Twelfth Australasian Fluid Mechanics Conference
The University of Sydney, Australia 1995

MODELLING THE KORTEWEG-DE VRIES EQUATION FOR THE DESCRIPTION OF NONLINEAR INTERNAL WAVE TRANSFORMATION IN THE OCEAN

Peter E. Holloway[1], Efim Pelinovsky[2], Tatyana Talipova[2], Belinda Barnes[1]

[1]Department of Geography and Oceanography
University College, University of New South Wales
Australian Defence Force Academy
Canberra ACT 2600, Australia

[2]Institute of Applied Physics
Russian Academy of Science
Nizhny Novgorod, Russia

ABSTRACT

Long internal waves in the ocean, generated by tidal flow over the sloping topography of the continetal slope, are often observed to form into nonlinear waves as they propagate. These take the form of internal bores and associated internal solitary waves such as solitons. In this paper a numerical solution to a modified Korteweg-de Vries (K-dV) equation is employed to study the evolution of an inital waveform into bores (or shocks) and internal solitary waves with the model applied to conditions on the Australian North West Shelf (NWS) where such nonlinear waves are observed. It is shown that the K-dV model explains many of the features of the observed waveforms and that the waveforms that evolve are strongly dependent on vertical profiles of the background density and shear flow. In particular temporal and spatial variability in the nonlinear coefficient of the K-dV equation is seen as important in contributing to the nature and variability in the nonlinear internal wave field.

INTRODUCTION

Non-linear effects on oceanic internal waves are seen to manifest themselves in many different regions of the World Ocean, including the Australian North West Shelf (NWS). Observations, eg. Holloway (1987) and Smyth and Holloway (1988), show a typical picture of a long internal wave ($\lambda \sim 20km$) evolving into shocks (bores or internal hydraulic jumps) and as groups of short period waves of soliton-like form. In an attempt to explain the observed evolution of internal waves, a non-linear theory is used, based on a numerical solution to the modified Korteweg-de Vries (K-dV) equation. The model includes observed ocean stratification and background shear flow varying horizontally over slowly varying water depth and allows for dissipation through a quadratic friction term.

Observed time series of isotherm displacements and onshore currents are shown in Figure 1 from a mooring located in 109m water depth at the outer edge of the continental shelf on the NWS. The figure shows a variety of nonlinear wave forms including bores on both the leading and trailing faces of the long internal tide as well as short period (approximately 10 min, close to the buoyancy period) internal solitary waves.

K-dV MODEL

The model used considers an initial waveform for an internal wave propagating over topography with variable depth ($H(x)$) in the presence of a background density profile, which produces a background buoyancy frequency profile ($N(z)$), and a background shear current ($U(z)$). The initial waveform can be arbitrary but in experiments presented here is taken to be sinusoidal. The modified K-dV equation is employed to describe the nonlinear internal-wave evolution assuming the waves are long (wavelength exceeds the water depth) and are small amplitude (amplitude is small compared to the water depth).

The K-dV equation, including quadratic dissipation, is written as (eg. Zhou and Grimshaw (1989)),

$$\frac{\partial \eta}{\partial t} + c\frac{\partial \eta}{\partial x} + \alpha\eta\frac{\partial \eta}{\partial x} + \beta\frac{\partial^3 \eta}{\partial x^3} + \frac{c}{2Q}\frac{dQ}{dx}\eta + \frac{kc^2}{\beta}\eta|\eta| = 0 \tag{1}$$

where $\eta(x,t)$ is the wave displacement,

$$Q = \frac{c^2 \int_{-H}^{0} (c-U)\,(d\Phi/dz)^2\,dz}{c_0^2 \int_{-H}^{0} (c_0-U_0)\,(d\Phi_0/dz)^2\,dz} \tag{2}$$

and comes from the variable depth where values with index "0" are the values at some origin x_0 and k is the friction coefficient. The parameters α, β and c

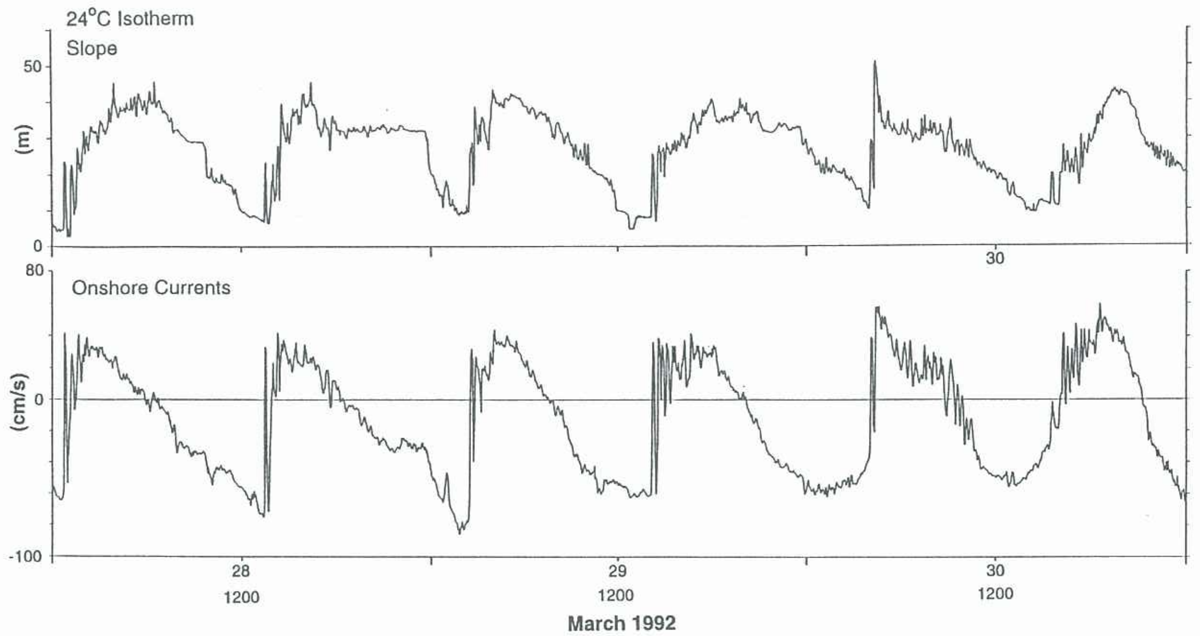

Figure 1. Observed time series of isotherm displacements and of onshore currents at a location in 109m water depth at the outer edge of the continental shelf on the NWS.

are coefficients of nonlinearity, dispersion, and phase speed of long internal waves, respectively. They are determined by the background density and horizontal velocity profiles and are defined as (in the Boussinesq approximation)

$$\alpha = \frac{3}{2}\frac{\int_{-H}^{0}(c-U)^2\,(d\Phi/dz)^3\,dz}{\int_{-H}^{0}(c-U)\,(d\Phi/dz)^2\,dz} \tag{3}$$

$$\beta = \frac{1}{2}\frac{\int_{-H}^{0}(c-U)^2\,\Phi^2\,dz}{\int_{-H}^{0}(c-U)\,(d\Phi/dz)^2\,dz} \tag{4}$$

where z is a vertical coordinate, positive upwards. The phase speed and vertical structure of vertical-displacement amplitude of a wave mode $\Phi(z)$ are determined by the solution of the eigenvalue problem

$$\frac{d}{dz}[(c-U)^2\frac{d\Phi}{dz}] + N^2(z)\Phi = 0\,,\ \Phi(-H) = \Phi(0) = 0 \tag{5}$$

The numerical scheme is described by Holloway et al. (1995).

Numerical solutions of (1) to (6) are found once stratification and background velocity are defined over a bathymetric cross-section. The model assumes that the internal wave can be described by a modal function and the waves are first mode. This neglects effects of steep topography and the propagation of waves along characteristics. The coefficients in (1) are also assumed constant but are allowed to vary slowly with distance such that at each value of x, local values are used.

DISTRIBUTION OF COEFFICIENTS IN THE K-dV EQUATION

The coefficients c, α and β are first calculated, without the effects of background shear flow, from two different data sets. The first is the climatic summer average for the NWS region from Levitus (1982) and the second from temperature/salinity surveys from the region carried out in January, 1995 (Holloway, Craig and Furnas, unpublished data). Results are plotted in Figure 2 showing the cross shelf distribution of the coefficients.

The values of c and β are very similar for both climatic data and 1995 data and show a decrease in values as the water depth decreases. The value of α is similar for the two data sets in deep water but there are significant departures on the shallower shelf. On the shelf the Levitus data does not represent the surface and bottom mixed layers (this data has a single profile for deep water that is truncated for shallower depths) which are seen in the 1995 observations. Of significance is the change in sign of α from negative in deep water to positive in shallow water, with the zero point at about 180 m water depth. This change in sign will have a large influence on the nature of any solitons that are formed. The change in sign requires a change in polarity of the solitons. In addition the strong spatial variability seen in α on the shelf will produce spatial variability in internal solitary waves.

MODEL RESULTS

Model runs were made with an initial sinusoidal wave with period of 12hr and amplitude 30m originating at a depth of 1382m (corresponding to the most offshore observation point) with no frictional dissipation ($k = 0$). These results showed the waveform steepen-

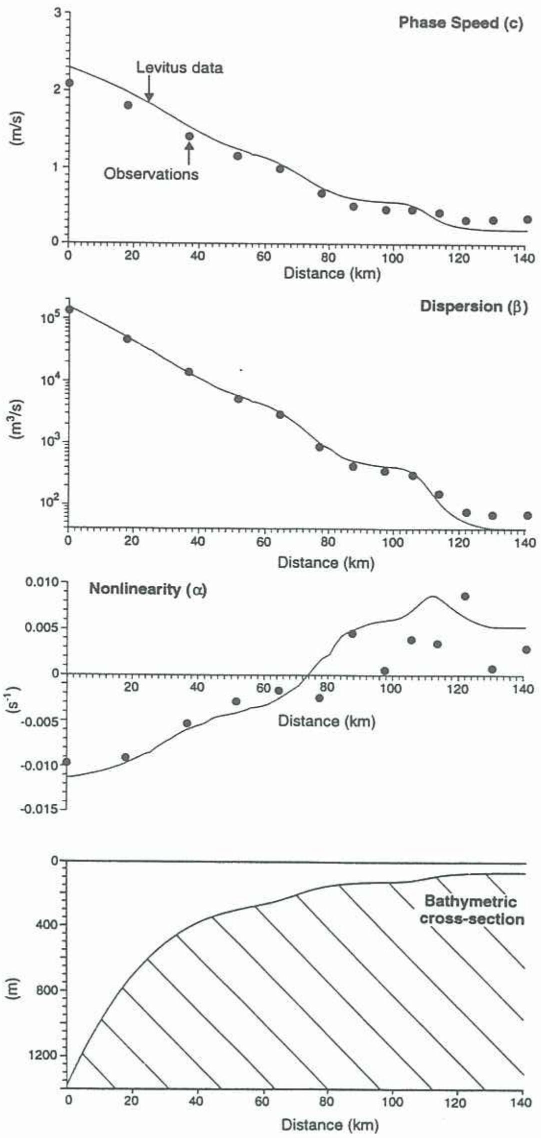

Figure 2. Cross-shelf distribution of coefficients of the K-dV equation calculated from Levitus Climatic data and from a temperature/salinity survey from January 1995.

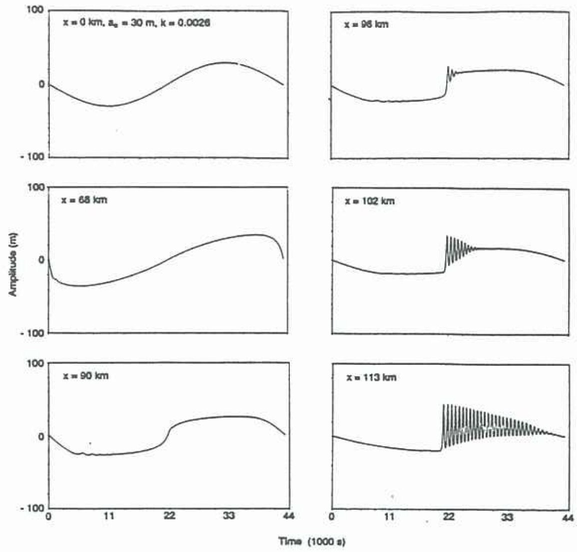

Figure 3. Modelled wave forms using cofficients from the Levitus data, with initial amplitude of 30m and $k = 0.0026$.

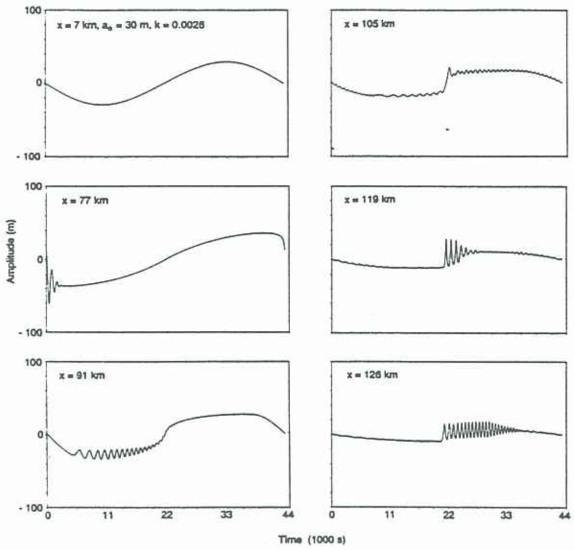

Figure 4. Modelled wave forms using cofficients from the 1995 data, with initial amplitude of 30m and $k = 0.0026$.

ing to form a bore and solitons, but the waves were much larger than observations. Results presented here are for $k = 0.0026$ which provides sufficient dissipation to provide good agreement between modelled and observed bore and solitary wave amplitudes.

Figure 3 shows modelled waveforms with c, α and β calculated from Levitus data and Figure 4 shows results from 1995 data. For the Levitus data, a bore forms after the wave has propagated 90km so that it is in a region of positive α. Solitons then form on the leading face of the wave and grow in number as the wave continues to propogate shorewards. The amplitude of the soliton envelope decays almost linearly away from the bore. A very smooth and regular wave pattern is seen with maximum soliton amplitude of about 50m, similar to observations. Results from the 1995 data produce a different wave evolution. A bore forms more rapidly, before 77km, in the region where α is negative. The solitons then propagate to the point of zero α and change form into a tail of oscillatory (cnoidal) waves. A second shock then forms at 105km and solitons evolve from this. The nature of these solitons varies somewhat as they propagate through a region of strongly varying α. Reducing the initial wave amplitude causes the bore to form later and hence in a region of positive α, so that the solitons formed propogate without a change in polarity.

INFLUENCE OF CURRENT SHEAR ON THE MODAL FUNCTION

The solutions above show the strong dependance of the wave evolution on the distribution of α. Equally

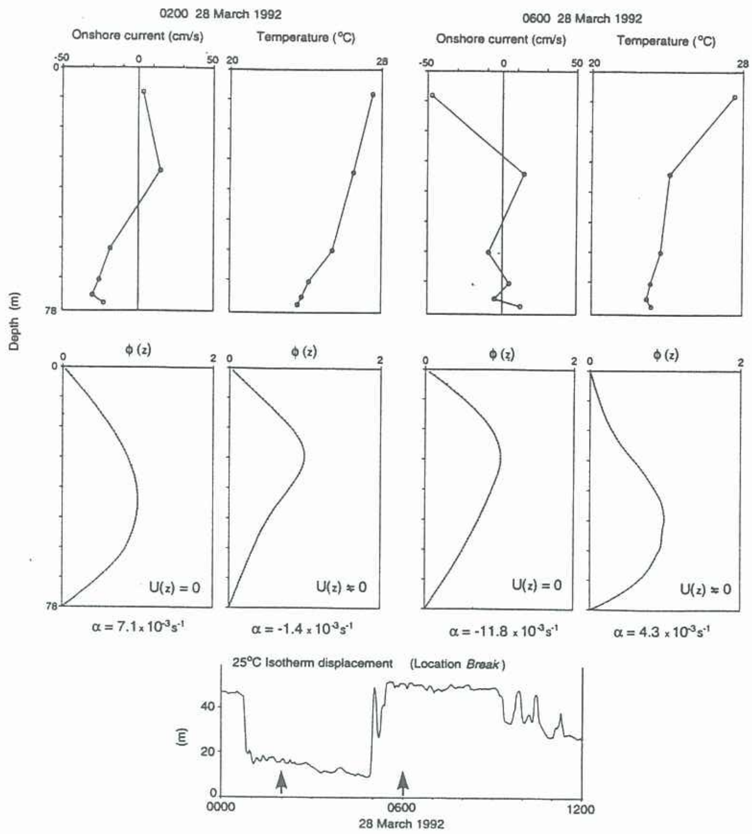

Figure 5. Velocity and temperature profiles for times ahead and behind a bore along with vertical modal functions calculated with and without the influence of background current, and corresponding values of the nonlinear coefficient (α).

it is found that for shallow water at the top of the slope, background shear has a large influence on the modal structure and hence on the value of α. This is demonstrated by the results plotted in Figure 5. Modal functions are plotted for two different times, ahead and behind a bore, with temperature and velocity profiles as shown. If the background flow is neglected, the modal functions show a maximum wave amplitude below mid-depth for the depressed stratification (0200 hr) and the reverse for the raised stratification (0600 hr). Values of α are positive and negative respectively, as could be anticipated. However, the shear strongly distorts the modal functions and reverses the signs of α, hence changing the nature of solitary waves that occur in these regions.

DISCUSSION

The application of weakly nonlinear theory, using a first order K-dV equation including frictional dissipation, reproduces many observed features of bores, solitons and short period oscillatory waves that occur as an internal tide propogates across the continental slope and shelf. However, many of the waves are strongly nonlinear with heights exceeding half the water depth. In these cases weakly nonlinear theory is not strictly valid and future modelling should include temporal and spatial variability in the K-dV coefficients.

REFERENCES

Holloway P.E., 1987, "Internal hydraulic jumps and solitons at a shelf break region on the Australian North West Shelf". *J. Geoph. Res.*, v. C92, 5405 - 5416.

Holloway, P.E., E. Pelinovsky, T. Talipova, B. Barnes, 1995, "A nonlinear model of internal tide transformation on the Australian North West Shelf". *J. Phys. Oceanogr.* (submitted).

Levitus S., 1982, "Climatological atlas of the World Ocean". NOAA Professional Paper 13.

Smyth N.F., Holloway P., 1988, "Hydraulic jump and undular bore formation on a Shelf Break". *J. Phys. Oceanogr.* v. 18, 947 - 962.

Zhou X., Grimshaw R., 1989, "The effect of variable currents on internal solitary waves". *Dyn. Atm. Oceans*, v. 14, 17 - 39.