figure 1. Activity diagram which is a plot of Fr_t versus Re_t. The straight lines indicate transition zones for cases where $Pr \approx 700$. The line from **A-B** is $Re_t = 15$ and indicates where inertia dominates over viscous forces, **A-C** is $Fr_t = 1$ and indicates the transition from inertial dominated to buoyancy influenced turbulence, and, **A-D** is $Fr_g = 4$ and indicates the transition from buoyancy influenced turbulence to non-turbulent (or fossil) internal wave motions. Also shown are experimental data sets. The data are from the salt stratified water tunnel experiments of * Stillinger et. al. (1983), x Rohr (1985) and + Itsweire et. al. (1986), the thermally-stratified wind-tunnel experiments of ◇ Tavoularis and Corrsin (1981), Δ Lienhard and Van Atta (1990), □ Yoon and Warhaft (1990) and o Jayesh et al (1991), and ■ the present study of thermally stratified fresh water reservoirs.

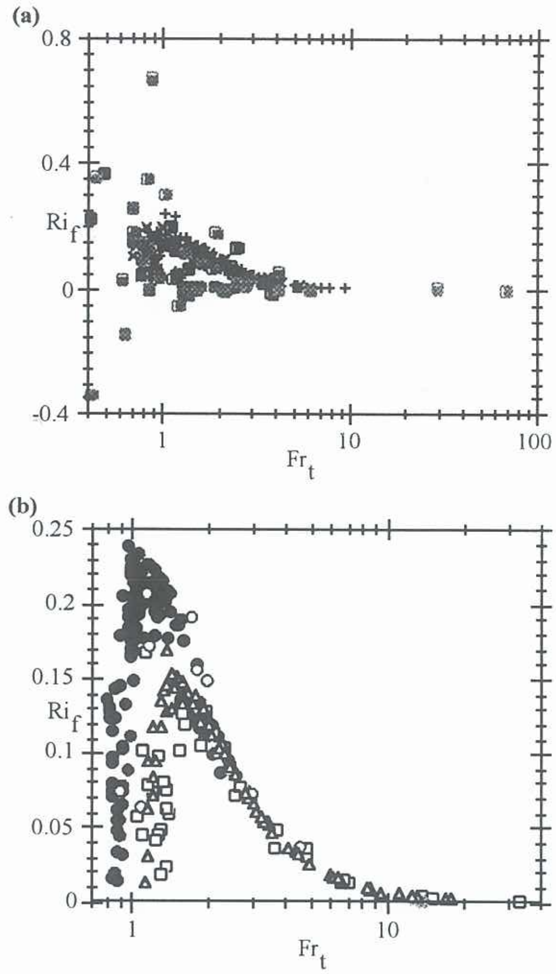

Figure 2. Flux Richardson number, Ri_f, estimates presented as a function of the turbulent Froude number, Fr_t for (a) the water based experiments and (b) for the numerical results and wind tunnel studies. See Figure 1 for symbol details.

The Ri_f vs Fr_t estimates derived with the flux profiler instruments in thermally stratified water columns show considerable scatter. Since we do not know the history or the position within the events that the data were sampled from we have included all the estimates which satisfied the previously mention criteria; resulting in both up and down gradient estimates being present (Figure 2.b.). Given the above observation that the scatter in the estimates increases with a reduction in the ratio L_s/L_c the spread in data is not a surprising result, since for our profiling probe data $2.2 < L_s/L_c < 100$.

The scatter observed in all these the results show that the greater the number of samples the more certain the estimate will be. This result was also shown, and to some extent quantified, by Ivey *et al* (1995). They found that only relatively few samples would be required to obtain estimates of Ri_f accurate to 50%.

To ascertain whether any discernible trends existed between Fr_t and Ri_f in the thermally stratified water measurements data were binned together over small integer ranges of Fr_t (each bin containing at least 5 points) and an average Ri_f derived. For comparative purposes this was also performed with the laboratory and numerically derived estimates. The results, plotted in Figure 3, clearly show that all experimental conditions follow a similar trends even though there was considerable scatter in the original field data. The caption to Figure 3 shows the binning ranges that were used and the error bars indicate one standard deviation. The figure reveals that the Ri_f estimates for the thermally stratified water tunnel data is best predicted by that which applied to the salt stratified water tunnel experiments, for the range of data available. However, significantly more data is required to refine the numerical constants. This result means that the mixing efficiency diffusion coefficient for density (as well as salt, heat and contaminants) may be readily derived within a large range of environments given that Fr_t can be estimated.

CONCLUSIONS

By using new profiling instruments capable of recording vertical velocities and density on microscales we were able to determine mixing efficiencies within

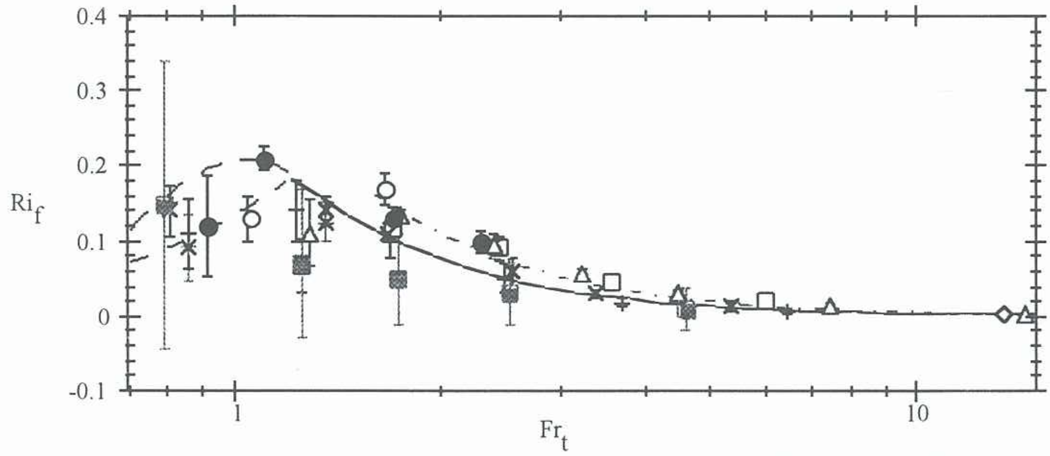

Figure 3. Plot of the average Ri_f verses average Fr_t estimates within a given Fr_t bin range. The error bars indicate the standard deviations of the Ri_f estimates. The bins ranges used are for * Stillinger et. al. (1983) 1-1.5, 1.5-4, x Rohr (1985) 0.5-1, 1-1.5, + Itsweire et. al. (1986) 0.5-1, 1-1.5, 1.5-2, 2-3, 3-4, 4-10, ◇ Tavoularis and Corrsin (1981) 13-15, Δ Lienhard and Van Atta (1990) 1-1.5, 1.5-2, 2-3, 3-4, 4-5, 5-10, 10-20, □ Yoon and Warhaft (1990) 1-1.5, 1.5-2, 2-3, 3-4, 4-10, o Jayesh et al (1991) 0.5-1.5, 1.5-2, ● Ivey *et al* (1995) 0.5-1, 1-1.5, 1.5-2, 2-3, and ■ present thermally stratified fresh water data 1-1.5, 1.5-2, 2-3, 3-4, 4-10. The solid and dotted lines represent the theoretical curves predicted by Ivey and Imberger (1991) for salt stratified water and thermally stratified air, respectively.

thermally stratified water columns. By considering simple scaling arguments, and combining these new findings with historical data, it was possible to derive simple relationships capable of predicting the efficiency of mixing based on readily measurable quantities. This now means that we can estimate the vertical diffusion rate for the most commonly encountered, stratified flow, conditions. The results also show that mixing events within thermally stratified reservoirs is very intermittent in nature.

Significantly more work is required to increase the sample set size to better determine the numerical constants required to relate the mixing efficiencies to readily measurable quantities. Further studies need to be performed to investigate Re_t dependencies, which were beyond the scope of this investigation.

ACKNOWLEDGMENTS

This work was partly supported by Australian Research Council Grant A89531551.

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