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STABILITY OF 'QUASILINEARIZATION' FOR THE ROTATING POROUS DISK WITH UNIFORM SUCTION OR BLOWING

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ABSTRACT

In the literature, we can find the numerical solution of the flow around a flat disk rotating around its own revolution axis when the fluid at infinity is at rest. The velocity is expressed by a set of three unknown functions of a dimensionless variable ζ. The problem is to obtain a solution for a set of three non-linear differential equations with five boundary conditions. The first solution is given by Cochran [1]. It is obtained in the form of a power series near the disk, an asymptotic series for large values of ζ and a link between these two solutions. The second solution is given by Sparrow *et al* [5] using some specific routines implemented on an IBM computer.

This paper presents different numerical solutions in the case of the adjunction of a uniform blowing (positive or negative value) through the porous disk. To compute the three components of velocity, we use Radbill's quasilinearization method [3].

With an adequate set of boundary conditions, 'quasilinearization' converge, in all times, to an acceptable physical solution for the velocity and the temperature

EQUATIONS OF MOTION

Like von Karman [2], we stipulate that the components of velocity and pressure are expressed with the next functions depending on a dimensionless variable ζ.

$$\varsigma = \frac{z}{\sqrt{\nu . \omega^{-1}}} \; ; \; U(\varsigma) = \frac{u_r}{r.\omega} \; ; \; V(\varsigma) = \frac{v_\theta}{r.\omega} \; ; \; W(\varsigma) = \frac{w_z}{\sqrt{\nu.\omega}} \; ; \; P(\varsigma) = \frac{p}{\rho.\omega.\nu} \quad (1)$$

Inserting these functions in the Navier-Stokes equations, we find a system of three simultaneous equations

$$W' = -2.U \; ; \; U'' = U'.W + U^2 - V^2 \; ; \; V'' = V'.W + 2.U.V \quad (2)$$

The equation for pressure and temperature ($\mathcal{P}r \;\#\; 0.71$ (cf [4])) may be integrated as soon as the velocity is known. The results shoud be at the measure of accuracy of the velocity.

$$T'' = \mathcal{P}r \,.\, W.T' \quad (3)$$

$$P - P_0 = -\frac{1}{2}.\left(4U + W^2\right) \quad (4)$$

QUASILINEARIZATION

We consider the vector $\Phi = \left[W , U , U' , V , V'\right]^t$ and two succesive solutions $\Phi^{(k)}$ and $\Phi^{(k+1)}$. To compute a solution, we first transform the non-linear equations $\Phi' = F(\Phi)$ equivalent to (2) for the flow field into a linear system of five ordinary equations using a development analogous to Newton-Raphson's method, limited to the first order, around an estimated previous solution. This development is :

$$\frac{d\,\Phi^{(k+1)}}{d\varsigma} = F\left(\Phi^{(k)}\right) + \sum_{j=1}^{5}\left(\varphi_j^{k+1} - \varphi_j^{k}\right) \left.\frac{\partial F}{\partial \varphi_j}\right|_{\varphi_j=\varphi_j^k} \tag{5}$$

which gives a linear matrix system.

$$\frac{d\,\Phi^{(k+1)}}{d\varsigma} = A\,.\,\Phi^{(k+1)} + B\left(\Phi^{(k)}\right) \tag{6}$$

We can use [3] the standard methods for linear differential equations and look for a set of a particular solution Φ_p and two independent solutions Φ_{hm} for the homogeneous part of (6).

The new estimated solution is given by the the linear combination (7) where the two parameters λ_j are fitted to verify the boundary conditions at infinity.

$$\Phi^{(k+1)} = \Phi_p + \lambda_1.\Phi_{h1} + \lambda_2.\Phi_{h2} \tag{7}$$

Equation (4) is linear and homogeneous. Its solution is obtained with the knowledge of W. A solution only needs to look for two independent solutions.

PHYSICAL CONDITIONS

For the linear system (6), we know the conditions of no-slip on the disk, and the intensity (vs) of the positive blowing or negative suction through it

$$\Phi(0) = \left[0\,,\,1\,,\,U'(0)\,,\,vs\,,\,V'(0)\right]^t \tag{8}$$

and, at a large distance from the disk, we may stipulate that the fluid is at rest or irrotational.

$$\Phi(\varsigma_\infty) = \Phi_\infty = \left[W_\infty\,,\,U_\infty\,,\,U'_\infty\,,\,V_\infty\,,\,V'_\infty\right]^t \tag{9}$$

fluid at rest : $U_\infty = 0\;;\;V_\infty = 0$ (10)

irrotational motion : $U'_\infty = 0\;;\;V_\infty = 0\;;\;V'_\infty = 0$ (11)

For the dimensionless temperature, the boundary conditions are : $T(0) = 1\;;\;T_\infty = 0$ (12)

RESULTS AND DISCUSSION

COCHRAN's SOLUTION

A fourth order scheme as Runge-Kutta needs the two initial slopes for U and V. A multiple shooting method to fit two various conditions at infinity is tedious and expensive as compared to a linear combination (7). For vs = 0, our characteristics are represented on table 1 (step size h =0.01 , $\eta_\infty = 16$).

Using this solution as first initialisation, we can compare theaccuracy of our numerical results.

TABLE 1

COMPARISON BETWEEN OUR NUMERICAL RESULTS AND RESULTS OF [2] AND [4]

vs = 0			[2]	[4]
$U'(0)$	0.51022		0.510	0.510
$V'(0)$	-0.61594		-0.6159	-0.616
W_∞	-0.88445		-0.8845	-0.886
P_∞	-0.39113	(equ. (4))		+0.393

These results for these derivatives are summarized in figures 1 and 2. For the blowing, it is necessary to increase the value of ζ_∞. For a suction, we see the diminution of the boundary layer and we must shorten the step size. The greatest value of $U'(0)$ is near $\zeta = 0.25$ (vs=0.). For the greatest values of suction, the fluid is nearly rotating around the vertical axis r = 0 because the inclination of the streamlines decrease towards 0 . It is possible to improve our results using Richardson's correction which provides a better accuracy using two results obtained with two different step sizes.

Figure 1 is of great interest to minimize number of iterations. On figure 2, with vs becoming increasingly negative, the inflow goes directly into the porous surface.

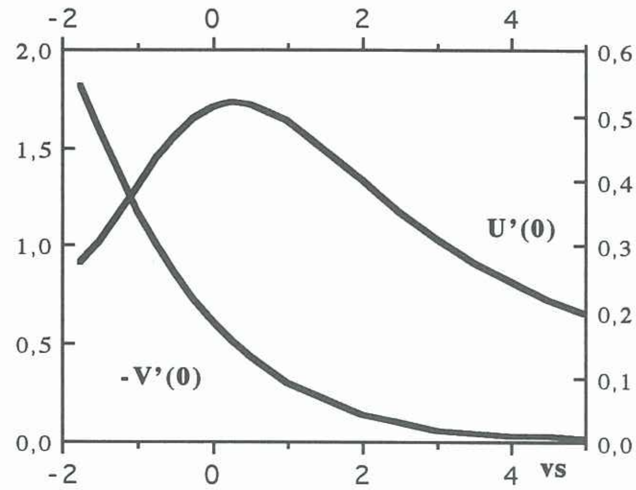

FIG 1
REPRESENTATIVE OF THE SLOPES U' AND V' FOR DIFFERENT MASS FLUXES vs

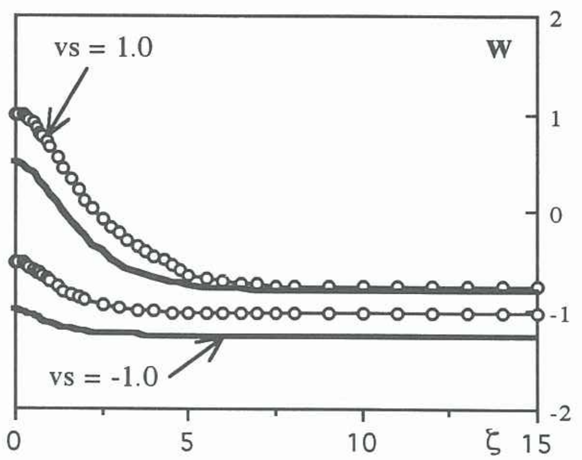

FIG 2
REPRESENTATIVE AXIAL-VELOCITY W FOR DIFFERENT vs = -1.0 , -0.5 , 0.5 , 1.0

SECOND SOLUTION

We pretend not to know Cochran's solution and start the computation with these values :$U'(0) = 0.508$ and $V'(0) = -0.604$ to compute the characteristics of the flow field with a constant blowing (vs > 0) or suction (vs < 0). These initial conditions with conditions (10) can give a second numerical solution for the matrix system (6).

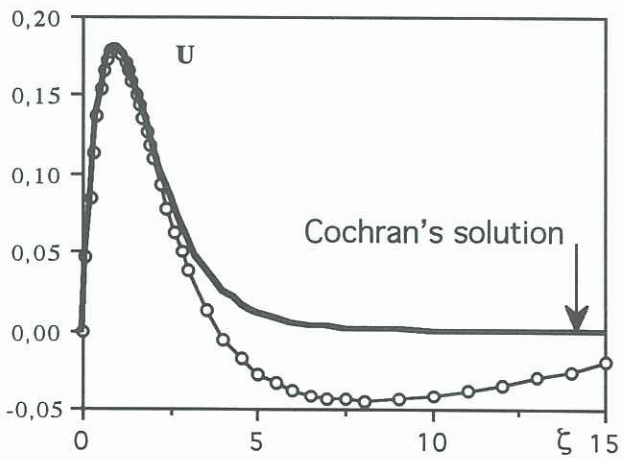

FIG 3
DIFFERENCE BETWEEN OUR TWO NUMERICAL SOLUTIONS FOR THE RADIAL VELOCITY

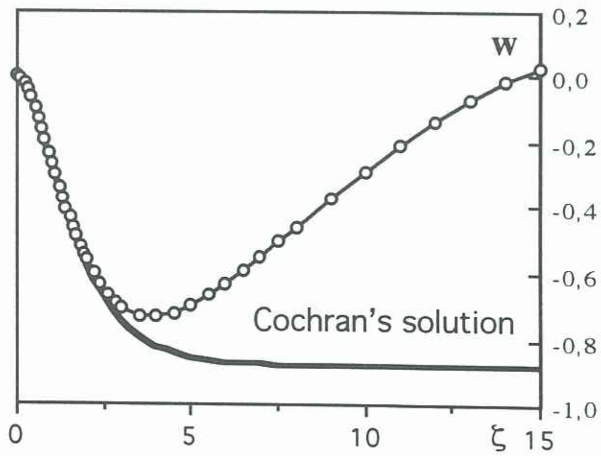

FIG 4
DIFFERENCE BETWEEN OUR TWO NUMERICAL SOLUTIONS FOR THE AXIAL VELOCITY

To obtain this new solution or the precedent depends on the three following criteria :(a) the initial guess solution ; (b) a set of two final conditions at infinity ; (c) the value of infinity: ζ_∞.

Cochran's solution is characterized in the fact that the components U and V and their derivatives must be asymptotically equal to 0 at infinity. But, at the upmost, we can specify two conditions for our scheme (cf (8)). Then, the conditions (a) are of primary importance and we may use the results of figure (1) to minimize the number of iterations and obtain Cochran's solution.

Figure 5 and 6 represents three numerical solutions ($\zeta_\infty = 16.0$; vs = -1.5 ; step size 0.01).We can see that the profiles of V and T are close together. To appreciate the influence of the boundary conditions we give thereafter some numerical results with used boundary conditions. These numerical properties are given in table 2.

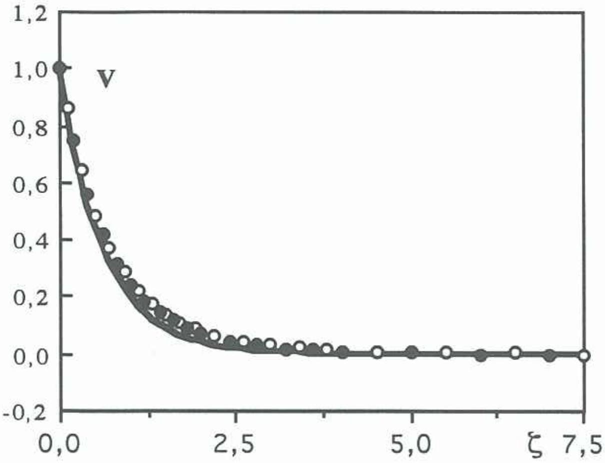

FIG 5
PROFILES OF TANGENTIAL VELOCITY

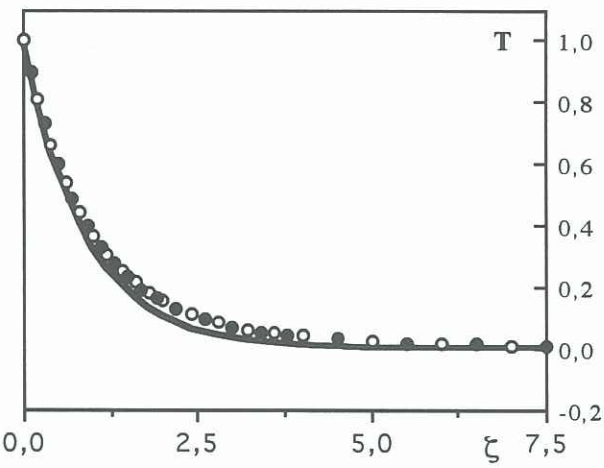

FIG 6
PROFILES OF TEMPERATURE

TABLE 2

SOME IMPORTANT PROPERTIES OF THE SOLUTIONS

COND. = 0	U'_0	$-V'_0$	U_∞	U'_∞	V_∞	V'_∞	W_∞	T'_0
U_∞, V_∞	0.2121	1.4721	<1.E-12	7.782E-3	<1.E-12	4.01E-4	7.20E-3	-1.0260
U'_∞, V'_∞	0.3068	1.5799	-2.99E-6	<1.E-12	-2.47E-6	<1.E-12	-1.6194	-1.1019
U_∞, V'_∞	0.2167	1.4642	<1.E-12	7.86E-3	-1.72E-3	<1.E-12	7.83E-3	-1.0282

CONCLUSION

Radbill's scheme is very adequate to solve a system of linear or non-linear differential equations with boundary conditions. This method is still useful for the dual problem of a disk at rest in a uniform rotating swirling flow (cf [4], chap XI). This scheme is more stable in the second case (11). A coarse estimate with (2) at infinity shows that if asymptotically $U'_\infty = 0$ then the others conditions at infinity are verified.We have obtained a second numerical solution.and a shooting method can oscillate between these solutions.

REFERENCES

[1] W. G. COCHRAN - The flow due to a rotating disk. Proc. Cambridge Phil. Soc. 30 - p 365-375 , (1934)

[2] T. von KARMAN - Über laminare und turbulence Reibung. - ZAMM 1, p 233-252 (1921)

[3] J. R. RADBILL - Application of Quasilinearization to Boundary-Layer Equations - AIAA Journal - Vol 2 - N° 10 - p 1860-1862 (1964)

[4] H. SCHLICHTING - *Boundary-Layer Theory* - McGraw-Hill - 6e edition (1968)

[5] E. M. SPARROW & J. L. GREGG - Mass transfer, flow and heat transfer about a rotating disk - Trans. of the ASME - p 294-302 (1960)