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JET IMPACT IN COLLAPSING BUBBLES

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ABSTRACT

The effect of the asymmetry caused by buoyancy on the growth and collapse of a bubble near a rigid vertical wall is examined using a fully three-dimensional boundary integral method. These results are compared with the predictions of the Kelvin impulse approximation for the direction of the jet which penetrates the bubble during its collapse. The direction of the jet determines the position of its impact and is therefore an important factor in the damage causing mechanism of bubble collapse.

INTRODUCTION

The subject of bubble dynamics has long been an important field of research due to the damage caused to hydraulic machinery by cavitation bubbles as well as the intentional damage inflicted on marine vessels by underwater explosions. Recent research has focussed on the effect of the high speed liquid jets which are often observed to penetrate collapsing bubbles after the initial expansion phase. Damage to nearby structures is observed to be related to the flow generated by the jet impact on the opposite side of the bubble and the details of the jet motion are therefore important for the understanding of this aspect.

Numerical studies of the jetting phenomenon have been carried out during the past decade using the boundary integral method for both axisymmetric cases (Blake, Taib and Doherty (1986), Best and Kucera (1992)) and for fully three-dimensional geometries (Chahine (1990), Harris (1992)). Complementing the numerical simulations has been the development of an analytical approach using the concept of the Kelvin impulse which was applied to bubble dynamics by Benjamin and Ellis (1966), this work being extended by Blake and Cerone (1982), Blake (1988) and Best and Blake (1994).

The present paper presents a description of a new fully three-dimensional boundary integral algorithm (3DBIM) for bubble dynamics using radial basis functions for the surface approximations. This is followed by a brief account of the Kelvin impulse theory as applied to the prediction of the jet direction during bubble collapse. It is shown that the 3DBIM can clarify and extend the Kelvin impulse results regarding jet impact during bubble collapse near a rigid structure.

MATHEMATICAL FORMULATION

For the range of bubble dynamics under consideration here the fluid flow can be assumed to be inviscid, incompressible and irrotational in a domain Ω, bounded by $\partial\Omega \equiv S \cup \Sigma$, where S is the free surface of the bubble and Σ denotes the other boundaries. Cartesian coordinates are chosen with gravity acting vertically downwards in the negative z direction. The motion is thus defined by a velocity potential $\Phi(\boldsymbol{x}, t)$ ($\boldsymbol{x} = (x, y, z)$) which satisfies Laplace's equation

$$\nabla^2\Phi = 0, \qquad \text{in } \Omega,$$

together with the kinematic and dynamic boundary conditions on the bubble surface,

$$\frac{D\boldsymbol{x}}{Dt} = \nabla\Phi, \qquad \boldsymbol{x} \in S,$$

$$\frac{D\Phi}{Dt} = \frac{1}{2}|\nabla\Phi|^2 - \alpha\left(\frac{V_0}{V}\right)^{\lambda} - \delta^2(z - z_0) + 1,$$

where D/Dt is the derivative following a fluid particle. On fixed, rigid boundaries the condition

$$\partial\Phi/\partial n = 0,$$

is applied.

The equations have been non-dimensionalised by scaling lengths with respect to the maximum bubble radius, R_m, time with respect to $R_m(\rho/\Delta p)^{1/2}$

and pressure by $\Delta p = p_\infty - p_c$, with p_c the constant vapour pressure of the cavity and p_∞ the hydrostatic pressure at the depth at which inception of the bubble occurs. The parameter $\alpha = p_0/\Delta p$ can be regarded as indicating the strength of an explosion where the bubble contains a non-condensible gas in addition to the vapour, so that the pressure p_b exerted by its contents is

$$p_b = p_c + p_0 \left(\frac{V_0}{V}\right)^\lambda,$$

where λ is the ratio of specific heats and p_0 an initial pressure.

The main interest of the present study is to investigate the effects of the buoyancy parameter,

$$\delta = \left(\frac{\rho g R_m}{\Delta p}\right)^{1/2}$$

and the non-dimensional distance, γ, from a solid boundary on the direction of the jet as it penetrates the bubble during the final stages of collapse, since this will then determine the location of the jet impact on the opposite surface of the bubble.

To complete the formulation, initial conditions for the bubble shape and position, along with the values of Φ and $\nabla\Phi$ on the bubble surface are required. Those given by Best and Kucera (1992) are used in the examples presented here. These are,

$$R_0 = 0.1, \qquad \Phi_0 = -2.5806976$$

and an initial radial velocity of 25.806976 for a cavitation bubble which starts as a sphere of radius R_0 and

$$R_0 = 0.1651, \qquad \Phi_0 = 0,$$

with initial velocity zero for an explosion bubble with $\alpha = 100$, $\lambda = 1.4$.

NUMERICAL METHOD

The boundary integral method has become a standard technique for the numerical simulation of potential flows with a free surface. It is based on the solution of the following integral equation at each time step, coupled with integration of the free surface boundary conditions.

$$c(\boldsymbol{x}) = \int_S \left(G\frac{\partial \Phi}{\partial n}(\boldsymbol{x}') - \Phi(\boldsymbol{x}')\frac{\partial G}{\partial n'}\right) dS'$$

$$c(\boldsymbol{x}) = \begin{cases} \Phi(\boldsymbol{x}) & \boldsymbol{x} \in \Omega, \\ \frac{1}{2}\Phi(\boldsymbol{x}) & \boldsymbol{x} \in S \end{cases}$$

where the Green's function is of the form,

$$G(\boldsymbol{x}, \boldsymbol{x}') = \frac{1}{4\pi}\frac{1}{|\boldsymbol{x} - \boldsymbol{x}'|}.$$

An important aspect of free surface computations using this approach is the approximation of the surface normals at a given number of nodes on the bubble together with the tangential component of the surface velocity at each node — the normal component having been obtained by solving the integral equation. An element based approach leads to numerical difficulties in constructing a smooth solution, while direct polynomial interpolation to the scattered surface nodes fails for particular arrangements of the nodes. The 3DBIM used in the present work employs radial basis functions so that an interpolant to a function $f(\boldsymbol{x})$ is represented as

$$s(\boldsymbol{x}) = \Sigma_{j=1}^{m} a_j \psi(|\boldsymbol{x} - \boldsymbol{x}_j|) + \Sigma_{j=1}^{K} b_j q_j(\boldsymbol{x})$$

where $\{q_i\}$ forms a basis for the space of polynomials of order not exceeding K. ψ is chosen as the multiquadric,

$$\psi(|\boldsymbol{x} - \boldsymbol{x}_j|) = \sqrt{|\boldsymbol{x} - \boldsymbol{x}_j|^2 + c^2},$$

with c a constant. A full account is given in Blake *et al.* (1995).

THE KELVIN IMPULSE

The application of the Kelvin impulse of a transient cavity to predicting the direction of jet impact is described in Best and Blake (1994). In non-dimensional form the Kelvin impulse (scaled with respect to $R_m^3(\rho\Delta p)^{1/2}$) is defined as,

$$\mathbf{I} = \oint_S \Phi \mathbf{n} dS,$$

The theory is developed for a cavitation bubble with a constant internal pressure in terms of a small parameter ϵ which is $O(1/\gamma)$. In addition it is assumed that the buoyancy parameter is $O(\epsilon)$ and that the bubble does not depart greatly from its original spherical shape. With these assumptions the Kelvin impulse can be approximated in terms of complete and incomplete Beta functions with an error $O(\epsilon^4)$. The components $\mathbf{I}^\Sigma$ and $\mathbf{I}^g$ resulting from the effects of the solid boundaries and buoyancy respectively, are given here for reference.

$$\mathbf{I}^\Sigma(t) = \begin{cases} \frac{\sqrt{6}\pi}{9}\boldsymbol{\Gamma}\left[B_r(\frac{7}{6},\frac{3}{2}) - \frac{1}{2}\mu B_r(\frac{3}{2},\frac{3}{2})\right], \\ \qquad 0 \le t \le T/2, \\ \frac{2\sqrt{6}\pi}{9}\boldsymbol{\Gamma}\left[B(\frac{7}{6},\frac{3}{2}) - \frac{1}{2}\mu B(\frac{3}{2},\frac{3}{2})\right] \\ -\frac{\sqrt{6}\pi}{9}\boldsymbol{\Gamma}\left[B_r(\frac{7}{6},\frac{3}{2}) - \frac{1}{2}\mu B_r(\frac{3}{2},\frac{3}{2})\right], \\ \qquad T/2 \le t \le T, \end{cases}$$

$$\mathbf{I}^g(t) = \begin{cases} \frac{2\sqrt{6}\pi}{9}\delta^2\left[B_r(\frac{11}{6},\frac{1}{2}) + \frac{1}{2}\mu B_r(\frac{13}{6},\frac{1}{2})\right]\mathbf{e}_z, \\ \qquad 0 \le t \le T/2, \\ \frac{4\sqrt{6}\pi}{9}\delta^2\left[B(\frac{11}{6},\frac{1}{2}) + \frac{1}{2}\mu B(\frac{13}{6},\frac{1}{2})\right]\mathbf{e}_z - \\ \frac{2\sqrt{6}\pi}{9}\delta^2\left[B_r(\frac{11}{6},\frac{1}{2}) + \frac{1}{2}\mu B_r(\frac{13}{6},\frac{1}{2})\right]\mathbf{e}_z, \\ \qquad T/2 \le t \le T. \end{cases}$$

T is the period of the motion, $r = R^3$ where R is the radius of the spherical approximation to the bubble shape and $\boldsymbol{\Gamma}$ and μ are determined by the solid boundaries.

If the bubble is not greatly distorted from its spherical shape then the direction of the Kelvin impulse is related to the direction of translation of its centroid and subsequently the direction of the jet. The expression for the impulse can therefore be used to determine a 'zone of attraction' around a submerged structure within which the bubble jet will be directed towards the structure. However, the parameter ϵ is not small close to the structure and the behaviour of the bubble jet in this region is addressed below.

RESULTS

In order to compare the predictions of the Kelvin impulse theory with the 3DBIM the motion of a bubble near a vertical fixed, rigid wall is considered. This case provides a simple way to introduce asymmetry by means of the buoyancy force. $\boldsymbol{\Gamma}$ and μ are evaluated as

$$\boldsymbol{\Gamma} = -\frac{1}{\gamma^2}\mathbf{e}_\gamma, \quad \mu = \frac{1}{2|\gamma|},$$

where $\mathbf{e}_\gamma$ is a unit vector normal to the boundary and directed away from it.

The case of a cavitation bubble with $\gamma = 4$, $\delta = 0.25$ shows good agreement between the 3DBIM and the Kelvin impulse. The direction of the centroid motion and the jet is at an angle of approximately 78^o to the horizontal for the 3DBIM compared to 80^o for the Kelvin impulse prediction during the collapse phase. As the point of inception of the bubble is moved closer to the vertical wall the agreement between the two methods becomes less good, as is expected since ϵ increases.

Figure(1) shows the expansion and collapse of a cavitation bubble for $\gamma = 1.5$, $\delta = 0.25$ computed with the 3DBIM. For most of the period of motion the bubble remains almost spherical departing from this shape only during the final stage of collapse. A flattening of the underside of the bubble can be seen at $t = 1.9429$ in response to the influences of the wall on its right and the buoyancy force. The direction of translation of the centroid at this time is at an angle of 23^o to the horizontal while the Kelvin impulse approximation gives 43^o. However, the jet which forms at the end of the collapse phase can be seen to move vertically to pinch off the upper end of the bubble rather than in the direction of the centroid. Buoyancy therefore acts to change the behaviour of the jet in this case and the motion of the bubble centroid does not provide an indication of the jet direction.

Reducing the effect of buoyancy produces the result shown in figure(2) for $\gamma = 1.5$, $\delta = 0.1$. Here the flattening of the bubble towards the end of its collapse occurs on the side away from the wall and the jet and the centroid directions of motion are nearly the same (approximately 5^o to the horizontal). The Kelvin impulse gives an angle of 8.6^o for this case. The bubble shape is close to being axisymmetric and thus when the asymmetry introduced by buoyancy is small the jet direction is approximately in the direction of the centroid translation.

A computation carried out for an explosion bubble for the parameters of figure(1) shows very similar behaviour but with a lengthening of the bubble period.

These computations show that the effect of buoyancy can influence the direction of the jet that penetrates the bubble. A strong buoyancy force can cause the jet to travel vertically rather than towards the wall so that its impact is likely to be less damaging to the structure.

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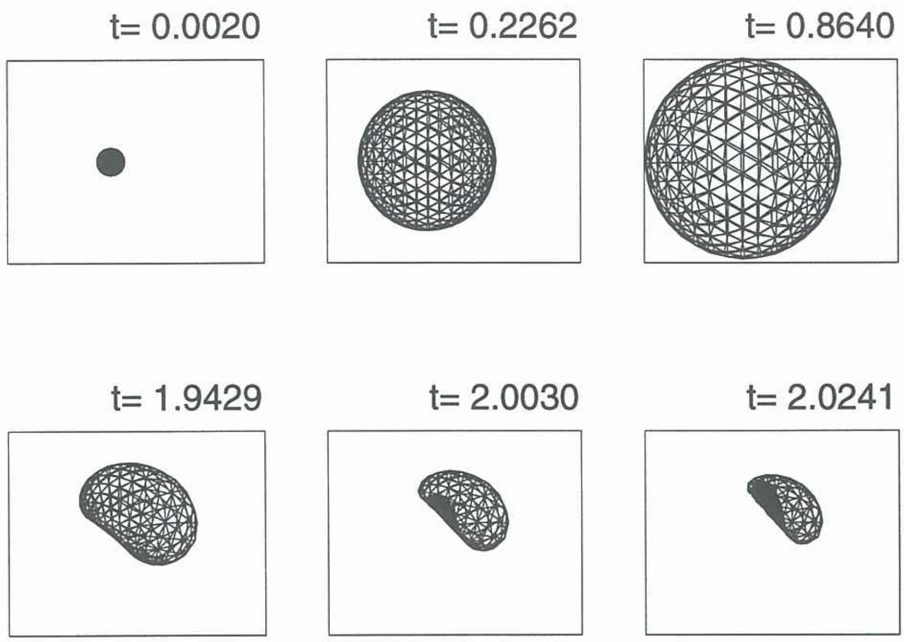

Figure 1: The growth and collapse of a cavitation bubble for $\gamma = 1.5$, $\delta = 0.25$. The wall position is given by the right hand side of the frame.

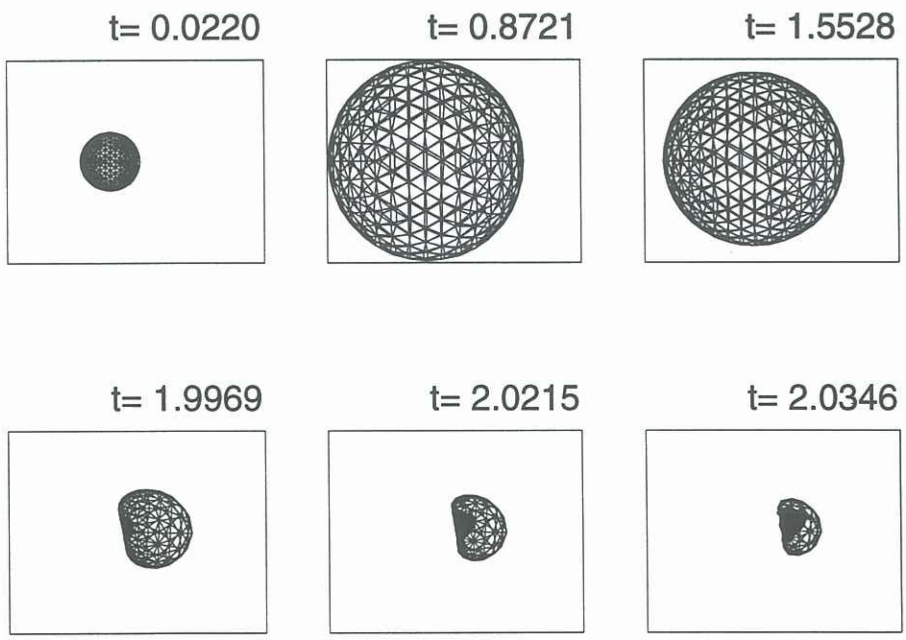

Figure 2: The growth and collapse of a cavitation bubble for $\gamma = 1.5$, $\delta = 0.1$. The wall position is given by the right hand side of the frame.