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PARTICLE FLUX THROUGH SEDIMENT FINGERS

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ABSTRACT

Experiments were carried out to examine the fingering convection process occurring at the interface between a layer of warm, particle-laden water overlaying a layer of cold, fresh water. Two types of particles were used: silicon carbide, SiC, particles with nominal diameters of 4.5 µm, and titanium dioxide, TiO_2, particles with estimated diameters of 0.3 µm. In the formation stage, the sediment fingers were quite irregular, with large caps or vortex rings at their tip, and were descending rapidly. When fully developed, the sediment fingers, having approximately the same length and size, appear in a regular array all along the interface. Concentration measurements were made with six optical fiber probes arranged in the vertical direction in the lower layer. Results show that the particle flux due to finger convection for density ratios between 2 and 9 is two to three times that due to sedimentation.

INTRODUCTION

Fingering instability is a well-known phenomenon in a double-diffusive fluid layer in which the faster-diffusing component (heat) is stabilizing and the slower-diffusing component (solute) is destabilizing (Turner 1973). Once established, finger convection is an efficient process for the transport of both heat and solute. The same phenomenon occurs when the slower-diffusing component is replaced by a suspension of dense solid particles of small diameter, even though the particles have a finite sedimentation velocity but nearly zero Brownian diffusivity. This was shown in the experiments conducted by Houk and Green (1973), in which a suspension of taconite tailings and clay in warm water was introduced above a layer of cold, fresh water. Sediment fingers with enlarged caps or vortex rings were seen to descend rapidly into the lower, fresh-water layer. The authors were able to estimate the particle flux by measuring the descent rate of individual vortex-tipped fingers.

Later, Green (1987) showed that the same phenomenon occurs in a surface density current of warm, particle-laden water flowing over stationary, cold, fresh water. This is the model process of a silt-laden river discharging into an estuary with a salt wedge, or into a lake at a lower temperature (Eisma 1993). By assuming the particle flux can be estimated by the salt flux expression given by Schmidt (1979) for a heat/salt system, Green was able to estimate the parameter ranges within which transport of particles by fingering is of importance. Most recently, Green and Diez (1995) showed that the vertical transport of plankton is greatly enhanced by the fingering process.

In all the experiments reported by Green and his co-workers, the durations were quite short, typically less than 5 min from the start of the experiment. Consequently, all the data and flow visualization results were for the initial formation period of the sediment fingers. In this paper, we report the results from experiments of much longer durations, typically 30 min or longer. It will be shown that the fully developed sediment fingers are a regular array of long, slender cells of approximately the same size, much like the steady-state salt fingers. Particle flux through the sedimentation fingers was measured by the use of optical fiber probes, and the results are different from those suggested by Houk and Green (1973).

EXPERIMENTS

All experiments were carried out in a tank made of 1-cm-thick plexiglass, 20 × 40 × 50 cm tall. The tank was divided into two halves, each 25 cm in height, by a sliding barrier made of aluminum. Two types of small particles were used. One was SiC grinding power with a nominal diameter of 4.5 µm. It is expected that the standard deviation in size was approximately 1.5 µm, a value determined by Koyaguchi et al. (1993) for comparably sized particles. The density of SiC is 3.217 g cm^{-3}. The other particle was TiO_2 power with an approximate diameter of 0.3 µm, determined by a simple sedimentation experiment. Its density is 4.17 g cm^{-3}. These two particles were selected because of the large difference between their respective terminal velocities: 0.25×10^{-2} cm/s for SiC particles and 0.16×10^{-4} cm/s for TiO_2 particles.

The particle concentration, C, of the suspensions, defined as the ratio of the total mass of the particles to that of the suspension, was chosen such that the relative increase in the density of the suspension due to the addition of particles, $\Delta\rho/\rho$, is a constant at 5.6×10^{-4}, the value used by Green (1987) in the surface density current experiment.

For the experiments with SiC particles, the initial temperature differences, ΔT, between the top particle-laden layer and the lower fresh layer were set at 25°C, 12°C, and 5.5°C, with corresponding density ratios of $R_\rho = 9.4$, 4.5, and 2.0, where $R_\rho = \alpha\Delta T/(\Delta\rho/\rho)$, $\alpha = (1/\rho)(\partial\rho/\partial T)$. For the TiO_2 particles, only the experiment with $R_\rho = 9.4$ was performed.

At the start of each test, the bottom half was first filled with fresh water at room temperature and the barrier was closed. Then, approximately 11 kg of warm suspension was poured into the top half of the tank to a depth of 14 cm. Immediately, a 5-cm-thick Styrofoam board was placed on the free surface to prevent evaporation and minimize heat loss. For the purpose of flow visualization and observation, the sides of the tank were not insulated. The suspension was left standing for approximately one minute before the barrier was withdrawn.

Observations

As the barrier was withdrawn, an internal wave motion was immediately generated at the interface due to the combined effects of the void being filled, the wake motion downstream of the 1.5-mm-thick plate, and the scraping of the boundary layer by the end wall as the barrier was withdrawn. With TiO_2 suspension, this wave motion was damped in approximately 5 min. With SiC suspension, because of the vertical motion generated by sedimentation at the interface, the wave motion was damped in 2-3 min.

In the TiO_2 suspension, when the fingers were in the formation stage, vortex rings were seen forming at the tip of individual fingers, as observed by Houk and Green (1973) and Green (1987). In fact, these vortex rings descended much more rapidly than the fingers because of self-induction, and they eventually separated from the fingers. At later times, the fingers become more organized. At 11 min, as shown in Fig. 1, the fingers were approaching the same length due to the shear motion of the large-scale convection in the bottom layer. At that time, the average length of the fingers was approximately 3.5 cm, extending below the position of the barrier. Toward the left end of this photo, some fingers can be seen to extend upward into the TiO_2 suspension. Because of the light scattering characteristics of TiO_2 particles, it is very difficult to see the fingers in the upper layer. At 16 min, the fingers were well organized, with an average length of 4.5 cm; the width of each individual finger was between 2 and 3 mm. These fingers persisted and were growing slowly in length and width until the end of the experiment, approximately 45 min, when the TiO_2 concentration became uniform throughout the tank.

In the experiments with SiC particles, the vortex rings at the beginning of the experiment were not as clearly defined as in the TiO_2 case. The finger forming processes, however, were similar to the previous case. Unlike the previous case, the fingers in the upper layer were as visible as those in the lower layer.

Flux Measurements

Six optical fiber probes were used to determine the particle density of the lower layer as a function of time. Each probe is made of 2.25-mm-diameter optical fibers, with a gap of 6 mm maintained by a plexiglass fiber holder. When the probe is deployed in a particle suspension, the attenuation of light through this gap can be calibrated to measure the particle concentration in the fluid. These same probes were used by Koyaguchi et al. (1993) to determine the concentration of SiC particles in experiments modeling convection in magmas. A more detailed description of the probes is given in their paper. For the present experiment, the six probes were mounted 3.5 cm apart on a vertical stand. The topmost probe was 4 cm below the sliding barrier.

The data-reduction procedure for the SiC particle experiment with initial $R_\rho = 9.4$ is illustrated in Fig. 2. From the concentration measurements, the total mass of particles suspended in the lower layer at every 5-min interval was calculated by assuming uniform concentration in the horizontal direction (the masses are depicted as crosses in Fig. 2). These data points show a rapid increase of the mass of particles suspended in the water for the first 15 min, followed by a lower rate of increase. At 40 min, the end of the experiment, all the particles (9.17 g measured initially) from the upper layer had been transported into the lower layer. This means that, at that time, 4.47 g of particles had been deposited on the bottom of the tank. By assuming the rate of particle deposition to be proportional to the concentration of the suspension at 3.5 cm above the tank bottom, the total mass of particles transported into the lower layer was calculated as a function of time. This is shown as open circles in Fig. 2. At early times, the rate of deposit was very small because concentration of the suspension near the bottom was nearly zero. The straight line was determined by a linear regression of all the data points. The mass flux of the particles in this case was 4.9×10^{-6} g/cm^2-s and was constant throughout the experiment. This is approximately 2.5 times the mass flux by sedimentation of 2.0×10^{-6} g/cm^2-s. Using the correlation recommended by Schmidt (1979), the salt flux in a heat/salt system under the same conditions would be 3.8×10^{-6} g/cm^2-s.

DISCUSSION OF RESULTS

The same data reduction procedure was applied to the other two cases with SiC particles, and the results are shown in Fig. 3. Only the straight line for the $R_\rho = 9.4$ case is shown in the figure to avoid clutter. Data points for the $R_\rho = 4.5$ case follow the same straight line up to 30 min. Thereafter, the mass flux rate was reduced. This is attributed to the reduction of the temperature difference across the two layers. The double–diffusive effects that were responsible for driving the convection became less important and the flux rate was reduced to 2.3×10^{-6} g/cm^2-s.

For the $R_\rho = 2.0$ case, the initial particle flux was higher than in both of the two previous cases. During the experiment, it was quite evident from visual observations that the initial fingers with large caps descended rapidly into the lower layer and reached the bottom of the tank in 2-1/2 min. The data in Fig. 3 show that the flux rate was noticeably reduced to a lower value after 15 min. The average mass flux for the first 15 min was 5.8 g/cm^2-s and was 1.9×10^{-6} g/cm^2-s from 15 to 35 min. Apparently, for this case with a lower initial temperature difference across the layers, the double-diffusion mechanism was effective only for the first 15 min or so.

The results for TiO_2 particles are also shown in Fig. 3. The flux rate was constant throughout the experiment. At the end of the experiment, 45 min, the concentration of the lower layer was the same as that in the upper layer, and finger convection had ceased. The mass flux was 1.64×10^{-6} g/cm^2–s.

TABLE 1. MASS FLUX OF PARTICLES

Particles	R_ρ	Mass Flux of Particles, $F_p \times 10^6$ (g/cm^2–s)			
		Experimental		Sedimentation	Salt[a]
		Initial	Final		
SiC	9.4	4.9	4.9	2.0	3.75
	4.5	4.9 (< 30 min)	2.3	2.0	3.75
	2.0	5.8 (< 15 min)	1.9	2.0	3.75
TiO_2	9.4	1.6	1.6	0.008	3.37

All the mass flux results are summarized in Table 1. The measured mass flux values are also compared with the flux by sedimentation and the salt flux based on the correlation recommended by Schmidt (1979). The slightly smaller value for TiO_2 is due to a slightly higher value of $\beta = \rho^{-1}\partial\rho/\partial C$. The results show that, for the SiC particles, when finger convection is active, the mass flux is two to three times that due to sedimentation alone. For sub-micron particles, such as in the case of TiO_2, the particle flux due to fingers far exceeds that due to sedimentation alone. However, using the salt flux through the salt fingers as an estimate would over-predict by about 100%.

The principal sources of error in these experiments are the heat loss to the surroundings and the assumption that the concentration is horizontally uniform. The effect of heat loss was estimated by using the vertical temperature distribution at t = 21 min and 58 min in an experiment with TiO_2 particles. A heat balance shows that there was an average temperature drop of 1.8°C in 38 min throughout the upper layer due to heat loss. This change in the ΔT across the layer may affect the time when the fingering convection becomes ineffective, but it will not affect the general trend of the data.

The more serious error is the assumption of horizontal uniformity of the measured concentration. During the early stages of the experiment, it was quite evident that this assumption is not true. At later times, the suspension in the lower layer appeared to be quite uniform. In order to assess the validity of the assumption of uniformity, two experiments were run under the same conditions for SiC particles with a nominal diameter of 9 μm. After 10 min, the measured concentrations at all six vertical positions became essentially constant with time. The average concentration of the lower layer was measured at 1.24×10^{-4} in one experiment and at 1.03×10^{-4} in the other, a 20% relative error. From this result, it is conjectured that the concentration measurements, and thus the mass flux reported here, will have an error of ±10% during the first 15 min of the experiments. These experiments were not presented and discussed here because the finger structure was not clearly exhibited. Further experiments are needed to determine the mechanism of particle transport in the large particle case.

CONCLUSIONS

1. The fully developed sediment fingers have approximately the same length and width, and they appear as a regular array all along the interface. Only in the early formation stage are these fingers seen to have large caps or vortex rings at their tips and to be descending rapidly, as reported by Houk and Green (1973).
2. For the SiC particles, at R_ρ = 9.4, the mass flux of particles remains essentially constant throughout the entire experiment, and its value is approximately 2.5 times that due to sedimentation flux.
3. At $R_\rho \leq 4.5$, the mass flux of SiC particles is the same or slightly larger (for R_ρ = 2.0) than the value for the R_ρ = 9.4 case in the first part of the experiments. In later stages, the mass flux reduces to a value comparable to the sedimentation flux. The transition point occurs earlier as R_ρ is reduced.
4. For sub-micron particles, as in the case of TiO_2, measured mass flux is a constant throughout the experiment, and its value is approximately half of the salt flux through the salt fingers with the same initial relative density increase.

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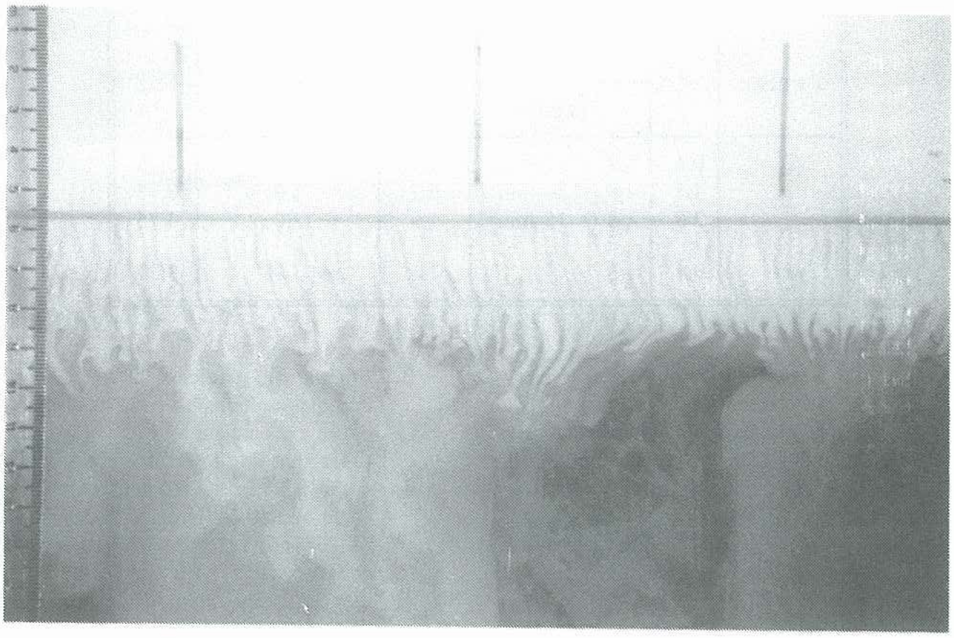

Fig. 1. Sediment fingers with TiO_2 particles at $t = 11$ min. Vortex rings can be seen at tips of fingers. Some fingers can be seen to extend above the position of the barrier.

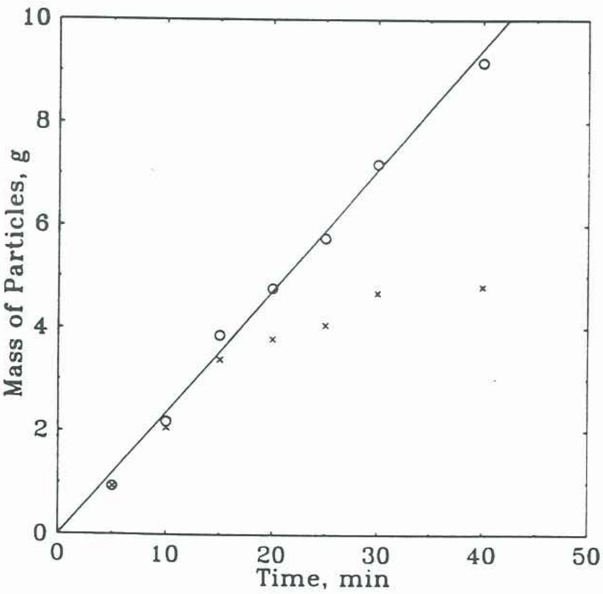

Fig. 2. Experimental results for SiC particles at $R_p = 9.4$. ×, mass of particles in suspension in the lower layer; ○, total mass of particles in the lower layer; —○—, linear regression line.

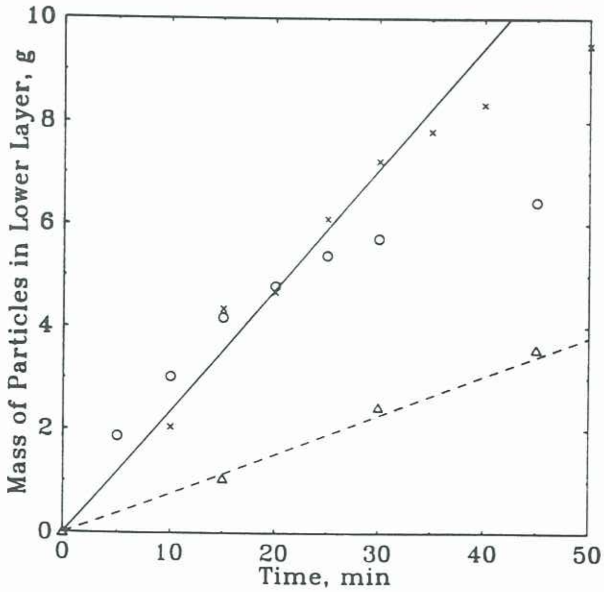

Fig. 3. Results of all four experiments. —, linear regression line for SiC particles at $R_p = 9.4$; ×, SiC particles at $R_p = 4.5$; ○, SiC particles at $R_p = 2.0$; Δ, TiO_2 particles at $R_p = 9.4$.