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THE TORSIONALLY DRIVEN CAVITY: A TEST CASE FOR COMPARISON BETWEEN EXPERIMENT AND PREDICTION

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ABSTRACT

Numerical and experimental investigations of vortex breakdown in a torsionally driven cylindrical cavity flow have been undertaken. It is demonstrated that the approach to numerical convergence is strongly dependent on whether grid compression near the rotating lid is included. In the case of a uniform mesh, for the Reynolds number considered, increasing the grid resolution leads to the appearance of initially fewer recirculation regions and then an increase. This non-monotonic route to convergence was not observed when the Ekman layer was better resolved using grid compression at the rotating lid. Measurement uncertainties associated with rheometers leads to an uncertainty in the viscosity and hence the Reynolds number of the flow. It is demonstrated that comparison of predicted flow patterns at nominally the same Reynolds number can be misleading; what is termed validation in these circumstances is at best coincidental.

INTRODUCTION

It is common, and commendable, to find published papers increasingly containing validating experiments for numerical predictions. In the absence of such validation, predictions may lack conviction due to the numerous possible sources of error, such as inadequate modelling, incorrect coding, and insufficient mesh resolution.

Where experimental results are available for validation, or otherwise, of predictions, conclusions reached follow a certain pattern. Divergence between observation and prediction is usually described as experimental error (by numericists) or simplistic modelling (by experimentalists). As the two sets of results converge, analysis of possible errors diminishes until in the case of precise agreement, little analysis of errors is presented and it is assumed that both sets of results are correct.

It is the purpose of this paper to demonstrate that the 'holy grail' of perfect experimental validation sought by numericists can result in some selectivity of presentation and possibly deluded confidence in the results. Specifically, we show that 'inaccurate' experimental results can match 'inaccurate' predictions. The particular flow investigated, the torsionally driven cylindrical cavity flow, is simple in geometry and well defined; it has fixed or well specified rotating boundaries and is of a steady nature at the Reynolds numbers considered. This flow has been studied widely in recent times, both experimentally and computationally (e.g., Escudier, 1984, 1988; Lopez, 1988; Brown and Lopez, 1990; Lopez and Perry, 1992). One of the important characteristics of the flow is the appearance of one or more bubble-type vortex breakdown regions as the Reynolds number is increased.

Problems in interpreting another aspect of the torsionally driven cavity have been highlighted in a previous paper (Hourigan *et al.*, 1995); the investigation of streaklines for steady swirling flow in a lid-driven cylinder just prior to vortex breakdown was presented. It was demonstrated that the appearance of regions of wiggles on the otherwise seemingly straight central dye streakline is due to small, and almost inevitable, offsets in the injection of the dye at the stationary end disk. Previously, the appearance of these wiggles was thought to have been due to unsteadiness or asymmetry in the flow.

In the following, we demonstrate that the use of a uniform grid system for the numerical modelling of the swirling flow can lead to a non-montonic approach to the converged solution. An apparent matching of prediction and observation can occur at relatively low grid resolution. A further complication arises in that the viscosity of the fluid used for the experiments is not known to better than a 5% error; this translates to a comparable uncertainty in the Reynolds number. Significant changes in the flow structure can appear for changes in the Reynolds number of this order. It is therefore problematical whether an 'accurate' numerical prediction will be 'validated' by an observation. We show that an 'inaccurate' prediction can match an 'inaccurate' measurement; conversely, an accurate prediction may be 'invalidated' by an observation.

EXPERIMENTAL EQUIPMENT

A brief outline of the design of the swirling rig used is presented here.

The radius of the cylinder was 0.070 m. The rig included a water bath which reduced optical distortion and also maintained a stable temperature in the working fluid. The rotating lid at the bottom was driven by an electric motor with a choice of reduction gearbox or direct belt drive to give a wide range of speeds up to over 1400 rpm.

Photographs were taken using a Nikon FM2 35 mm camera fitted with a 58 mm Nikkor Noct lens. Dye injection was used to visualise the structures. The dye was a small quantity of flourescein powder dissolved in some of the working fluid and was injected by means of a hypodermic syringe through a small diameter hose leading to a hole of diameter 0.5 mm in the centre of the fixed lid. Care was taken to ensure that the dye was injected at a rate that did not disturb the main flow. Illumination was by a Coherent Highlight argon-ion laser which was piped to the rig by a fibre optic cable and expanded into a light sheet by a cylindrical lens.

The fluid used for the present work was a solution of glycerol in water with 76% by weight of glycerol. The density of the fluid was 1197.6 kg m^{-3}. The viscosity was measured using a Contraves Rheomat 108 and was 0.044 ± 0.001 Pa s.

The Reynolds number for the swirling flow is defined as $\mathrm{Re} = (\rho \Omega R^2)/\mu$, where ρ is the density, Ω the angular speed of the lid, R the radius of the cylinder and μ the fluid viscosity.

NUMERICAL METHOD

The Galerkin finite-element method was used to obtain the solution of the Navier-stokes equations for axisymmetric flow in cylindrical coordinates. Two grid systems were used; one consisted of uniform rectangular elements; the other possessed the added feature of a sinusoidal compression of the first five nodes near the rotating lid. This compression was included to test the effect of better resolving the Ekman layer. The penalty formulation with biquadratic Lagrangian interpolation for the velocity field and discontinuous bilinear Lagrangian interpolation for the pressure field was employed. The nonlinear set of equations was solved by Newton iteration, with the stopping criterion being when the norm of the velocity differences was less than 10^{-6}.

RESULTS AND DISCUSSION

Figure 1 shows flow visualisation results for a height to radius ratio of 3.5. The Reynolds number increases by slightly less than 5% yet there is a significant difference in the flow structure. At $Re = 2.957 \times 10^3$ the visualisation shows only one clearly identifiable (and

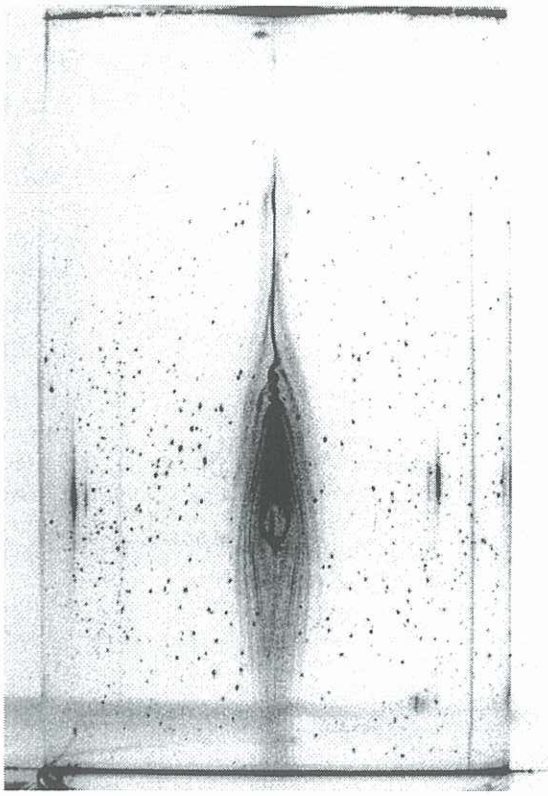


Fig. 1(a) $\mathrm{Re} = 2.957 \times 10^3$

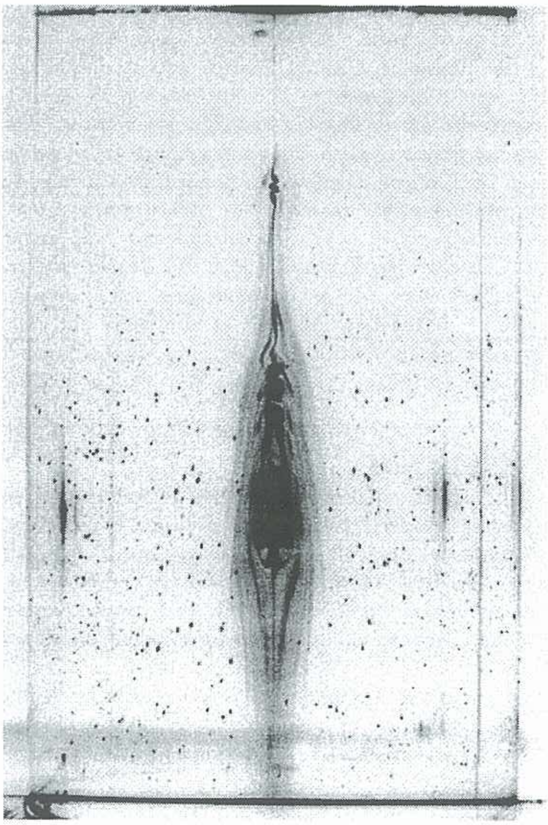


Fig. 1(b) $\mathrm{Re} = 3.031 \times 10^3$

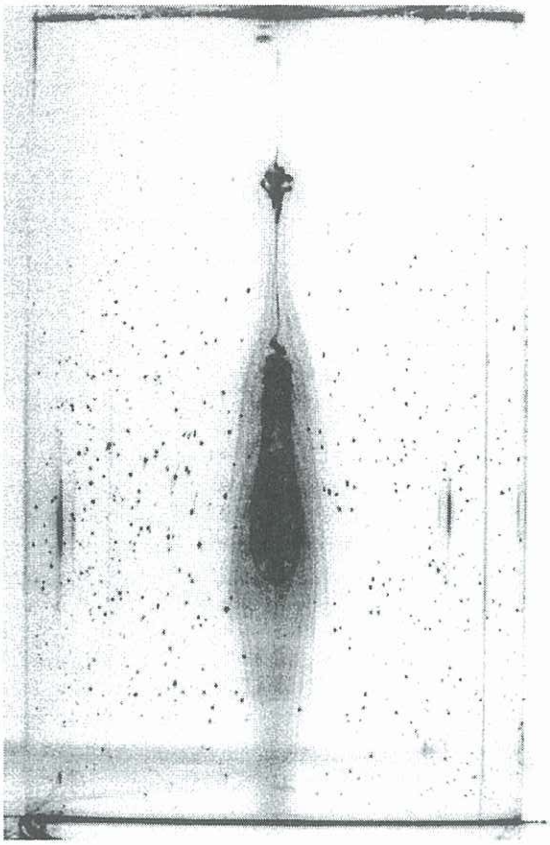


Fig. 1(c) Re = 3.092×10^3

Fig 1. Photograph (reverse image) of observed streaklines for different Reynolds numbers. Bottom lid is rotating, top lid is fixed. Dye is injected nominally at the centre of the top lid.

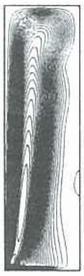
weak) recirculation zone. About a quarter of the way down the centre of the cylinder and also just above the recirculation zone are pre-breakdown spirals similar to those reported by Hourigan et al (1995). These develop into recirculation zones as the Reynolds number increases.

Figure 2 shows the streamlines obtained for the case of a uniform grid of increasing number of nodes. It is seen that as the number of nodes increases, the top recirculation bubble disappears and reappears again when a higher number of nodes are used. Figure 3 shows the corresponding results for the compressed grid with Fig. 3d illustrating the resolved prediction. Note that the recirculation pattern, for an unresolved prediction, in Fig. 2c (at $Re = 2.95\times10^3$) quite closely matches the experimental result at $Re = 2.957 \times 10^3$, Fig. 1a.

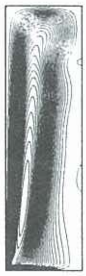
The converged result of Fig. 3d implies that the experimental Reynolds numbers are slightly too low although this amount would still be within the experimental error quoted. Hence care has to be taken when comparing computations with experimental results with non-zero experimental error as this can lead to erroneous conclusions about the correctness of simulations when the convergence process is not completely understood.

(a) Mesh (60 × 30)

(b) Mesh (100 × 50)



(c) Mesh (120 × 60)



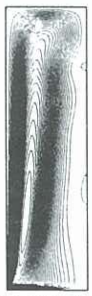
(d) Mesh (280 × 140)

Fig. 2 Predicted half-flow streamlines for increasing mesh sizes with a uniform mesh. $Re = 2.95 \times 10^3$.

(a) Mesh (60 × 30)

(b) Mesh (100 × 50)

(c) Mesh (120 × 60)



(d) Mesh (280 × 140)

Fig. 3 Predicted half-flow streamlines for increasing mesh sizes with a mesh compressed near the rotating lid. $Re = 2.95 \times 10^3$.

The plot of maximum streamfunction as a function of the number of nodes employed is shown in Fig. 4. It is seen that the approach to convergence is nonmonotonic for a uniform grid; in fact a nominally correct value of streamfunction (based on the converged compressed grid result) and hence flowfield is obtained at a much lower number of nodes than required for true convergence. If the flowfield at this resolution was compared to a 'correct' flow visualisation one may be fooled into thinking that one had also a 'correct' simulation when this is clearly not the case.

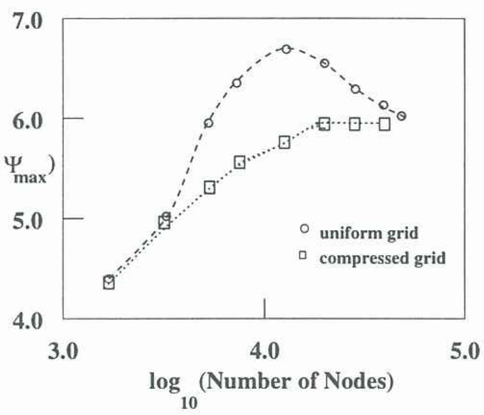


Fig. 4 Plot of predicted maximum streamfunction value versus number of nodes for the uniform grid and compressed grids.

The Ekman layer on the rotating lid has a thickness of the order $Re^{-1/2}$ times cylinder radius; in the present study, this thickness is less than one hundredth of the axial dimension of the cavity. Figure 2b is a case where the axial node spacing is approximately the size of the Ekman layer; it shows a solution with two recirculation zones similar to the fully resolved solution of Fig. 3d. It also has a maximum streamfunction value similar to the resolved solution. Further refinement of the mesh however leads to deviation of character of the flow and the maximum streamfunction value from the resolved case (e.g. compare the single recirculation zone of Fig. 2c with the two recirculation regions of the resolved case in Fig. 3d.) Eventually, as the Ekman layer begins to be better resolved with nodes appearing in the layer, the maximum streamfunction value for the uniform grid solution is seen to approach the resolved solution with a compressed mesh. The number and size of the recirculation regions also becomes similar (compare Fig. 2d to Fig. 3d).

Finally, the highly inaccurate solution of Fig. 2a for a coarse mesh shows 3 recirculation regions, similar to the observation at a higher Reynolds number in Fig. 1c. This demonstrates that matching predicted and observed flow patterns alone without considering the accuracy of the predictions or the uncertainty in the experiments can lead to illusionary validation.

CONCLUSIONS

An experimental and numerical study has been undertaken of the torsionally driven cylindrical cavity flow. It is found that the typically used uniform mesh may not resolve the Ekman layer; however, there is an apparent matching for coarser meshes with the final resolved solution. As the mesh is further refined, the predicted flow characteristics, such as number and size of recirculation regions, begin to differ significantly from the resolved solution. Further mesh refinement leads to a return to the characteristics of the fully resolved solution. Using a compressed mesh where nodes appear in the Ekman layer at much coarser meshes, the solution approaches the resolved solution faster and in a monotonic manner.

The validation of the predicted flow patterns with experimental observations can be illusionary if account is not made of experimental errors in determining, say, the viscosity of the fluid used. For water/glycerol mixtures, the consequent error in determining the Reynolds number may be as large as 5%; the characteristics of the flow can change dramatically over even such a small Reynolds number difference. This leads to the possibility of having an inaccurate prediction validated by an observation at an imprecisely determined Reynolds number.

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