Twelfth Australasian Fluid Mechanics Conference
The University of Sydney, Australia 1995

VORTICITY GENERATION AND FLOW SEPARATION AT FREE SURFACES

R.W. Cresswell, B.R. Morton and P. Blanchonette
Centre for Dynamical Meteorology and Oceanography
Monash University
Clayton, Victoria
AUSTRALIA

ABSTRACT

The physical mechanism whereby vorticity is generated at a free surface is discussed and is shown to be acceleration of surface fluid layers relative to underlying fluid due to stress gradients brought about by the boundary condition that the free surface viscous stress be zero. Flow separation in the conventional sense may be said to occur when sufficient vorticity is generated such that the velocity jump across the boundary layer approaches the free stream velocity. However, numerical calculations of flow separation in an axisymmetric free surface jet entering water show that the maximum advection away from the boundary layer occurs around the region where the normal gradient of vorticity changes sign. This is the point at which vorticity generation changes sign (because the diffusive flux of vorticity in the normal direction changes sign) and so signals the end of the region in which vorticity of the correct sign for the jet shear layer is being generated. We suggest that this may be a meaningful definition of flow separation.

INTRODUCTION

When a homogeneous fluid is bounded by an idealized free surface then all vorticity generation which occurs at the surface arises due to the boundary condition which constrains the viscous stress to be zero at the surface. The vorticity generated here is of primary importance to 'deep' flows (where the influence of solid boundaries on the flow may be neglected) as without vorticity the flow will remain irrotational with the interior motion determined solely by the motion of the free surface. In these circumstances there can be no turbulence within the fluid and no mixing other than by molecular diffusion can occur. Similarly, if the vorticity remains contained within the boundary layer at the free surface there then the majority of the flow remains irrotational. Hence an understanding of the physical mechanisms of free surface vorticity generation and flow separation are of the utmost importance to all but the most basic free surface flows.

The main focus of this paper will to explain the physical mechanism whereby vorticity is generated at a free surface as a consequence of the vanishing of the viscous stress at the surface. However, as we have stated, the phenomenon of flow separation is a crucial one and so we must spend some time discussing free surface flow separation. Unfortunately, the definition of flow separation much less its explanation is still a subject of much debate, and we cannot hope to answer the questions raised concerning flow separation over solid surfaces much less extend those ideas to free surfaces in this paper. Perhaps by the time this paper is presented we may have the answers to some of the questions we shall raise here.

Although velocity (momentum) is the primary variable in fluid mechanics, it is often more instructive to view a problem in terms of the curl of the velocity field, i.e. the vorticity, than in terms of velocity itself. The velocity field is not Galilean invariant, which is to say that structures apparent in one inertial frame may appear to be absent when the flow is viewed from another inertial frame. The vorticity field, however, is invariant under all such transformations and as such provides a usable and often useful picture of the physical flow. One situation in which this becomes evident is that of flow separation. The traditional picture of flow separation in which the existence of reverse flow behind an obstacle is synonymous with flow separation is clearly not a physically reasonable one, although the centre of mass frame of the obstacle involved is a natural choice for separated flow past a bluff body. The problem of unsteady flows involving separation from free surfaces make it clear that the 'separation bubble' picture is inappropriate as the choice of reference frame is no longer clear because the free surface is changing shape, and the use of streamlines becomes somewhat dubious in the case of a time-dependent flow.

Several definitions of flow separation have been proposed (see, for example, Simpson, 1989, for a review) including the vanishing of the wall shear (and hence the vorticity) which is associated with the appearance of reverse-flow. While this definition has been shown to be inappropriate itself for defining flow separation (because in unsteady flow the surface vorticity often changes sign with no breakdown of the boundary layer character and hence no 'separation') it will be seen that for steady flows where flow reversal does in fact take place, the point of zero vorticity is coincident with the point of flow reversal for both flows over solid surfaces and flows at free surfaces. In recent years a definition of flow separation based on the vorticity field has been proposed (Page, private communication) in which the existence of a 'tongue' of vorticity reaching out from the boundary layer defines flow separation. This results in a criterion for separation where $\partial\zeta/\partial r = 0$, where ζ is the out of plane component of the vorticity for a 2D flow and r is the direction normal to the surface. However, this gives a very different picture of flow separation to the traditional view. Figure 1, for instance, shows the vorticity field around a stationary, undeformable, cylindrical bubble which shows a tongue of vorticity extending into the flow. The accompanying streamline plot shows that there is no evidence of any flow reversal behind the bubble. In this case there appears to be little discernible effect of the vorticity field on the flow field, although on closer inspection we see that the flow is in fact retarded behind the bubble implying incomplete pressure recovery and hence drag associated with some form of wake. The most sensible definition of flow separation however would seem to be that point at which the advection of vorticity in the direction normal to the boundary layer dominates over, or becomes comparable with, diffusion in the same direction. A useful tool has been available for some time now for studying the point at which the boundary layer breaks down - namely the boundary layer equations themselves. The boundary layer equations become invalid in a region where the assumptions upon which they are based break down. This results in the appearance of a singularity in the solution to the boundary-layer equations (Goldstien, 1948). This point has often been taken as defining the position of the flow separation, whereas there is no such singularity in the solutions of the Navier-Stokes equations themselves which are everywhere smooth and so it may be misleading to think of separation occurring at a single 'point', as the phenomenon would appear to be spread over a finite region.

We shall now consider the physical process whereby vorticity is generated at a free surface, and then discuss two examples of free surface flows where vorticity generation is of importance, namely the drop-formed vortex ring and the steady state example of a free surface jet impinging on and penetrating through a water surface.

FREE SURFACE VORTICITY GENERATION

At an idealized free surface the continuity of force across the surface leads to the condition that the surface stress in the liquid and air be equal, i.e.

$$\sigma_{xy} = \mu_w \left\{ \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right\} = \mu_a \left\{ \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right\}. \quad (1)$$

so that assuming $\mu_w >> \mu_a$ then this may be approximated by

$$\sigma_{xy} = \mu_w \left\{ \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right\} = 0 \quad (2)$$

which leads to the conclusion that the z—component of the vorticity, ω_z, can only be zero at a free surface if both $\frac{\partial v}{\partial x}$ and $\frac{\partial u}{\partial y}$ are zero also. This is the case for a stationary plane free surface, but not otherwise. For instance, at an axisymmetric curved free surface such as that appropriate to the free surface jet penetrating a water surface (figure 2) we can write out the azimuthal vorticity component in toroidal coordinates ξ, η, ζ (figure 3) where instantaneously $\zeta = 0$ coincides with a free surface, as follows (Cresswell and Morton, 1995)

$$\omega_\xi = \left\{ \frac{2U_\eta}{r_2} + \frac{2}{a}(\cosh\zeta - \cos\eta)\frac{\partial U_\zeta}{\partial \eta} \right\} \quad (3)$$

where U_η and U_ζ are the velocity components in the η and ζ directions respectively. The term involving $\frac{\partial U_\zeta}{\partial \eta}$ above corresponds to the terms involving change of shape of the surface and so may be neglected for a steady-state flow. Hence we find that for the above geometry and for a steady state flow

$$\omega_\xi = \frac{2U_\eta}{r_2} \quad (4)$$

which is to say that the vorticity at the free surface is twice the tangential velocity times the surface curvature. Note however that this does not correspond to a mechanism for the generation of vorticity, merely a boundary condition on its value at the free surface and is a direct consequence of the continuity of force through the surface. The stationary free surface is able to diffuse the vorticity in the direction normal to the surface (from where it may be advected in the normal direction), and both diffuse and advect it in a direction tangential to the free surface. However, the value of vorticity at the surface remains determined by the boundary condition on free surface stress and so vorticity must be generated to account for any rate of change of vorticity implied by the balance of terms in the vorticity equation. The question then is how this vorticity is generated. Quite simply, we know that the stress at the surface must remain zero and this determines the required level of vorticity. Any departure from this value just inside the surface itself results in a stress there. The closer this stress exists to the surface, the higher the stress gradient is. As the gradient of stress is a force term in the Navier-Stokes equations, surface layers are accelerated relative to underlying layers so generating vorticity. All of this generation must occur at the surface itself where the boundary condition applies and not, of course, in the interior of the fluid.

For the boundary layer to separate in the conventional sense the velocity jump across the layer must become equal to the free stream velocity and so the surface velocity (and via equation 4 the surface vorticity) must drop to zero. We see immediately that diffusion alone cannot account for this as the surface vorticity dropping to zero implies that the normal gradient of vorticity at the surface has become negative and hence the diffusive flux of vorticity is in the direction towards the surface rather than away from it. Hence advection must play an important role in moving vorticity from regions early on in the development of the boundary layer (where $\nu\frac{\partial \omega}{\partial n}$ is positive) to regions later on when $\nu\frac{\partial \omega}{\partial n}$ is negative.

Penetrating Jet with Free Surface

In order to gain a clearer picture of the processes involved we shall consider an axisymmetric jet bounded by a free surface impinging on a plane water surface (figure 2). A finite element calculation of this flow has been performed with a view to examining the region of separation. It should be pointed out however that these numerical results are to some extent bogus as they do not calculate, but rather impose, the free surface shape. As such surface tension effects have been circumvented for the moment. The streamline plot of figure 4 appears to show a separated region and a recirculating flow. Figure 5 shows a vorticity contour plot with the position of the dividing streamline marked. Clearly the majority of advection of vorticity out of the boundary layer and into the mean flow is occurring before the position of the dividing streamline. Figure 6 shows the variation of vorticity, ω_ξ, tangential velocity, U_η, the normal gradient of normal velocity, $\frac{\partial U_\zeta}{\partial \zeta}$, and normal gradient of vorticity, $\frac{\partial \omega_\xi}{\partial \zeta}$ around the curve (denoted by the line of nodes marked on figure 5). It is clear that the region of peak ejection of fluid from the boundary layer, indicated by the maximum of the $\frac{\partial U_\zeta}{\partial \zeta}$ curve is close to the point at which $\frac{\partial \omega_\xi}{\partial \zeta}$ changes sign.

Drop-formed vortex rings

The drop-formed vortex ring is a highly unsteady flow phenomenon involving vorticity generation and flow separation. In order to form a vortex ring a free-ended vortex sheet is required which becomes a detached ring of vorticity. As the flow is one in which the shape of the free surface is constantly changing during the drop coalescence process, definitions of separation involving stagnation points on the surface become imprecise, although the net result of the physical process - the production of a vortex ring - is unequivocal and clearly independent of the frame of reference of the observer. From photographic studies (figure 7) in which the receiving fluid has been marked with passive tracers (in this case 40 μm PVC spheres) it can be seen that at an early stage in the coalescence process the drop fluid appears to be 'subducted' below the receiving fluid. In other words, all surface fluid elements from above appear to have been advected towards a separation line where they have left the surface and become part of the interior. If we are to take this as being a separation point, we are forced to define the said point as being the point of maximum gradient of velocity in the direction along an inwardly directed normal. We are interested in advection away from the surface so even for an unsteady case we must consider a frame of reference which is instantaneously moving at the velocity of the surface in the direction normal to the surface itself. In this frame the surface normal velocity is always zero and the normal gradient of normal velocity is our measure of the rate at which (rotational) fluid particles are being advected into the interior of the fluid.

CONCLUSION

Vorticity is generated at a free surface through the action of viscous forces and the requirement that the viscous stress be zero at the surface (for a continuum model of the surface, eg a two fluid model, it is the fact that the viscous stress tensor is no longer symmetric which leads to a source term in the vorticity equation). Any departure in vorticity just beneath the surface from the value at the surface implies finite viscous stress just beneath the surface and so a stress gradient in the surface fluid layers, accelerating them relative to underlying fluid. If we take the surface stress to be zero, then the generation of vorticity becomes equal to the viscous diffusion of vorticity away from the surface. Flow separation is a much less clear picture, although our (albeit preliminary) results show that the phenomenon is one in which all variables vary smoothly and continuously with no dramatic 'ejection' of fluid from the boundary layer. Fluid is advected normal to the boundary layer over a region which has a peak somewhere near the point at which the normal gradient of vorticity changes sign.

REFERENCES

R W Cresswell and B R Morton
Drop-Formed Vortex Rings - The Generation of Vorticity
Physics of Fluids **7** p.1363 1995

S Goldstein
On Laminar Boundary-Layer Flow Near a Position of Separation
Quart. J. Mech. Appl. Math. **1** p.43 1948

R L Simpson
Turbulent Boundary-Layer Separation
Ann. Rev. Fluid Mech. **21** p.205 1989

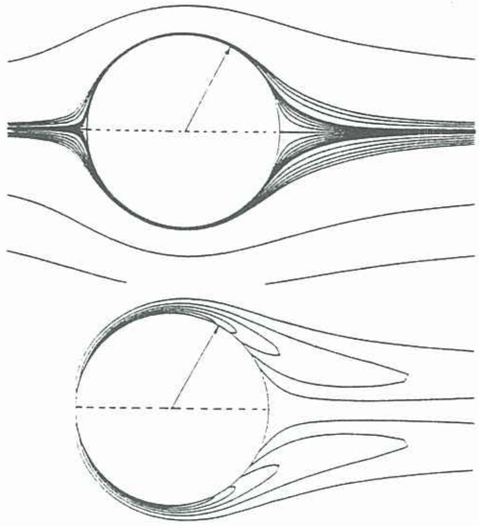

Figure 1: Streamlines and vorticity contours for flow around a cylindrical bubble at Re=100. $\partial\zeta/\partial r = 0$ at the indicated position, but there is no change of sign of ζ and no evidence of reverse flow

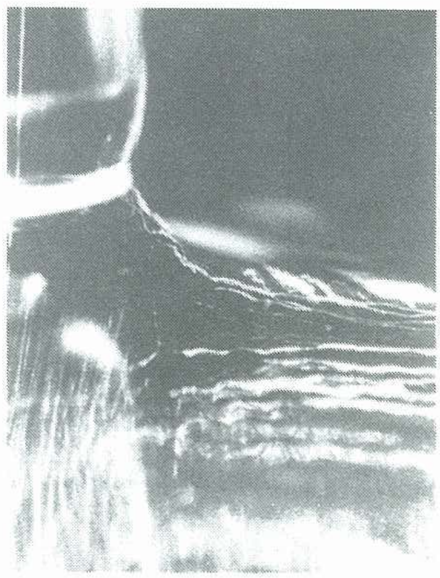

Figure 2: Streakline photograph of an axisymmetric jet with a free surface penetrating a pool.

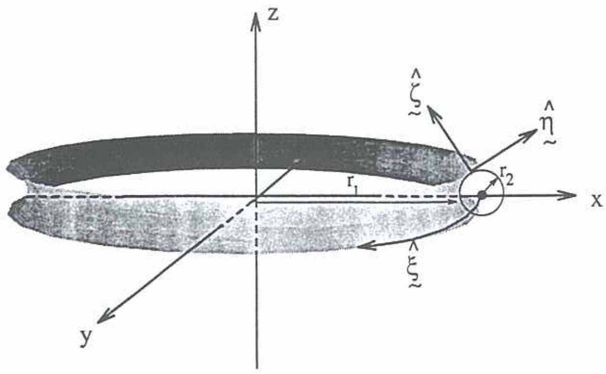

Figure 3: Toroidal coordinate system appropriate to the axisymmetric jet. Here r_1 and r_2 are the principle radii of curvature when the free surface coincides with a surface of constant ζ.

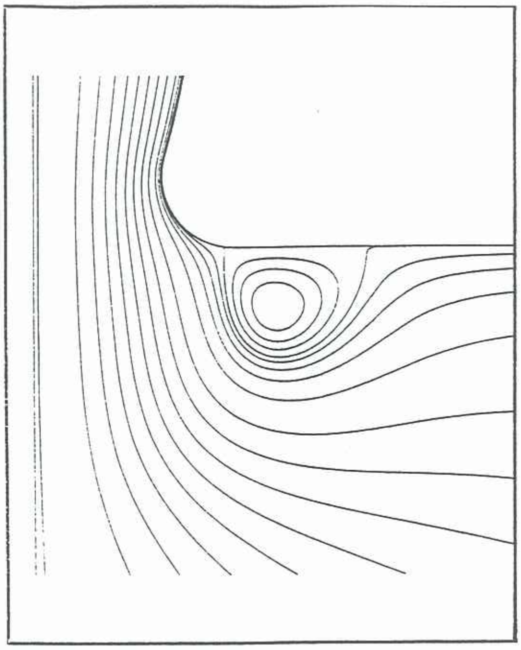

Figure 4: Streamlines of an early stage in the transient development of an axisymmetric jet entering water with a zero surface stress boundary condition.

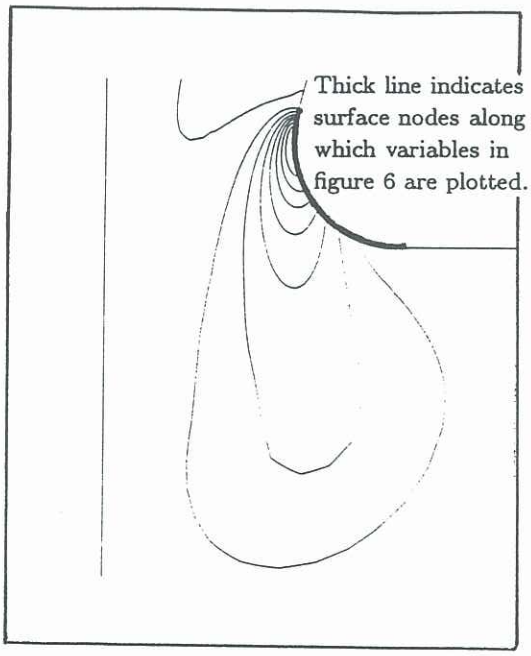

Figure 5: Contours of vorticity for the streamline plot shown in the previous figure. The arrow marks the position of the separated streamline.

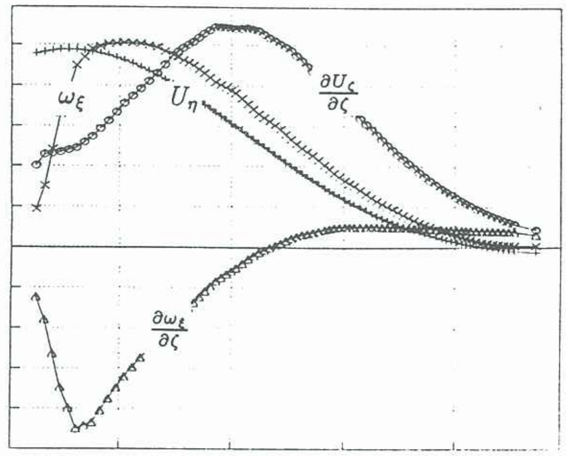

Figure 6: Variation of ω_ξ, U_η, $\frac{\partial U_\zeta}{\partial \zeta}$ and $\frac{\partial \omega_\xi}{\partial \zeta}$ along the line of surface nodes marked in the previous figure.

Figure 7: Photograph of a drop penetrating the surface of water. The particles discernible on the outside surface of the drop denote the position of the separation line where drop fluid is subducted below receiving fluid.