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NATURAL VENTILATION OF AN ENCLOSURE CONTAINING ONE POSITIVE AND ONE NEGATIVE SOURCE OF BUOYANCY

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ABSTRACT
This paper describes theoretical modelling and experimental verification of the flow and stratification in a naturally ventilated enclosure containing two point sources of buoyancy of opposite sign. The plumes from the two sources produce a vertical density profile consisting of three distinct, fully mixed layers. The paper describes a theoretical model that predicts the depths and densities of these layers. The positions of the interfaces between the three layers were found to be a function only of the effective area, A*, of the enclosure openings the height of the enclosure, H, and the ratio of the strengths of the two sources of buoyancy, B_1/B_2. An important entrainment mechanism that results when the weaker plume impinges on a density interface is also modelled. The experimental technique involves the use of a water-filled enclosure with salt solution injected at one point and a mixture of methylated spirits and water at a second. The situation under consideration has particular relevance to the prediction of thermal stratification in building spaces with both cold and warm surfaces (or sources of air) present.

INTRODUCTION
Natural ventilation of enclosures is an important topic in several areas of engineering, particularly in the building industry. Often it is preferable to use naturally ventilated systems as opposed to mechanical ventilation, both from the point of view of reducing the energy consumption and also to minimise the level of airborne pollutants within a building. Natural ventilation is also frequently used in industrial buildings where contaminants from hot processes must be diluted sufficiently to meet occupational health and safety limits. The sizing of vents in the building to ensure correct air flow under the critical conditions of low wind speed is important and is relevant to the discussion that follows.

Previous work by Linden *et. al.* (1990) determined that a single point source of buoyancy inside an enclosure naturally ventilated with high-level and low-level openings, so that displacement ventilation occurred, results in the formation of two layers of fully mixed fluid within the enclosure. Using the concept of a "point source of buoyancy" on the floor of the enclosure to represent a heat source and the work by Morton *et. al.* (1956) on entrainment into plumes, Linden *et. al.* (1990) showed that the fluid in an enclosure of height H stratifies into two well-mixed layers. The lower layer is at ambient temperature and there is a hot layer of uniform temperature above a horizontal interface at z=h. The non-dimensional height, ξ=h/H, of the interface between the cool lower layer of air and the upper level in the enclosure is given by

$$\frac{A^*}{H^2} = C^{3/2}\left(\frac{\xi^5}{1-\xi}\right)^{1/2}, \qquad (1)$$

where A^* is the "effective" area of the top and bottom openings of the enclosure, H is the height difference between the top and bottom openings and ξ = h/H. The constant $C = \frac{6}{5}\alpha\left(\frac{9}{10}\alpha\right)^{1/3}\pi^{2/3}$ where α is the entrainment constant for the plume.

The effective area A^* of the openings is defined as

$$A^* = \frac{c_d a_t a_b}{\left(\frac{1}{2}\left(\frac{c_d^2}{c}a_t^2 + a_b^2\right)\right)^{1/2}}, \qquad (2)$$

where a_t and a_b are the areas of the top and bottom openings, respectively, and c is the pressure loss coefficient associated with the inflow through a sharp edged opening. The discharge coefficient, c_d, through a sharp edged opening is employed here to account for the vena contracta arising at the downstream side of the upper vents. The density of the lower layer is equal to the surrounding ambient fluid.

In this paper we consider a more complex situation where both a positive and a negative source of buoyancy are present in a naturally ventilated enclosure under steady state conditions.

THEORETICAL ANALYSIS

The situation under consideration here is shown schematically in figure1. A source of negative buoyancy, B_1, is located on the ceiling of the enclosure and a source of greater magnitude but opposite sign is located on the floor. It is assumed that the plumes are sufficiently far from each other that they do not interact. We denote the ratio of the buoyancy fluxes by $\psi \equiv B_1/B_2 \leq 1$. For Boussinesq plumes we can reverse the relative strengths of B_1 and B_2 by reversing gravity. We take both B_1 and B_2 as positive and derive the conservation equations with due regard to the directionality of the plumes. Three layers of fluid of different densities are formed as shown - this configuration has been confirmed by our experiments.

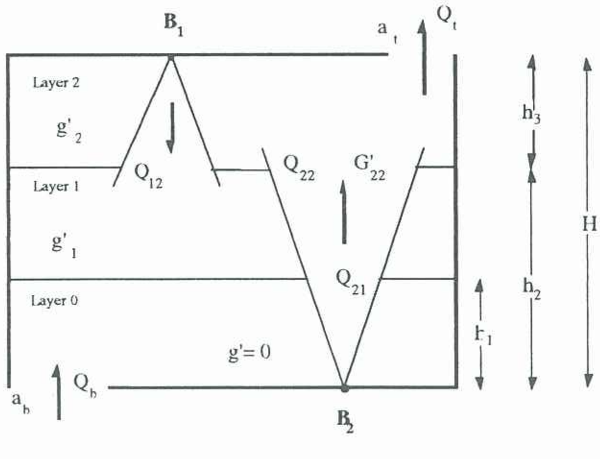

Figure 1. Ventilated enclosure with two buoyancy sources of opposite sign.

A number of relations for conservation of volume flux and buoyancy within the enclosure and the individual plumes may be identified. The volume flowrate, Q, and buoyancy, G', of the two plumes at any given cross section are determined using the entrainment assumption and equations derived by Morton et al (1956) using "top hat" velocity and density profiles. In the case of the stronger plume developing in layer 0 of ambient fluid, for example,

$$B_2 = G_2' \, Q_2 = \text{constant}, \tag{3}$$

$$Q_{21} = C\,(B_2 \, h_1^5)^{1/3}. \tag{4}$$

$$G_{21}' = \frac{1}{C}(B_2^2 \, h_1^{-5})^{1/3}. \tag{5}$$

Here the first subscript denotes the source from which the plume originates and the second indicates the location of the cross-section of interest (at either the first or second interface, subscript 1 or 2, respectively). There is a flow induced through the top and bottom openings of the enclosure due to the buoyancy of layers 1 and 2. The volume flux through the openings is given by

$$Q_t = Q_b = A^* \left(g_2' \, (H - h_2) + g_1' \, (h_2 - h_1)\right)^{1/2}. \tag{6}$$

We wish to find the heights of the two interfaces, h_1 and h_2, and so we need to determine a function $f(\psi)$ that we define as

$$\frac{h_2}{h_1} = 1 + f(\psi) \tag{7}$$

There are also a number of volume and buoyancy conservation relations that apply to the enclosure under steady state conditions (full details are to be published in Cooper and Linden, 1995). One of the important assumptions made in this analysis is that buoyant layers 1 and 2 in figure 1 are fully mixed and have densities equal to the densities of the plumes entering, ie $g_1' = G_{12}'$ and $G_{22}' = g_2'$. This has been justified experimentally in previous work by others including Linden et al (1990). From the conservation relations in the enclosure and in the plumes themselves, the following expression is obtained.

$$\frac{A^*}{H^2 \, C^{3/2}} = \left(\frac{\xi_1^5}{(1-\psi)(1-\xi_1) - \psi^{2/3}\left(\dfrac{\xi_1}{1-\xi_1(1+f(\psi))}\right)^{5/3} f(\psi)\xi_1} \right)^{1/2}. \tag{8}$$

The model of the stratification in the enclosure also needs to account for the fact that the stronger plume, B_2, passes through the lower interface and therefore develops in a non-uniform environment. Consequently, we treat the development of the stronger plume in layer 1 as if it arises from a "distributed" source, ie one with finite mass flow and momentum. Forced plume theory similar to that of Morton (1959) is used in this regard. The stronger plume then has a somewhat reduced buoyancy flux, B_2', relative to its immediate environment as it passes through layer 1.

$$\frac{B_2'}{B_2} = \psi + \psi^{2/3}\left(\frac{h_1}{h_3}\right)^{5/3}, \tag{9}$$

This leads to the means by which we determine the volume flowrate Q_{22} and hence to evaluation of $f(\psi)$ in the following form

$$f(\psi) = \frac{3}{5}\left(\frac{B_2}{B_2'}\right)^{1/5}\left(\frac{B_2}{B_2'} - 1\right)^{3/10} \int_a^b \frac{dt}{[t^2 + 1]^{1/5}}, \tag{10}$$

where

$$b = \left(1 + \psi^{1/3}\left(\frac{h_3}{h_1}\right)^{5/3}\right)\left(\frac{B_2}{B_2'} - 1\right)^{-1/2}$$

$$\text{and } a = \left(\left(\frac{B_2}{B_2'} \right) - 1 \right)^{-1/2}.$$

Equations (8) and (10) may then be solved numerically to predict the heights of the interfaces between the buoyant layers shown in figure 1. In a manner similar to the situation of a single plume, it can be seen from equations (8) and (10) that *the height of the two interfaces is independent of the magnitudes of the buoyancy source strengths* but is determined solely by the geometry of the enclosure and the *ratio of the strengths* of the buoyancy sources, $\psi = B_1/B_2$.

An additional flow mechanism that we must now consider has an important influence on both the height of the interfaces in the enclosure and on the flow therein. The weaker plume, B_1, passes through the interface between layers 1 and 2. It does not then simply mix with the fluid in layer 1 but also continues downward as a weak jet before impinging on the density interface between layers 1 and 0. This impingement results in the entrainment of ambient fluid from layer 0 into layer 1 which, in turn, results in an increase in the height of the interface between the two buoyant layers (ie an increase in h_2).

The volume flowrate of ambient fluid, Q*, entrained by this mechanism has been estimated using the results of Kumagai (1984) who studied entrainment resulting from plume impingement on a density interface in some detail. In the present situation the entrainment volume flux is a function of the Froude number, Fr*, of the plume (or, strictly speaking, the weak jet) impinging on the lower interface and results in a correction to the upper limit, b, of the integral in equation (10). This Froude number is equal to $w_{11}/\sqrt{r_{11} \cdot g_1'}$ where w_{11} and r_{11} are the velocity and radius of the plume from B_1 when it arrives at $z=h_1$. Here we assume that the plume has developed only through fluid of density equal to that of the upper layer (g_2') from the source at $z=H$ to $z=h_1$ as an approximation to the complex situation to that is found in reality. The Froude number, Fr*, is then a function of ψ and the non-dimensional interface heights.

$$Fr^* = \left(\frac{5}{8\alpha}\right)^{1/2} \left(\frac{1-\varepsilon_2}{1-\varepsilon_1}\right)^{5/6} \left[\left(\frac{h_3}{h_1}\right)^{5/3} \left(\psi^{-2/3} - \psi^{2/3}\right) - 1 \right]^{-1/2} \quad (11)$$

The integral limit b is then found to be:

$$b = \left(1 + (1+\Omega)\psi^{1/3} \left(\frac{h_3}{h_1}\right)^{5/3} \right) \left(\frac{B_2}{B_2'} - 1 \right)^{-1/2} \quad (12)$$

where

$$\Omega = \frac{Fr^{*3}}{1 + 3.1Fr^{*2} + 1.8Fr^{*3}} \quad . \quad (13)$$

Results of the solution of (8) and (10) incorporating the correction for entrained volume flux, Q*, for a case where $A^*/H^2 = 0.003$ are shown in figure 2.

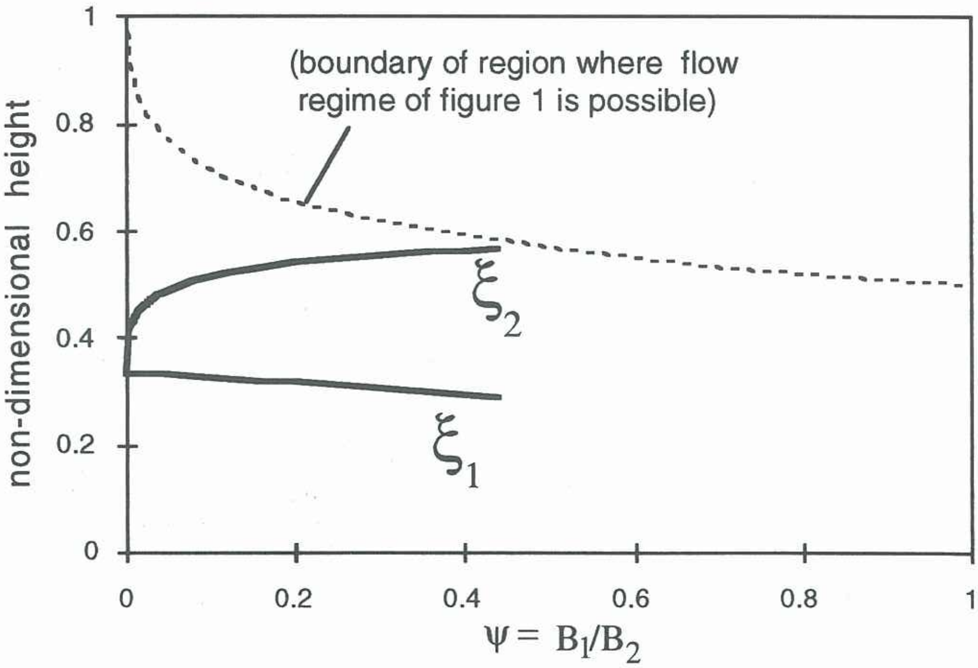

Figure 2. Theoretical prediction of interface heights for sources of opposite sign. ($A^*/H^2 = 3 \times 10^{-3}$)

The flow regime in the enclosure in the case of two plumes of opposite sign has three possible configurations depending on the magnitude of ψ. The three layers shown in figure 1 only exist in this configuration up to a limiting value of ψ. At this limiting condition a second flow configuration applies where the weaker source has sufficient buoyancy flux to supply Layer 1 with fluid of density equal to that of the ambient fluid as shown schematically in figure 3. In this case the lower interface at $z = h_1$ disappears. This limiting condition is indicated by the dashed line in figure 2 and the ratio of plume sources strengths is given by

$$\frac{h_1}{h_3} = \left(\psi^{-2/3} - \psi^{1/3}\right)^{3/5}. \qquad (14)$$

For values of ψ greater than this critical condition, the negatively buoyant plume is sufficiently strong to form a layer of fluid denser than ambient at the bottom of the enclosure corresponding to the third flow configuration which is illustrated in figure 4. In this case there is a two-way flow through the bottom vent, a_b, and ambient fluid from this source rises as a plume to supply fluid for the middle layer of density somewhat greater than ambient.

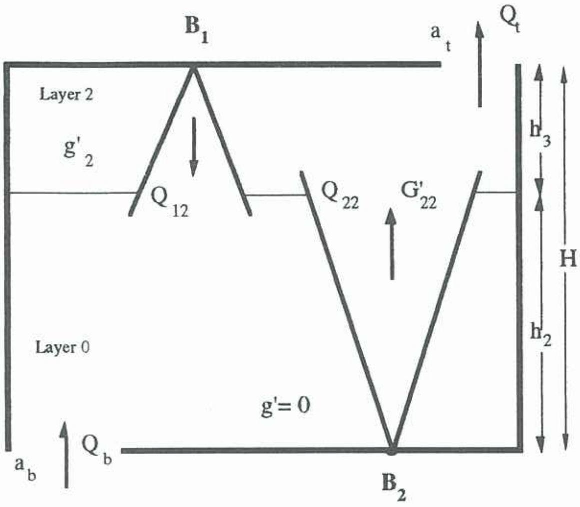

Figure 3. Ventilated enclosure with buoyancy sources of opposite sign at the limiting value of B_1/B_2.

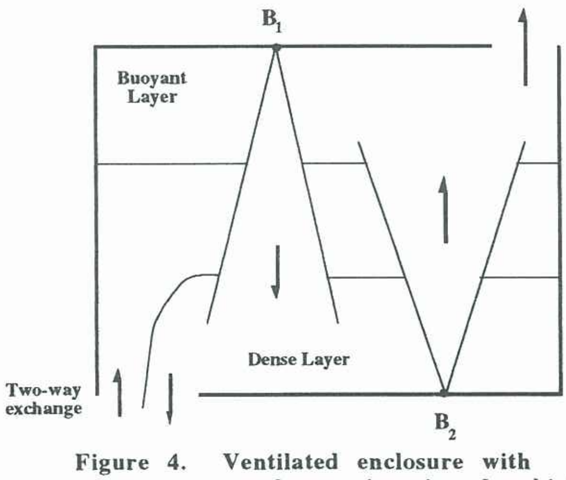

Figure 4. Ventilated enclosure with buoyancy sources of opposite sign for high values of B_1/B_2.

EXPERIMENTAL RESULTS

Experiments were carried on an acrylic enclosure 250mm in height and 200x300mm in plan. The enclosure was suspended in a large tank of water which acted as ambient fluid. The positive source of buoyancy was fed by a mixture of methylated spirits and water and the negative source with common salt solution. The results of experiments for two values of non-dimensional vent area are shown in Figure 5. Only the flow configuration illustrated in figure 1 was investigated quantitatively, however, the flow regimes shown in figures 3 and 4 were confirmed qualitatively .

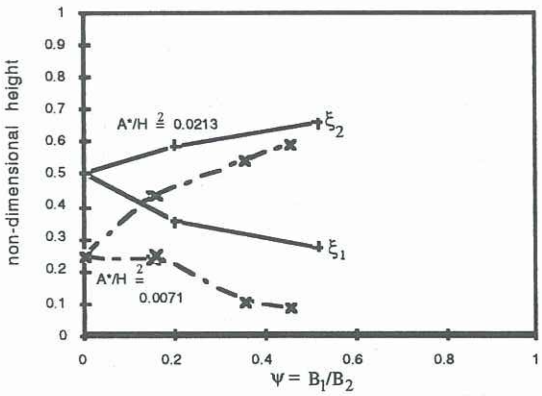

Figure 5. Non-dimensional interface heights ξ_1 and ξ_2 for the lower and upper interface, respectively, plotted against ψ, the ratio of the buoyancy fluxes, for two sources of opposite sign.

CONCLUSIONS

In an enclosure with two buoyancy sources of opposite sign for low values of the parameter B_1/B_2 three layers are formed. However, there is a limiting condition for this configuration; i.e. at a given value of B_1/B_2 the two layers of density differing from that of the ambient fluid merge to form a single layer. Higher values of B_1/B_2 produce a more complex flow pattern with two of the three layers having density less than the ambient fluid. The heights of the interfaces between the three layers are successfully predicted using plume theory.

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