EFFECTS OF BOUNDARY LAYER TRANSITION ON VORTEX SHEDDING FROM THICK PLATES WITH FAIRED LEADING EDGE AND SQUARE TRAILING EDGE

M.K. Bull, Y. Li and J.M. Pickles

Department of Mechanical Engineering
University of Adelaide
Adelaide, SA 5005
Australia

ABSTRACT

The paper presents results of an experimental investigation of the frequency of vortex-shedding from thick plates with elliptical leading edges and square trailing edges, over a range of flow velocities and chord-to-thickness ratios in which the boundary layer at the trailing edge of the plate may be laminar, transitional or fully turbulent. When the boundary layer at the trailing edge is laminar, the non-dimensional frequency varies linearly with Reynolds number (as previously found by Eisenlohr). A new (and different) linear relation is found for fully-turbulent boundary layers.

Modified non-dimensional frequency and Reynolds number parameters, involving the transition Reynolds number, are obtained which produce collapse of the vortex-shedding-frequency data over all states of trailing-edge boundary layer.

INTRODUCTION

The frequency of vortex-shedding from a plate with some form of faired leading edge and bluff trailing edge has been previously investigated by Eisenlohr (1986) and Eisenlohr and Eckelmann (1988). Their work was concerned mainly with laminar boundary-layer flow on thin plates with semicircular or elliptical leading edges and square trailing edges. Some effects of transition from laminar to turbulent boundary-layer flow were observed: for both forms of leading edge, the rate of increase of vortex-shedding frequency with flow velocity was observed to decrease following transition from a laminar to turbulent boundary layer on the plate. In the case of plates with semicircular leading edges, transition (attributed to flow separation at the leading edge of the plate) is also accompanied by a sudden jump in frequency, while in the case of elliptical leading edges transition is accompanied by a gradual change in slope of the frequency-velocity curve.

In both works, the ratio $C=c/t$ of plate chord c to plate thickness t was in the range $50 \leq C \leq 800$. For such plates the boundary layer thickness at the trailing edge is generally very much greater than the plate thickness, and it was found that a good correlation of data could be obtained in terms of the effective thickness variable $t'=(t+2\delta_*)$, where δ_* is the displacement thickness of the boundary layer at the trailing edge of the plate. For laminar boundary layers and both leading edges, Eisenlohr and Eckelmann obtained a least-squares fit to their frequency-velocity data given by

$$F_{t'} = 0.286\, Re_{t'} - 39.2 \qquad (1)$$

where $F_{t'} = ft'^2/\nu$, $Re_{t'} = Ut'/\nu$, U is the flow velocity and ν is the fluid viscosity. The present work shows that data correlations in term of these same variables, or modified forms of them, can also be obtained for transitional and turbulent boundary-layer flow on the plate.

EXPERIMENTAL DETAILS

All plates had a semi-elliptical leading edge with a ratio of major-axis to minor-axis of 5.0, and a square trailing edge. The plate thickness was t = 18.0 mm throughout. The models were set up in a two-dimensional jet issuing from a wind-tunnel contraction and confined between vertical side-walls 690 mm apart. For all models, the initial portion consisted of an aluminium plate with elliptical leading edge and chord-length of c = 150 mm; extensions to the plate chord length were made by adding a number of plates of rectangular cross-section at the trailing edge, to give chord-to-thickness ratios in the range $20 \leq C \leq 52$. The undisturbed flow velocity U was continuously variable over the range $1.2 \leq U \leq 23$ m/s. The corresponding Reynolds-number range based on plate thickness was therefore $1.44\times10^3 \leq Re_t = U\,t/\nu \leq 2.76\times10^4$.

Experiments were conducted with boundary-layer transition allowed to occur naturally. In this case the chordwise position of transition at a given flow velocity remained essentially fixed and independent of the extension of the plate chord length.

Experiments were also carried out with boundary layer trip wires of diameter 3.2 mm attached to both surfaces of the leading plate at a position 45 mm downstream of the leading edge. This gave a boundary layer effectively fully-turbulent at the plate trailing edge at all flow velocities over the test range.

Time signals corresponding to the velocity fluctuations at a position in the shear layer separating from one of the plate surfaces, just downstream of the trailing edge, were obtained by means of a hot-wire anemometer. Vortex-shedding frequencies were obtained from the strong local peak in the velocity spectrum generated by processing the time signal by a Rockland work station.

BOUNDARY LAYER DISPLACEMENT THICKNESS

To evaluate the effective plate thickness $t'=(t+2\delta_*)$, values are required of the displacement thickness of the boundary layer at the trailing edge of the plate, where the boundary layer may be in a laminar, transitional or fully-turbulent state. These have been obtained by calculation, based on the experimentally-determined position of the start of boundary layer transition, as defined by the Reynolds number $Re_{xtr} = Ux_{tr}/\nu$, where x_{tr} is the distance downstream of the plate leading edge at which boundary layer transition starts. Following Narasimha (1957, 1985), we identify x_{tr} with both the virtual origin of the turbulent boundary layer and the position of initial formation of turbulent spots.

For the Reynolds number based on plate chord $Re_c = U\,c/\nu < Re_{xtr}$, when the boundary layer flow on the plate is entirely laminar, the displacement thickness δ_{*L} is given by the Blasius relation

$$\frac{\delta_{*L}}{x} = \frac{C_{*L}}{Re_x^{1/2}} , \qquad (2)$$

with $x = c$, and $C_{*L} = 1.7208$. For $Re_c >> Re_{xtr}$, when the boundary layer flow at the trailing edge of plate is fully turbulent, the displacement thickness δ_{*T} can be adequately represented by the Prandtl relation

$$\frac{\delta_{*T}}{x} = \frac{C_{*T}}{Re_x^{1/5}} , \qquad (3)$$

with $C_{*T} = 0.0463$. When the boundary layer flow on the plate is partly laminar and partly turbulent, the displacement thickness in the transition region is given

$$\delta_* = (1-\gamma)\delta_{*L} + \gamma\delta_{*T} , \qquad (4)$$

by
where γ is the intermittency. Again following Narasimha (1957) and Dhawan and Narasimha (1958), we take

$$\gamma = 1 - \exp[-0.412\xi^2] , \qquad (5)$$

where $\xi = (x-x_{tr})/\lambda$ and λ, the length scale of the transition region, is defined as

$$\lambda = x_{\gamma=0.75} - x_{\gamma=0.25} . \qquad (6)$$

The relation between the length scale λ and x_{tr} is given by Dhawan and Narasimha (1958) as

$$Re_\lambda = 9\, Re_{xtr}^{3/4} , \qquad (7)$$

with $Re_\lambda = U\lambda/\nu$.

The displacement thickness of the transitional boundary layer at the trailing edge of the plate, where the intermittency is γ_c, is then obtained from equations (2), (3) and (4) as

$$\frac{\delta_*}{c} = \frac{(1-\gamma_c)\,C_{*L}}{\sqrt{\alpha}\,Re_{xtr}^{1/2}} + \frac{\gamma_c C_{*T}(\alpha-1)^{4/5}}{\alpha Re_{xtr}^{1/5}} , \qquad (8)$$

where $\alpha = Re_c/Re_{xtr}$, and where, from equations (5) and (7),

$$\gamma_c = 1 - \exp[-5.09\times10^{-3}(\alpha-1)^2 Re_{xtr}^{1/2}] . \qquad (9)$$

Equations (8) and (9) have been used to calculate δ_* for the purposes of determining t'.

EXPERIMENTAL RESULTS

Plots of the non-dimensional vortex-shedding frequency $F=ft^2/\nu$ as a function of the Reynolds number Re_t are shown in figure 1. In these plots, the laminar regime at low Reynolds number and the turbulent regime at high Reynolds numbers can be readily identified, with a continuous transition from one to the other at intermediate Reynold numbers. It is also clear that these non-dimensional variables do not produce a collapse of the data.

The data are replotted in figure 2 as $F_{t'}$ as a function of $Re_{t'}$, in the form used by Eisenlohr and Eckelmann (1988). The data conversion has made use of values of boundary-layer displacement thickness at the plate trailing edge calculated by the procedure detailed in the previous section.

The value of the transition Reynolds number Re_{xtr} has been determined either by direct observation of the signals from a hot-wire traversed in the streamwise direction close to the plate surface, at a fixed flow speed, or from the correlation between the transition Reynolds number and the Reynolds number, which we denote by Re_i, corresponding to the intersection of the extrapolated fully-laminar and fully-turbulent portions of the curves of vortex-shedding frequency against velocity. Both procedures are based on a judgement of the onset of transition, and in general the condition chosen

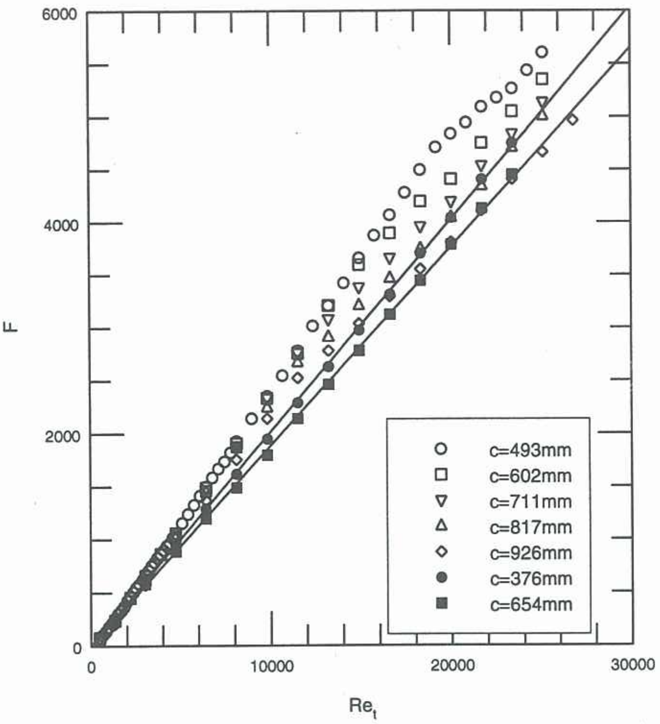

Figure 1: Non-dimensional vortex-shedding frequency F as a function of Reynolds number Re_t . Open symbols, natural transition; filled symbols, trip-wire data.

corresponds to an intermittency of about 0.05. The correlation between Re_{xtr} and Re_i is shown in figure 3.

The form of the variation of $F_{t'}$ with increasing $Re_{t'}$ is linear for the fully-laminar boundary-layer flow on the plate, while $Re_c < Re_{xtr}$. As Re_c is increased beyond Re_{xtr} , the transition region of the boundary layer moves progressively upstream from the trailing edge of the plate, and $F_{t'}$ departs from this linear variation; it finally approaches asymptotically a second linear dependence

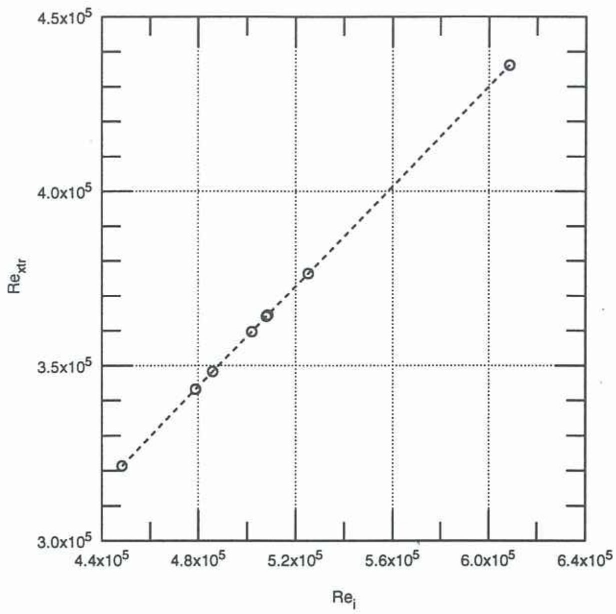

Figure 3: Correlation between Re_{xtr} and Re_i

on $Re_{t'}$, corresponding to fully-turbulent boundary-layer flow on the plate. The asymptotic values are consistent with the results for the fully-turbulent boundary-layer regime obtained from the tests with trip wires on the plate surfaces, which are also shown in figures 1 and 2. A least-squares fit to the trip-wire data gives

$$F_{t'} = 0.229\ Re_{t'} \ . \qquad (10)$$

The non-dimensionalisation used for figure 2 gives a good correlation of the frequency data when the boundary layer at the trailing edge of the plate is laminar or fully-turbulent, but not for transiton between the two. The reason for failure in the last case is that the beginning of the departure of $F_{t'}$ from the laminar-

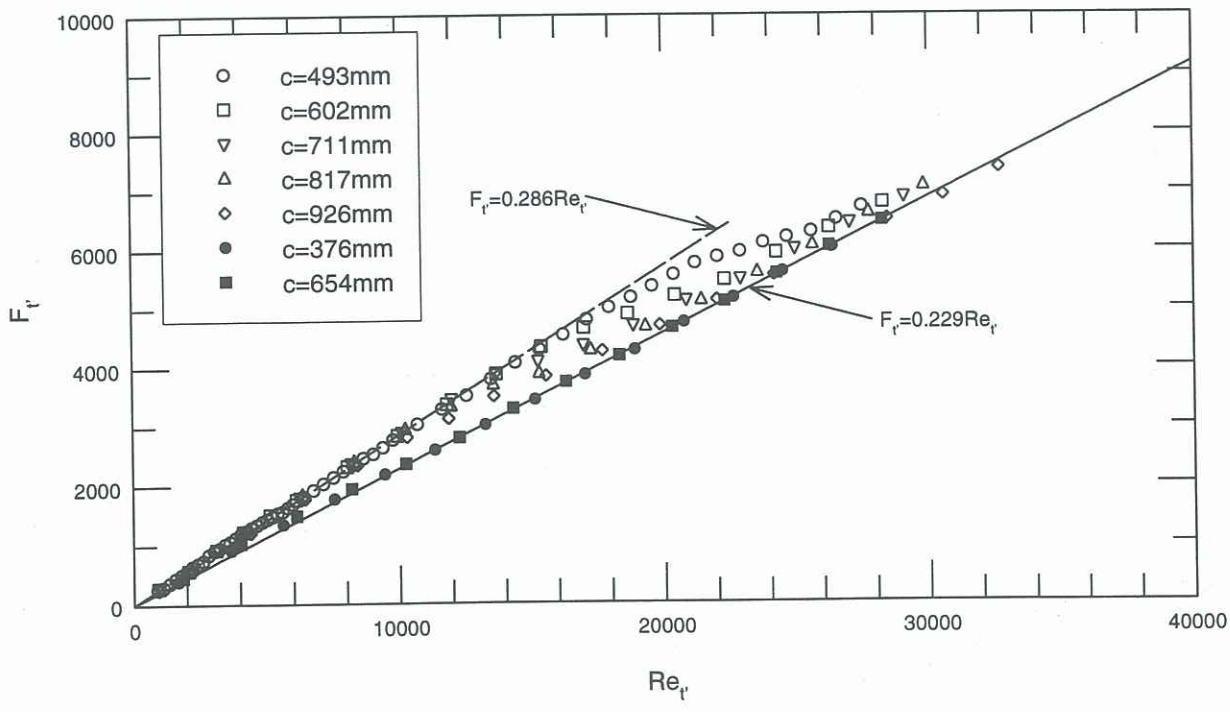

Figure 2: Non-dimensional vortex-shedding frequency $F_{t'}$ as a function of Reynolds number $Re_{t'}$. Open symbols, natural transition; filled symbols, trip-wire data.

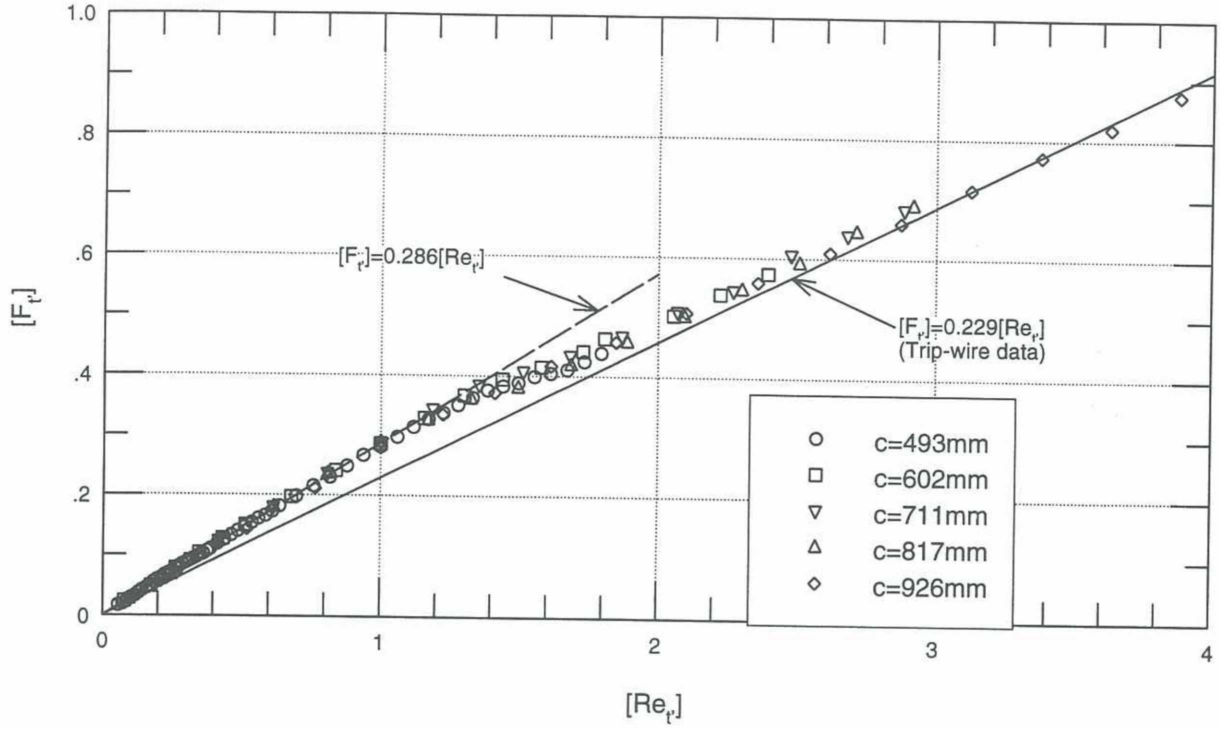

Figure 4: Non-dimensional vortex-shedding frequency $[F_{t'}]$ as a function of modified Reynolds number $[Re_{t'}]$.

boundary-layer relation occurs at $Re_{t'tr}$, the value of $Re_{t'}$ at which the onset of boundary-layer transition coincides with the trailing edge of the plate, which is a function of both Re_{xtr} and Re_c. $Re_{t'tr}$ is given by

$$Re_{t'tr} = Re_{xtr} \frac{t}{c} + 2C_{*L} Re_{xtr}^{1/2} . \qquad (11)$$

Normalised variables $[F_{t'}] = F_{t'} / Re_{t'tr}$ and $[Re_{t'}] = Re_{t'} / Re_{t'tr}$ produce a collapse of the data of figure 2 to essentially a unique relation, which is shown in figure 4. If the constant in equation (1) is neglected (valid for sufficiently large Reynolds numbers) the relations for laminar and fully-turbulent boundary layers are given respectively by

$$[F_{t'}]_L = 0.286 \, [Re_{t'}] \qquad (12)$$

and

$$[F_{t'}]_T = 0.229 \, [Re_{t'}] . \qquad (13)$$

CONCLUSION

The vortex shedding frequency in the wake of a flat plate with an elliptical leading edge and a square trailing edge depends on the boundary layer state at the plate trailing edge - whether laminar, transitional or fully turbulent. To a good approximation, the non-dimensional vortex shedding frequency $[F_{t'}]$ is a function of only Reynolds number $[Re_{t'}]$ over all boundary-layer states; the relation is linear at low and very high Reynolds numbers - when the boundary layer at the trailing edge is either laminar or fully-turbulent - but non-linear when the boundary layer is in a transitional state at the trailing edge of the plate.

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