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A STUDY OF THE TAYLOR HYPOTHESIS USING WAVELET ANALYSIS

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Abstract

Based on two-point single hot wire measurements the validity of the Taylor hypothesis has been investigated. The dependence of the convection velocity with respect to the scales of the turbulent motion was studied using a Wavelet analysis of the two hot wire signals. The cross correlation between the two signals was computed for a large range of signal frequencies and probe separations. It was verified that the Taylor hypothesis is valid, at least for turbulent structures of streamwise extent comparable to the boundary layer thickness. (The probe separations used were too large to resolve accurately the convection velocities of the smallest scales). The structure convection velocity was found to be independent of the turbulent scale and its value was found to be very close to the mean velocity.

1 Introduction

The Taylor frozen equilibrium hypothesis is frequently applied in turbulent research. If the turbulence level is low, the turbulent events do not significantly influence the mean velocity field and may therefore be assumed to be convected downstream at a rate equal to the mean velocity, U. As a consequence streamwise space derivatives for the small scale motion may be replaced by temporal derivatives using the approximation $\partial/\partial t = -U\partial/\partial x$. The Taylor hypothesis, together with the assumption of isotropic turbulence, has been used for most attempts to measure the dissipation rate ($\epsilon = 15\nu\overline{(\frac{\partial u}{\partial x})^2} = 15\nu\frac{1}{U^2}\overline{(\frac{\partial u}{\partial t})^2}$) (e.g. Saddoughi and Veeravalli, 1994). The hypothesis has also been extensively used in two-point space-time correlation measurements (e.g. Donovan *et al.*, 1994) and the study of the topology of turbulent structures (e.g. Antonia *et al.*, 1990), in order to convert time lags into streamwise separations.

In many flows the turbulence level can not be assumed to be low, and strong velocity gradients exist. Hence the application of the Taylor hypothesis may be questionable, and the correct convection velocity, U_c, difficult to determine. Considerable efforts have been undertaken to prove the validity of the Taylor hypothesis also for these flows. Piomelli *et al.* (1989) tested the hypothesis using a Large Eddy Simulation data base. The correlation coefficient between the time derivatives of the velocity fluctuations, $\frac{\partial u_i}{\partial t}$, and the streamwise space derivative, $U_1\frac{\partial u_i}{\partial x_1}$, was found to be very close to 1 for $y^+ > 30$. This supports the validity of the Taylor hypothesis for the dominant turbulent motions.

The study of coherent motions in turbulence has revealed that the convection velocity may indeed be dependent on the type of motion considered. Gan and Bogard (1991) found for a turbulent boundary layer that the sweeps observed near the surface were convected at a velocity which was higher than the local mean value, while ejections had a convection velocity which was lower. This is in agreement with the speculations by Perry and Abell (1977) that the large scale motion is convected at a rate close to the free stream velocity, U_e, while the small scale convection velocity may be closer to the local mean. The same conclusion was derived by Smits *et al.* (1989) based on two-point wall pressure correlations.

It appears that the validity of the Taylor hypothesis may depend on the type of motion of interest and the range of turbulent scales present in the flow. It therefore appears to be important to be able to study the scale dependence on the convection velocity. The Wavelet transformation (Hudgins, 1993) is a useful tool for this purpose, since it allows a velocity data sequence to be separated into a set of signals which contains only information at the selected scales. Here results from an experiment in a low Reynolds number turbulent boundary layer will be reported.

2 Experimental details

The measurements were made in a turbulent boundary layer at a Reynolds number based on the momentum thickness, Re_θ, of 1409. The free stream velocity was $U_e = 2.0$ m/s and the boundary layer thickness, $\delta = 94.7$ mm. Two single hot wires, made of 2.5 μm partly etched Platinum-10 % Rhodium Wollaston wire, were used. The length of the etched part was 0.5 mm for both probes. The probes could be traversed independently of each other both perpendicular to the surface and in the streamwise direction. The forward probe was mounted with its prongs vertically, entering the boundary layer from the outer flow. The downstream probe was mounted in the streamwise direction, directly behind the first. To minimize probe interference, the forward probe was made as a very open construction, with a distance between the prongs of 10 mm. The rear probe was made as small as possible, only slightly wider than the sensing element. The signals were filtered at 6.5 kHz and sampled at 13 kHz for 47 seconds. The high sampling rate was chosen to be able to accurately determine the time delays between the two signals.

The interference between the two probes was checked by comparing the mean and rms velocities obtained with the two probes separated in the streamwise direction by approximately 5 mm. The two signals were found to agree within 1 % in the mean and 1.7 % in the rms values. This is within the expected experimental scatter.

3 Results

Measurements were made at a number of y/δ positions using six different probe separations, L. This covered the range $0.026 < L/\delta < 0.42$. The mean convection velocity was first estimated by computing the conventional two-point cross correlation between the two u-signals.

$$\rho_{u_1u_2}(\Delta x,\tau) = \frac{\overline{u_1(x_1,t)u_2(x_2,t+\tau)}}{u_1'u_2'}$$

This verified the findings of Piomelli *et al.* (1989) that the Taylor hypothesis appears to be valid outside $y^+ > 30$ (Figure 1). Only very close to the surface does the overall convection velocity drop significantly below the mean velocity. Although y/δ was only 0.03 ($y^+ = 17.8$) at the measurement point closest to the surface, which is an order of magnitude smaller than the probe separations, the same U_c was measured for all separations, indicating that the drop in convection velocity is genuine for the dominant structures and not a result of insufficient spatial resolution.

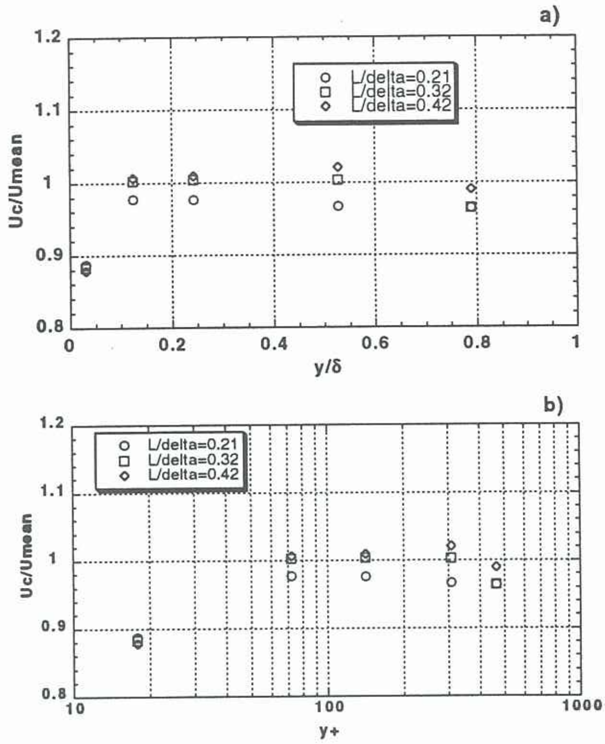

Figure 1 : Mean convection velocities based on two-point cross correlations.
a) Outer variables. b) Inner variables.

For each set of measurements a section of the signal from each probe was examined using the Haar wavelet. Since the Wavelet analysis is very CPU-time consuming, only a section of 16384 data from each probe were used, corresponding to about 1.26 seconds. 80 frequency scales, f_i, were selected, giving a range of frequencies of $9.41 < f_i < 8835$. This corresponded to a range of length scales of about $0.002 < x_i/\delta < 1.85$ at $y/\delta = 0.24$. (Here time has been converted to distance using the Taylor hypothesis, $x_i = -U/f_i$). The signals were then transformed back to the time domain, producing 80 pairs of velocity signals, each containing only the signal content from one specific length or frequency scale (see Figure 2). For each scale, i, the cross correlation function $\rho_{u_{1,i}u_{2,i}}(\Delta x,\tau)$ between the two u-signals was computed, from which the time delay was estimated. Figure 3 shows $\rho_{u_{1,i}u_{2,i}}(\Delta x,\tau)$ derived for one set of probe separations. A very distinct ridge corresponding to the peak correlation as function of the length scale may be observed. For large scale motions the ridge is quite wide and longer probe separations may be required to obtain consistent values for the time delay. At the smallest scales the correlation function becomes noisy due to the evolution of higher harmonics.

Therefore only the time delays τ_i for the intermediate scales were educed. Using the known probe separation the convection velocity as function of scale $U_{ci} = L/\tau_i$ was derived.

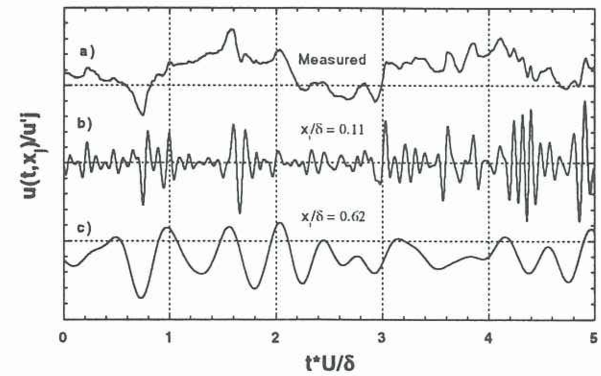

Figure 2 : Separation of the measured velocity signal into components corresponding to different Wavelet scales (data have been normalised using the rms values computed at the corresponding scales)
a) Original velocity signal b) x_j/L=0.11
c) x_j/L=0.62.

Vortex shedding from the prongs of the upstream probe was expected to contaminate the signal from the downstream probe when the probe separation became larger than the width of the upstream probe. Since this was assumed to mainly influence the small scale turbulent motion recorded, only scales larger than about one fifth of the probe separation were evaluated.

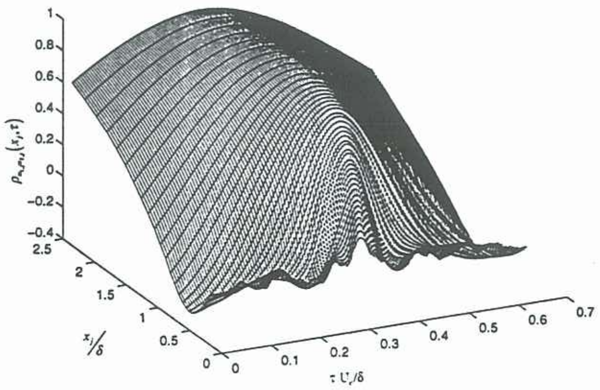

Figure 3 : Two-point velocity correlations, $\rho_{u_{1,j}u_{2,j}}(\Delta x, \tau)$ as function of time and Wavelet scales.

Figure 4 shows that the convection velocity is virtually constant for the range of scales examined. The velocity is very close to the mean velocity within the experimental scatter of about 5%. The scatter in the data was highest near the edge of the boundary layer and very close to the wall. It is believed that this is due to the intermittent nature of the velocity signals found here. Since the section of the time series examined was relatively short, it is possible that insufficient number of events were included to produce reliable estimates for the convection velocities. For the data analyzed, the scatter in U_{ci} observed at the innermost point ($y^+ = 17.8$) was too high to produce reliable results. Based on the velocities obtained from the overall convection velocities (Figure 1), it appears that a stronger dependence on the turbulent scales will be found close to the surface.

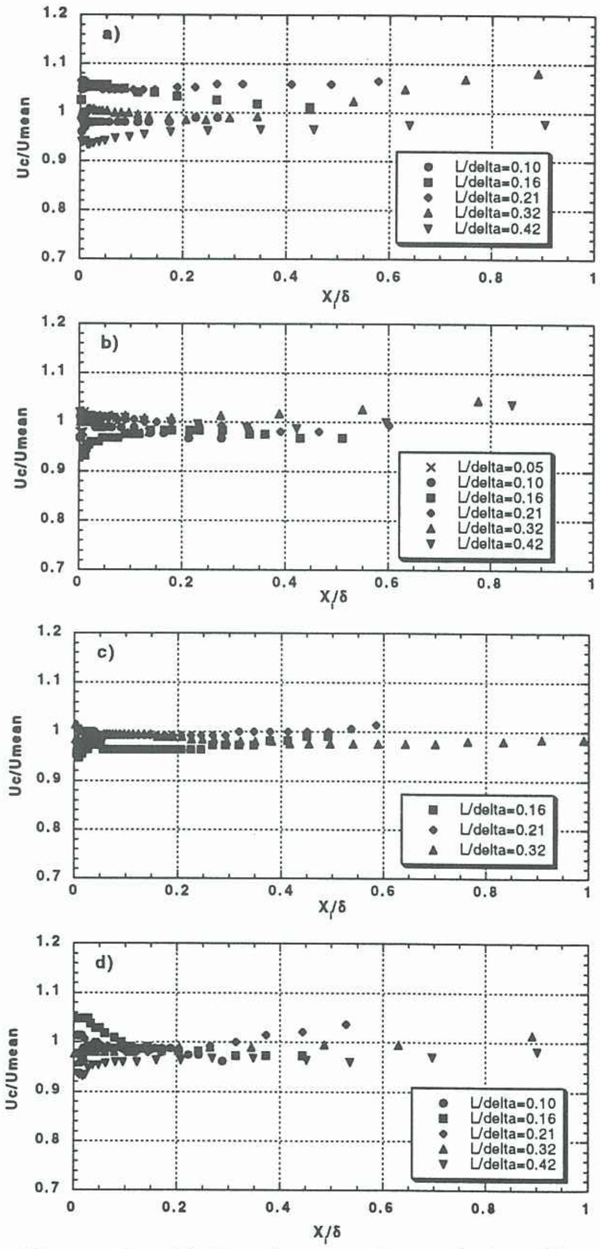

Figure 4 : Ratio of convection velocity, U_{ci}, to local mean velocity, U, as function of Wavelet length scale.
a) $y/\delta = 0.124$ b) $y/\delta = 0.243$
c) $y/\delta = 0.528$ d) $y/\delta = 0.792$

4 Conclusions

Two-point space-time correlations in a boundary layer have been examined to test the validity of the Taylor hypothesis. The results confirm that the hypothesis applies to the mean transport of turbulent quantities. The convection velocity for the fluctuations in the streamwise direction was found to coincide with the mean velocity, except very near the wall where a rapid decrease in the convection velocity is observed. Continuous Wavelet transforms were used in order to study possible scale dependent variations in the convection velocity. By breaking the signal up into sequences for a set of discrete wavelet length scales, it was shown that the hypothesis applies to all the scales of turbulent motion examined. The results in the buffer layer were inconclusive, but it appears that restrictions to the Taylor hypothesis applies here.

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