UNSTEADY NATURAL CONVECTION ADJACENT TO A SEMI-INFINITE VERTICAL PLATE AN EXPERIMENTAL STUDY

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INTRODUCTION

Experimentally derived results for the transient natural convection adjacent to an isothermal, semi-infinite, vertical plate whose temperature is instantaneously incremented by ΔT above that of the unstratified ambient fluid are presented. The motivation for the work is for later comparison with the unsteady convective boundary layers adjacent the vertical sidewalls of rectangular cavities . The emphasis here is given to the start up of the flow, and the transition to steady state for the classical semi-infinite plate problem.

EXPERIMENTAL SETUP

To model a semi-infinite plate, we modified an existing square cavity rig, (for full details of pre-existing rig see Patterson & Armfield 1990). Briefly the cavity had a 2-dimensional working region of 24x24cm and extended 50cm in the 3rd dimension. The two vertical sidewalls were of 1.15 mm thick copper, approximating perfect conductors relative to the water. All other walls consist of thick perspex, approximating perfect insulators. Behind both the copper plates are large water filled reservoirs whose temperatures are controllable with heater circulators and coolers. Pneumatically driven, raiseable gates make it possible to restrain the water of these reservoirs from making contact with the copper plates. The temperature of the air gap between the gate and the copper plate was maintained at the cavity temperature with water carrying copper tubing (heat exchanger) attached to the gate.

To simulate the semi-infinite plate, one copper wall was replaced with a composite perspex and copper wall, with a total length of 31.5 cm. The lower 7.5 cm of the wall was perspex, with a smooth transition to copper, simulating the leading edge of the plate.

The water within the cavity then became the ambient, its temperature being regulated by the reservoir behind the unused copper sidewall. The water of the reservoir behind the active copper plate at some raised temperature above the ambient was used to increase the copper plate temperature by ΔT after start-up (ie the raising of the gate).

Four fast response (7ms) thermistors were placed at distances of 1-3mm out from the plate (y-dir) and at heights of 3-21cm up the plate (x-dir), and another placed in the centre of the cavity. The sampling frequency in all experiments was 10 Hz. Three "flat type" thermistors were sealed to the copper plate, to gain the actual copper plate temperature.

RESULTS/ DISCUSSION

The results obtained are in the form of temperature time series. In this paper nine experiments are reported, in which the temperature difference achieved ranged from 1-6°C. Figure 1 illustrates typical behaviour, which is consistent over the full range of experiments. At each height, we see an initial period for which the temperature follows the analytical solution for a doubly infinite plate of the form:

$$T(y,t) = \Delta T erfc(\frac{y}{\sqrt{4\kappa t}})$$

Where y is the distance out from the plate, κ is the thermal diffusivity and t is time. This flow has no x-dependence in either temperature or velocity. The presence of the leading edge however forces a variation in x. Hence at each location along the plate there exists a finite time (prior to the arrival of information of the leading edge) for which the flow is governed by the one-dimensional, doubly infinite plate solution. Upon arrival of the leading edge signal the flow undergoes a complex adjustment, causing a conversion to the steady state, two dimensional boundary layer flow given by Ostrach (1982). This transition is characterised by a complex oscillatory signal.

The measurement of thermistor location is only accurate to 0.5 mm, however, for a given thermistor we can improve the estimate of its true distance out from the plate by substitution of both its final "steady" temperature and the measured distance up the plate into the numerically determined, steady-state Ostrach solution. This refined estimate for distance out from the plate was used for the analysis. Note that although the measured distance up the plate is only accurate to 0.5 mm the steady-state temperature profiles are not nearly as sensitive to the distance up the plate as they are to the distance out from the plate.

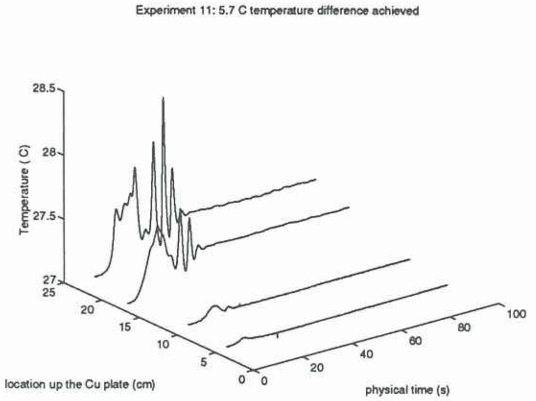

Figure 1: TEMPERATURE TIME SERIES FOR FOUR THERMISTORS AT VARIOUS LOCATIONS UP THE PLATE.

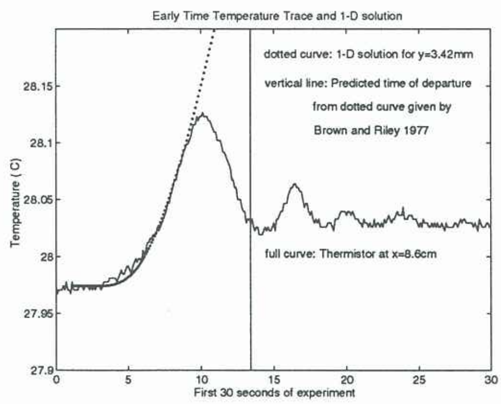

Figure 2: EARLY TIME TEMPERATURE TRACE AND 1-D SOLUTION.

Figure 2 shows the temperature time series acquired from a thermistor located 8.6 cm up the plate and 3.42mm out from the plate, and also the corresponding one dimensional solution for that y-location. We see that the temperature time series diverges from the one dimensional solution quite early, indicating that information of the leading edge has arrived at that time. Beyond this time of divergence convective effects set in. Exactly how that information is conveyed from the leading edge is poorly understood. Various models have been put forward by Goldstein & Briggs (1964), Brown & Riley (1973), Nanbu (1971) and Armfield & Patterson (1992). An early theory (Goldstein & Briggs, 1964) suggested that it travelled with the maximum velocity in the developing one dimensional boundary layer, whereby the penetration distance is given by

$$x_p = \int_0^{\tau} u_{\max}\, dt$$

where u_{max} is the maximum velocity of the 1-D solution and x_p is the penetration distance of information travelling at u_{max}. The arrival time of information of the leading edge had it travelled in the manner proposed is given by the vertical line in figure 2, which demonstrates that this model predicts the arrival time to be much later than that determined experimentally. This finding supports the argument put forward by Armfield & Patterson (1992) that the leading edge signal is in actual fact a travelling wave instability of the one dimensional unsteady boundary layer, which yields shorter arrival times.

A feature clearly distinguishable in all the time series is a group of nearly sinusoidal waves (refer again to figure 1) which exist just prior to the final steady state temperature. These waves can be seen to amplify with distance up the boundary layer. Their frequency, as determined from Fast Fourier Transform techniques (which compared very well with hand measured peak to peak estimates), is plotted in figure 3 versus ΔT, which illustrates that the frequency increases with ΔT. Similar plotting of ΔT versus distance up the plate produced widely scattered data, indicating that the frequency does not alter with distance up the plate. It is envisaged therefore that these waves are travelling instabilities of constant frequency on a transitional flow between the one dimensional, unsteady flow and the two-dimensional, steady state flow. The presence of these waves for all experiments demonstrates that they are intrinsically related to the transition mechanism

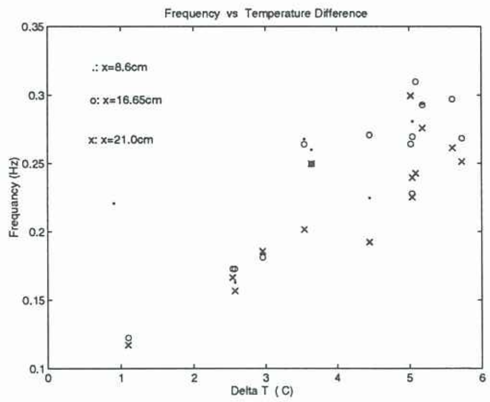

Figure 3: FREQUENCY OF WAVE GROUP VS APPLIED TEMPERATURE DIFFERENCE.

A model for the time to reach the final steady state temperature was given in an early scaling of the problem in rectangular cavities by Patterson and Imberger (1980). Extending their analysis to the semi-infinite plate, here using a local Rayleigh number based on distance up the plate, an x-dependent time scale is obtainable for reaching a conduction - convection balance in the heat equation:

$$\tau = \sqrt{x \frac{\Pr}{g \alpha \Delta T}}$$

Where x is the distance up the plate, g is the gravitational acceleration, α is the thermal expansivity, ΔT is the temperature difference and Pr is the prandtl number. In figure 4, the time to reach the steady state temperature, as given by that time immediately following the passing of the wave group is plotted vs this scaling estimate. This shows an excellent fit to a straight line, suggesting that the scaling is applicable, with a scaling factor of four.

CONCLUSIONS

Although more experiments and further analysis of the experimental data set reported here are underway, already much has been ascertained. These experiments confirm that estimates of the arrival time of information of the leading edge, based on the maximum flow velocity of the one dimensional boundary layer yields arrival time estimates which are too long. The frequency of the waves is a function of the applied temperature difference of the experiment only. This is consistent with the observations and calculations for the steady boundary layer reported by Gebhart and Mahajan (1975). The scaling estimate for time to reach the steady state, two dimensional boundary layer is confirmed by the data.

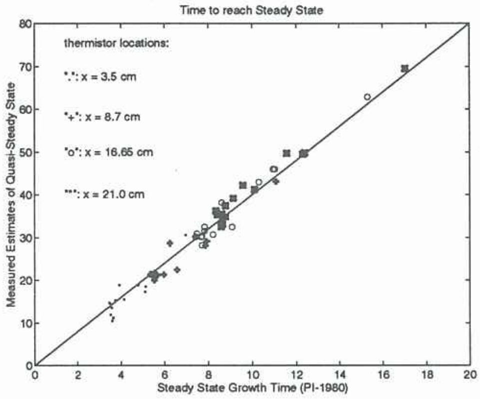

Figure 4: MEASURED TIME TO REACH STEADY STATE VS TIME SCALE FROM PATTERSON & IMBERGER 1980.

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