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SIMULATION OF MIXING FOR INCOMPRESSIBLE FLOW THROUGH A PERIODICALLY OBSTRUCTED CHANNEL

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ABSTRACT

Fluid mixing for constant volumetric flow through a channel containing periodic obstructions has been studied using a finite-difference simulation to determine fluid velocities, followed by the use of passive marker particle advection to look at fluid transport and mixing within the channel. For the geometry under consideration, Roberts (1994) showed there is a transition to time-periodic flow at a Reynolds number of around 100, based on the channel width and average velocity. These time-periodic flows show a marked increase in the mixing of fluid within the channel, relative to the steady flow. However it is not until Reynolds numbers of over 180 that efficient mixing of fluid throughout the channel takes place.

INTRODUCTION

Good mixing and heat/mass transfer are generally the crucial ingredients required for efficient continuous process engineering units. For sterile operation, the presence of impellers or flow obsructions leads to the possibility of the build up of degraded material, and to possible difficulties with cleaning. The use of turbulent flow through a pipe or channel to promote mixing or heat transfer could cause problems for shear sensitive materials, and provides little control over the rates of mixing or transport.

Figure 1 shows the channel geometry. Distance is scaled using the channel width, H. There are two geometrical parameters, the scaled height of the obstructions, T/H and the scaled spacing of the obstructions, L/H. For all of the calculations presented in this paper, $T/H = 0.25$ and $L/H = 1.5$. There is one flow parameter, the Reynolds number, $\mathrm{Re} = \rho \bar{V} H/\mu$, where ρ and μ are the fluid density and viscosity, respectively, and $\bar{V}$ the cross-sectional mean velocity. Velocities are scaled using $\bar{V}$ and times are scaled using $H/\bar{V}$.

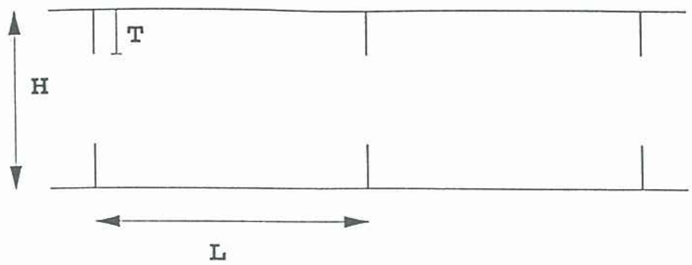

Figure 1. Channel geometry

FLOW PATTERNS

The flow velocities have been calculated using a finite-difference formulation of the vorticity transport and Poisson equations. The scheme is described in detail by Roberts (1994). A grid of size 64×61 over one cell of the channel was used, and the flow assumed to be two-dimensional and spatially periodic in the x direction. Numerical and experimental justifications for these assumptions are given in Roberts (1992).

For Reynolds numbers below 100, the flow is steady, and is characterised by slow moving recirculating regions forming downstream of each of the baffles. Figure 2 shows velocity vectors for Re $= 90$. Above a Reynolds number of 100, the flow becomes unsteady, due to the amplification of Tollmien-Schlichting waves by the presence of the obstructions. Figure 3(a-f) shows velocity vectors for Re $= 120$. The flow is time-periodic, with a dimensionless time period, $\tau \approx 2.18$ The flow is characterised by the motion of the recirculating regions downstream until they impinge on the next baffle. The figure shows flow over a period of $\tau/2$, with Figure 3(f) being the mirror across the channel centerline of Figure 3(a).

Similar transitions to time dependent flow in channels have been observed by Zhang and Tangborn (1994) for a mixed convection flow, and Ghaddar *et. al.* (1987) for flow in a grooved channel. For Reynolds numbers studied in this report (less than 210), the time dependent solutions are all time-periodic with τ a weak function of the Reynolds number. These flows are also symmetric in time, with $U(x, y, t + \tau/2) = U(x, 1 - y, t)$ and $V(x, y, t + \tau/2) = -V(x, 1 - y, t)$. U and V are the axial and cross-channel velocities, respectively.

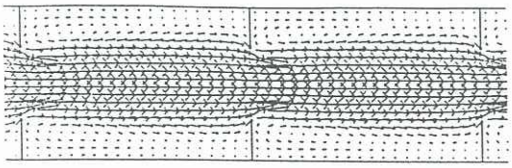

Figure 2. Velocity vectors, Re = 90

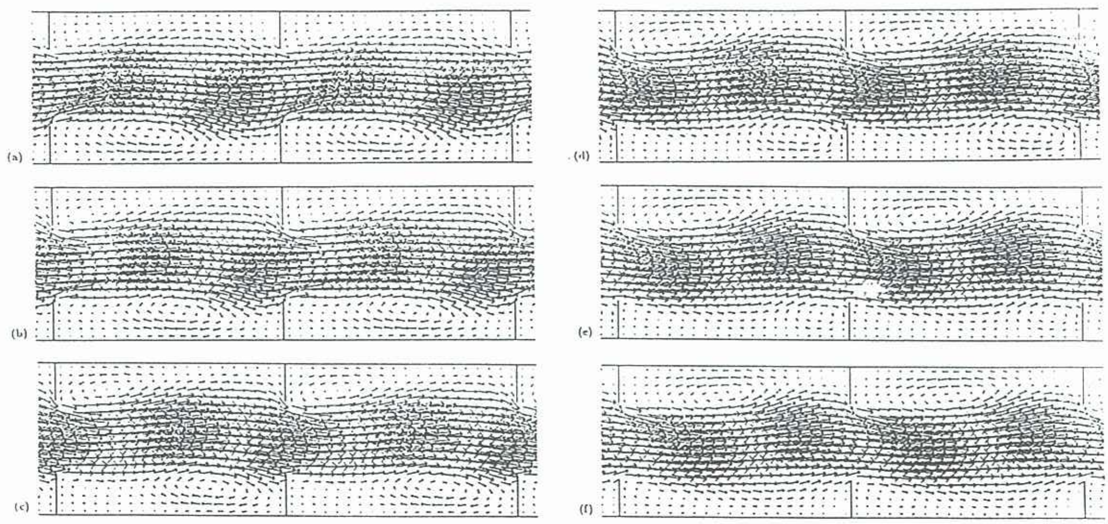

Figure 3. Velocity vectors, Re =120. (a) - (f) show one half-period of the flow.

PARTICLE TRACKING

We introduce ideal (massless) particles into this flow. The instantaneous particle velocity is just that of the local velocity field, so that the position of each particle is represented by $d\mathbf{X}(t)/dt = \mathbf{v}(\mathbf{X}, t)$, where $\mathbf{X}(t) = (x(t), y(t))$ is the position of the particle at time t. This ODE is solved independently for each particle using a fourth-order Runge-Kutta technique. The timestep used for the particle advection calculations is twice the timestep used for the calculations of velocity. For steady flow, the fluid particles simply follow the streamlines. When the flow is unsteady, the particle motion becomes more complicated.

Figure 4 shows the motion of a 'blob' of fluid in channel, described using 2500 particles. The initial position of the blob is shown in Figure 4(a). Figures 4(b)-(f) show the position of the blob at a scaled time of 40, for Reynolds numbers ranging from 90 to 210. Particles which had been swept downstream out of the visible channel were re-injected at the same local position, but in the cell which originally contained the blob. For the steady flow (Re = 90), the blob has been stretched due to its motion within the recircuating region, but has not been extensively mixed. For the time-periodic flows, at the lower Reynolds numbers (Re = 120, 150 and 180), while there is considerable mixing in parts of the channel, the particles in the blob have still not been able to mix with fluid in the top half of the channel. This restriction on the range of mixing has disappeared when Re = 210.

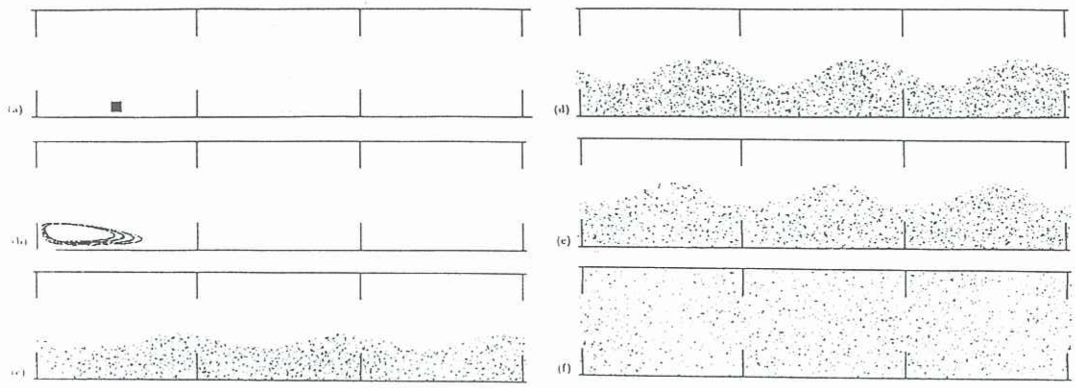

Figure 4. Motion of a 'blob' of fluid: (a) Initial position, (b) Re = 90, (c) Re = 120, (d) Re = 150, (e) Re = 180, (f) Re = 210.

Poincaré Sections of the flows have been determined in order to further investigate the mixing process. A detailed description of the generation of Poincaré sections for unsteady and spatially periodic flows is found in Ottino (1989). In the case of time periodic systems, the technique amounts to taking stroboscopic pictures at periodic intervals $\tau, 2\tau, 3\tau, \ldots\ldots$, etc, using an overlay technique which gives the particle's local position in a spatially periodic system (Howes *et. al.* 1991). Superimposing all pictures on one surface makes up the Poincaré section. Particles with regular movement each period form lines on a Poincaré section whereas chaotic particles become randomly distributed on the Poincaré section. Particles in a chaotic region show high rates of mixing, while those in regular regions do not. Furthermore, it is impossible for fluid to advect across regular regions. Heat and concentration can diffuse across regular regions, so they provide a barrier to transport and represent regions of inefficient mixing in the flow. These techniques have been recently used by Tangborn *et. al.* (1995) to investigate mixing in a two-dimensional mixed convection flow.

Figure 5 shows Poincaré sections for Re = 140, 160 and 180. The existance and size of these barriers can be clearly seen in this figure for the lower two values for the Reynolds number. These barriers take the form of regular regions that meander through the center of the channel, effectively dividing the fluid at the top and bottom of the channel. The width of the regular region decreases as the Reynolds number increases. When the Reynolds number is 180, there are no regular regions in the channel. However, the fluid still cannot fully mix across the entire width of the channel. Hence the lack of regular regions is a neccessary but not sufficient condition for effective mixing. Further tools will be required to study this barrier for mixing for the higher Reynolds number flow.

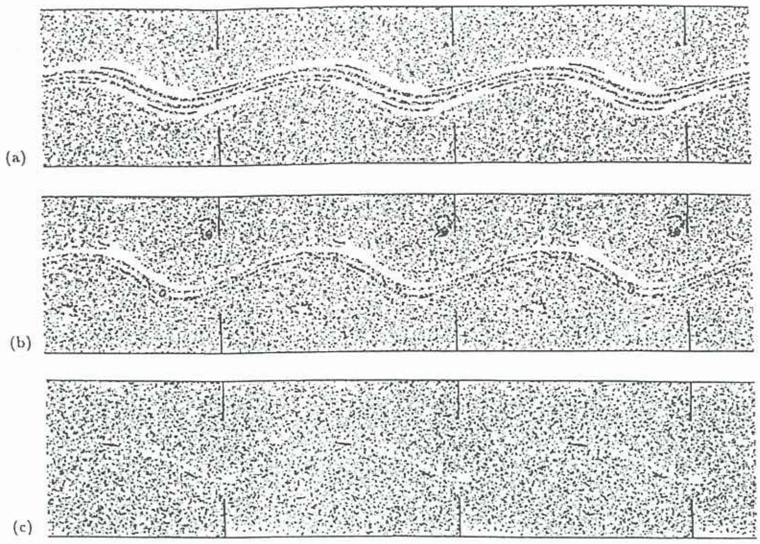

Figure 5. Poincaré sections for time-periodic flows: (a) Re = 140, (b) Re = 160, (c) Re = 180.

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