

## FLOW REGIMES IN THE SLICED-CYLINDER MODEL OF OCEAN CIRCULATION

Ross W. Griffiths

Research School of Earth Sciences  
Australian National University  
Canberra, A.C.T. 0200  
Australia

### ABSTRACT

A laboratory  $\beta$ -plane model allows examination of aspects of wind-driven ocean circulation at mid-latitudes, including the effects of a two-layer stratification. The model is similar to that used by Beardsley (1969) in his experiments with a homogeneous fluid, except that the basin radius-to-depth ratio used here is much larger. In order to facilitate a comparison with two-layer flow we report new results for the homogeneous case. The structure of a fast western boundary current, its separation, adjustment of the separated stream to the interior flow, and transitions between stable, periodic and aperiodic eddy-shedding regimes are investigated. In two-layer flow separation of the western boundary current from the western wall is found to coincide with outcropping of the lower layer to the surface. Eddy-shedding occurs as a part of the adjustment to either a slow equator-ward Sverdrup flow (in the homogeneous case) or an energetic current extension flowing across the interior of the basin (in the two-layer case).

### INTRODUCTION

Many aspects of wind-driven stratified circulation on the scale of ocean basins remain unclear. These include the causes of separation of western boundary currents from their continental boundaries and the factors affecting the position of separation. Likewise, the strongly nonlinear dynamics and path of the energetic current extension flowing into the basin interior are difficult to model, as is the matching of this current to the interior return flow.

Parsons (1969) suggested that boundary current separation corresponds to surfacing of the thermocline in the region of anticyclonic wind stress curl. By requiring that the equatorward longitudinally-integrated wind-driven Ekman flux is equal to the net poleward geostrophic flow in the upper layer across any latitude he showed that outcropping will occur when the value of a forcing parameter  $\lambda = L\tau_m / g\Delta\rho d_1^2$  exceeds a critical value  $\lambda_c$ , where  $L$  is the basin width,  $\tau_m$  is a scale for

the surface stress,  $\Delta\rho$  is the density difference between layers and  $d_1$  is the mean depth of the upper layer. In this model the lower layer is assumed stationary. Veronis (1973) added to this model a cyclonic sub-polar gyre and the possibility of a net poleward flux out of the basin. Numerical solutions by Huang & Flierl (1987) show how an extension of this model predicts a subtropical warm pool isolated to the south of the basin for sufficiently large forcing, and even to the southwest after separation from the eastern boundary under extreme forcing.

In the Parsons-Veronis hypothesis the boundary current extension was expected to follow the isopycnal outcropping. Pedlosky (1987) modified the model by considering the wind stress as applied only to the surface Ekman layer, so that transport in the Ekman layer is allowed to cross the outcropping density front. Hence some lower-layer water is carried into the upper layer, where it was assumed to be converted into upper layer water by warming. This modification resulted in a significant change in the global circulation pattern, in which outcropping of the interface is no longer associated with a separated boundary current flowing along the outcropping front. Pedlosky appealed instead to the local vorticity dynamics of the boundary current as the cause of separation.

Beardsley (1969) investigated homogeneous flow forced by a differentially rotating lid in the 'sliced cylinder' geometry, and his study has been followed by further experimental, analytical and numerical studies (Pedlosky & Greenspan, 1967; Beardsley & Robbins, 1975; Becker & Page, 1990). These have been successful in modelling some aspects of the mid-latitude circulation, particularly the interior Sverdrup balance closed by an intense western boundary current. However, even in this simple model there remains uncertainty over the mechanism responsible for separation of the western boundary current (Becker & Page, 1990). The only previous experiments with two-layer models (Krishnamurti & Na, 1978) were attempted in a somewhat different geometry in which the  $\beta$ -effect was simulated by a conical slope on both base and lid of



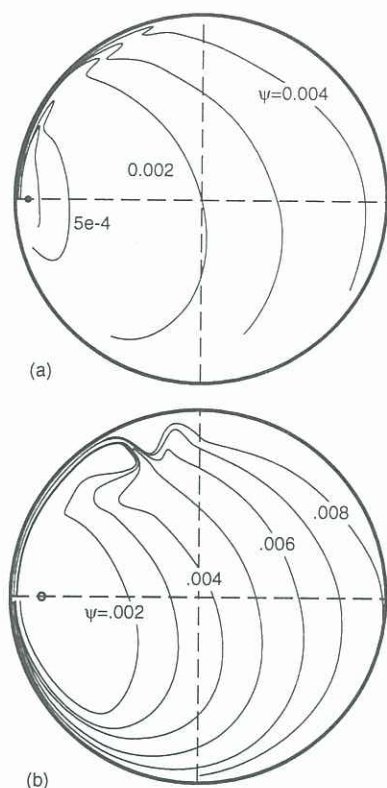


FIG. 1. EMPIRICAL STREAMFUNCTIONS FOR LINEAR AND NONLINEAR STABLE CASES.  $Ro=.011$  AND  $.067$ ;  $Re=2.0$  AND  $5.9$ . ( $\psi$  SCALED BY  $Ro\Omega DL^2$ )

a cylinder. Radial barriers served as meridional boundaries. Surfacing of the lower layer against the (differentially rotating) lid at the radial 'western' boundary was observed, but little quantitative information on separation and outcropping was obtained.

In new experiments in a planar  $\beta$ -plane model we examine the circulation in both one- and two-layer cases. Both base and lid are sloping in order to apply a  $\beta$ -effect in each layer, and the flow has a much smaller aspect ratio (depth/width) than those used previously in order that the boundary current width and eddy diameters be much smaller than the width of the basin. We particularly aim to explore the relationship between one- and two-layer flows, and to test the Parsons-Veronis hypothesis that the position of separation of the western boundary current coincides with the location of lower layer outcropping, which should depend on the wind stress forcing relative to the buoyancy restoring force on the density interface. To begin with, we concentrate on the flow regimes found in the homogeneous case.

#### APPARATUS AND METHOD

Experiments were carried out in a rotating cylindrical tank of diameter  $L=1.0$  m. The north-south gradient of planetary vorticity is simulated by a variation of the depth of each layer produced by planar sloping top and bottom boundaries. The lid rotates relative to the cylinder about an axis normal to the lid. The base slope  $\alpha_2$  was set at either  $\tan\alpha_2 = 0.05$  or  $0.10$ , while the lid was set to a number of slopes between zero and  $\tan\alpha_1 = 0.10$ . In one-layer cases the gradient of water depth is  $\alpha = \alpha_1 + \alpha_2$ . However, in order to impose similar values

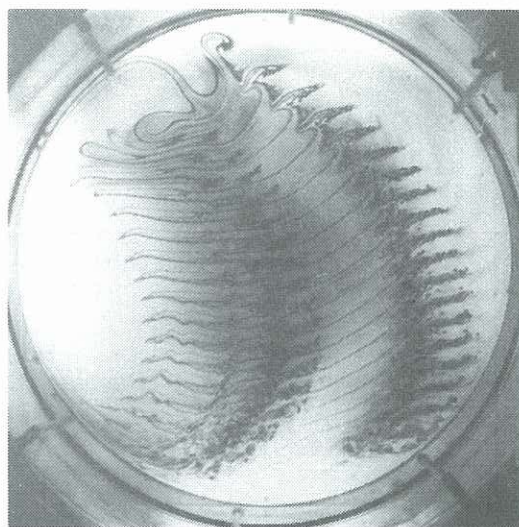


FIG. 2. PERIODIC EDDY SHEDDING IN AN HOMOGENEOUS FLOW (NORTHERN HEMISPHERE ORIENTATION,  $\Delta\Omega$  ANTICYCLONIC, WATER SHALLOWEST AT TOP OF IMAGE).  $Ro=0.053$ ;  $E=6.3\times 10^{-5}$ ;  $Re_E=6.7$ ;  $T=350$  ROTATIONS AFTER BEGINNING OF DYE RELEASE.

of  $\beta = (f/d_i)\tan\alpha_i$  in each layer of the two-layer case, the preferred conditions were  $\tan\alpha_2 = 0.10$  and  $\tan\alpha_1 = 0.05$ , with the ratio of upper to lower layer depths  $d_1/d_2 \approx 1/2$ . The total mean depth of water,  $D$ , was either  $15.0$  cm or  $12.5$  cm. Hence the aspect ratio  $D/L = 0.128$  or  $0.154$ , much smaller than the aspect ratios used by Beardsley (1969) ( $D/L = 0.502$ ) and Becker & Page (1990) ( $D/L = 1.17$ ). In our two-layer runs the ratio of mean depth-to-width of the *upper* layer is  $d_1/L \approx 0.04$ .

Tank rotation speeds  $\Omega = 0.5$ - $2.0$   $\text{rads}^{-1}$  and relative lid speeds  $\Delta\Omega = 0.0051$ - $0.16$   $\text{rads}^{-1}$  gave Rossby numbers  $Ro = \Delta\Omega/\Omega$  from  $0.0026$  to  $0.16$ , Ekman numbers  $E = \nu/\Omega D^2$  from  $3\times 10^{-5}$  to  $1.3\times 10^{-4}$  and  $E^{1/4} \sim \tan\alpha$ . Only anticyclonic lid motion was used. Hence the boundary current flowed from deeper to shallower water. For two-layer cases a layer of salt solution was carefully filled beneath a layer of fresh water so as to establish a sharp density interface, then left to approach solid body rotation before the lid motion was begun. A single density difference  $\Delta\rho/\rho = 0.005$  was used in order to reduce the number of variables. With  $d_1 = 4$  cm and  $\Omega = 1$   $\text{rads}^{-1}$  the baroclinic deformation radius was  $2.2$  cm.

A particularly effective but simple method of flow visualisation in these experiments is to bleed narrow streams of dye into the flow from  $1$  mm diameter syringe tubes positioned at chosen points in the tank. The most useful positions for these continuous dye sources were found to be within the boundary current. Hence they are positioned at the 'west', from where the dye lines are passively advected 'poleward' with the current. They enter the interior with the separated current, return through the interior of the basin, and eventually re-enter the western boundary current in the 'southern' region (using northern hemisphere directions). Dye reveals more information on the flow field than other techniques, such as streak photography, because the flow is unsteady and involves a large range of velocities ( $10^{-2}$  to  $1\text{ cm s}^{-1}$ ). In addition, measurements were made of velocity profiles in the boundary current and the interior by using a solution of Bromothymol blue PH indicator and electrode wires stretched from the centre of the tank to the 'west' wall. Temporal records of flow



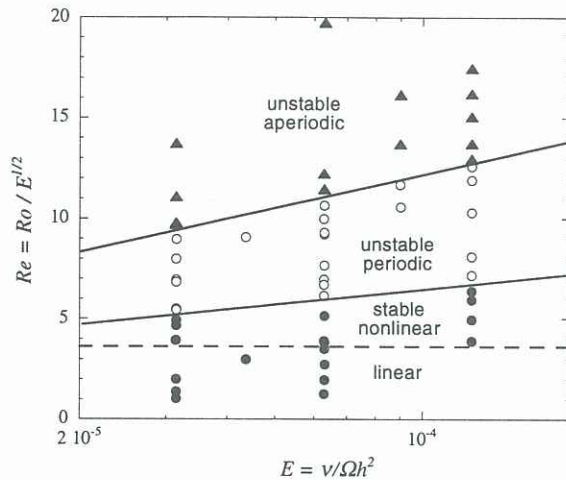


FIG. 3. STABILITY TRANSITIONS IN ONE-LAYER FLOW.

speed at selected points were obtained using over-heated thermistor probes (which resolved speeds of  $10^{-2}$   $\text{cm s}^{-1}$ ).

#### RESULTS FOR HOMOGENEOUS FLOW

As in earlier  $\beta$ -plane experiments, a fast, narrow boundary current develops on the 'western' side of the tank, flowing 'poleward' when the surface stress distribution imparts anticyclonic vorticity. For very small Rossby numbers circulation is centred to the 'west' of the basin, has 'north-south' symmetry about the centre, and streamlines diverge from the boundary current at all 'latitudes', consistent with linear theory (Pedlosky & Greenspan, 1967; Beardsley, 1969) and numerical solutions (Beardsley & Robbins, 1975; Becker & Page, 1990).

For sufficiently large forcing (apparently at a critical Reynolds number for the upper Ekman layer) the flow structure changes significantly (Fig. 1a, b). The current separates from the western wall as a narrow jet in which streamlines show no significant divergence. The jet penetrates a short distance into the interior before suddenly diverging to enter the broad slow interior drift 'equatorward'. Increasing the forcing slightly farther causes the flow to become unstable in this adjustment region (Fig. 2). We have defined instability here as the formation and shedding of *cyclonic* eddies to the west (closed anticyclonic eddies begin to be shed to the 'east' for still greater forcing). As instability is associated with divergence of the flow from the narrow separated jet to the full width of the interior of the basin, the unsteadiness leads to very extensive stirring of the passive tracer. The transition (Fig. 3) is well described by a critical value  $Re_c$  of the Ekman layer Reynolds number  $Re = Ro/E^{1/2}$ , where  $E = v/\Omega D^2$ . Beardsley (1969) and Beardsley & Robbins (1975) reported a similar instability, but placed the threshold under their experimental conditions (smaller aspect ratio) at larger values of  $Re$ . Another new observation is that there is a further transition from periodic eddy shedding to aperiodic flow at  $Re > Re^* > Re_c$  (Fig. 3). This transition is clear from both flow visualization and the records of flow speed past probes positioned in the boundary current and in the region of eddy-shedding (Fig. 4).

Velocity measurements in the interior of the homogeneous flows are consistent with  $v \sim E^{1/2} \tan \alpha$ , as

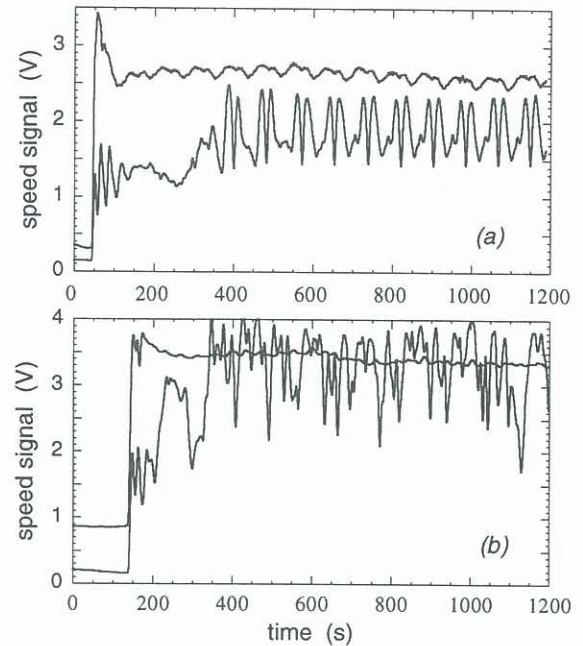


FIG. 4. FLOW SPEED IN THE BOUNDARY CURRENT (UPPER TRACE) AND EDDY-SHEDDING REGION, SHOWING SPINUP WHEN WIND STRESS IS TURNED ON AND ADJUSTMENT TO FINAL PERIODIC (A) OR APERIODIC (B) FLOW.  $\Omega = 2.0 \text{ s}^{-1}$ .

predicted for a Sverdrup balance between Ekman divergence and stretching of fluid columns as they are advected across contours of constant  $\Omega/D$ . The boundary current velocities and width are also consistent with those predicted for a Munk boundary layer.

The position of separation of the boundary jet is of particular interest for comparison with two-layer flows. Separation is meaningful only under conditions for which the current leaves the wall as a narrow jet. It occurs at  $\theta = 35^\circ \pm 3^\circ$  west of the shallowest point and is insensitive to conditions, showing only a possible small shift in the upstream direction ('equatorward') for larger forcing. This finding differs from the computations and experimental observations of Becker & Page (1990) which were for a cyclonic surface stress, a smaller aspect ratio and relatively small values of the slope parameter  $S = L \tan \alpha / D$ .

#### TWO-LAYER FLOWS

In our two-layer experiments the intense western boundary current extends throughout the depth, but velocities are smaller in the lower layer. The current separates from the western wall and forms a jet flowing into the interior (Fig. 6). Separation always coincides with outcropping of the lower layer to the surface at the wall. However, contrary to Parson's hypothesis, separation and outcropping occurs in an approximately fixed location, with only weak dependence on conditions. In addition, the lower layer flow separates from the wall at the same location. Thus, although separation occurs at slightly 'lower latitudes' than in the homogeneous case (on average at  $45^\circ \pm 7^\circ$  instead of  $35^\circ \pm 3^\circ$  from 'north'), the results indicate a primarily barotropic process.

A qualitative similarity between the one- and two-layer flows can also be seen downstream of separation. The jet loops first anticyclonically to smaller 'latitude'





FIG. 5. TWO-LAYER FLOW (SOUTHERN HEMISPHERE ORIENTATION,  $\Delta\Omega$  ANTICYCLONIC, WATER SHALLOWEST AT BOTTOM OF IMAGE). DARK DYE IN UPPER LAYER, LIGHT DYE IN THE LOWER LAYER.  $Ro=0.057$ ,  $\lambda=0.34$ .

and then cyclonically to flow 'poleward' again. This unstable anticyclonic loop follows the perimeter of an anticyclonic recirculation close to the western boundary.

The interior flow, on the other hand, is greatly altered by the effects of stratification. In the adjustment region the loop flow and eddy-shedding is highly baroclinic. Eddy shedding divides the upper layer jet transport between an eddy flux equatorward and to the west and a coherent stream flowing across the interior (Fig.5). This is a frontal stream coincident with outcropping of the density interface. It is unstable, with meanders continually developing into eddies. The front divides the upper-layer warm pool on the 'equatorward' side from a region on the 'poleward' side in which unstratified lower-layer water is in contact with the lid. Furthermore, the water carried by the cyclonic eddies shed from the loop does not continue to drift 'equatorward' in the western part of the basin as it does in one-layer experiments, but instead is advected slowly *eastward* some distance 'equatorward' of the outcropping front. In fact, flow in the interior of the upper layer pool is predominantly a slow anticyclonic gyre, reminiscent of the oceanic sub-tropical gyres. Both branches of the upper layer transport thus join a broad eastern boundary current that returns water 'equatorward', where it is again taken into the western boundary current.

In the eastern half of the basin there is a small flux of lower layer water into the upper layer as a result of the surface Ekman layer transport across the outcropping front in the manner discussed by Pedlosky (1987). This water is gravitationally unstable, mixes, and slowly adds to the upper layer volume. It is not yet clear to what extent this mixing contributes to the observed circulation.

Quantitative results for the two-layer flow will be given elsewhere. Here we simply note that the latitude of the outcropping front in the interior, unlike the separation position, does vary with surface stress, lid slope and upper layer depth: increasing  $\lambda$  shifts the front and the associated extension stream to deeper water (i.e. 'lower latitudes'). Thus the cyclonic loop and eddy-shedding represent an adjustment between the

narrow jet emerging from the fixed separation point and the forcing-dependent location of the extension stream. For  $\lambda < \lambda_c \approx 0.4$  there is no region of lower layer outcropping in the basin interior (only a local outcropping at the wall near the separation point). For sufficiently large forcing ( $\lambda > 0.8$ ) the upper layer also separates from the eastern boundary and the upper layer is then confined to a western 'low-latitude' pool, as in the numerical solutions of Huang & Flierl (1987). We note that the critical values of  $\lambda$  in this laboratory model are not expected to coincide with the theoretical values predicted for a cosine zonal wind stress in a rectangular basin with a stationary lower layer.

## CONCLUSIONS

A laboratory two-layer  $\beta$ -plane model reproduces many of the features of mid-latitude circulation and theoretical  $1\frac{1}{2}$ -layer models. The experiments model an isolated sub-tropical gyre, neglecting effects of a reversal of the wind stress curl and interaction with a cyclonic sub-polar gyre. Since the Ekman number, hence water depth, determines the boundary current width, quantitative aspects of the flow most relevant to the oceans are best investigated for small ratios of water depth to basin width. Further experiments are being carried out, using a broader range of conditions along with more detailed investigation of the lower layer motion and effects of a sloping side wall. Aspects of the flow to be studied in greater detail include the mechanism responsible for separation of the boundary current, the process of rapid adjustment from the separated jet to the broader interior flow, and the role of transport across the outcropping front.

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