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A SIMPLIFIED THEORY FOR THE TACKING OF A SAILBOAT

R. W. Bilger
Department of Mechanical & Mechatronic Engineering
University of Sydney
NSW 2006
Australia

ABSTRACT

Formulae are derived for the loss in boatspeed during the turning part of a tack and for the optimum radius for the turn. These results give insight into the strategy for steering and sailhandling during the tack and for design of the keel and rudder. Avoidance of flow separation at the bow is analysed and discussed.

INTRODUCTION

Tacking is the manoeuvre made by a sailboat during upwind sailing. The course is changed from an angle of about 40 degrees from the wind direction on one side, through the eyes of the wind, to an angle of about 40 degrees from the wind direction on the other side. In this way the sailboat can sail into the direction of the wind by a series of tacks in a zigzag course. The manoeuvre is controlled by changing the angle of the rudder to steer the sailboat through the desired change in heading. The sails flap during the period of change in wind angle and the boat loses speed during this period of negative driving force. The boat also loses speed due to the increased drag from the keel and rudder as these experience increased lift or side force to provide the centripetal force in the turn. It takes a while for the loss in boatspeed to be recovered and performance in the tack can be measured in terms of the loss in distance made good to windward compared with an ideal tack in which no speed is lost in an instantaneous tack.

Modelling the performance of sailboats in tacking and other such manoeuvres is a complex science which is as yet not fully developed. Computer codes that integrate the equations of motion for four degrees of freedom have been developed (Masuyama et al, 1993, Mattiske, 1993, Wellham, 1994) and are undergoing validation against field data. The four degrees of freedom are: longitudinal and lateral translation; and rotation in heeling and yaw (change of heading). These models are complex and do not give direct insight into the effects of design and other parameters. Our aim here is to analytically derive simple algebraic formulae which will give such insight. We confine ourselves to the turning part of the tack and make several simplifying assumptions.

ANALYTICAL FRAMEWORK

Figure 1 shows a sailboat in the turning part of a tack. For generality, the keel or forward foil is shown as being steerable. It is assumed that all parts of the sailboat are in simple rotation about the centre of the turn O with the centre of mass of the sailboat at C, so that the radius of the turn is R = OC. Typically, this radius is of the order of the length of the sailboat. The instantaneous speed of the boat is V_B and this is defined at C and will be normal to OC. The rate of rotation of OC is $\dot{\theta}$ and

$$\dot{\theta} = V_B / R \qquad (1)$$

The heading of the boat is in the direction CB where B is on the centreline near the bow. The rate of change of heading will also be $\dot{\theta}$, given that all points on the boat rotate around O. We consider trajectory coordinates centred at C. All the forces on the hull and its foils are reducible to a normal (lift) force along OC, denoted as L, and a drag force D in a direction normal to OC. We will consider the effects of these forces on the acceleration of the boat. It is noticed that the heading of the boat is not along the normal to OC: the boat must make leeway in the turn to generate the force L necessary to provide the centripetal acceleration in the turn. The contribution of sailforces is neglected in this simple model as their action more or less cancels out. Strategies to maximise their contribution are important

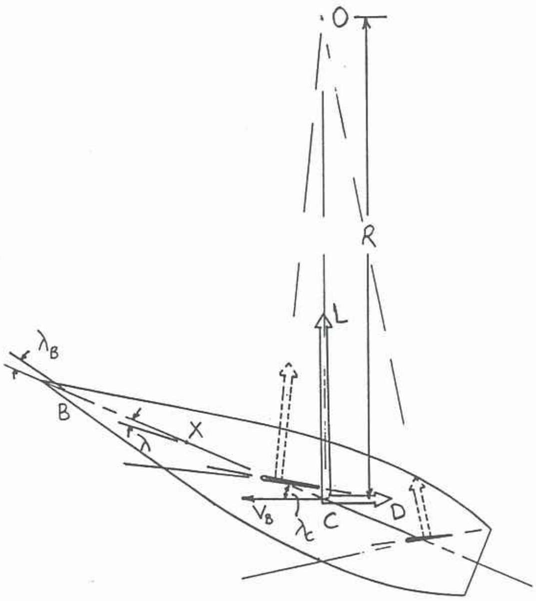

FIG 1. Forces and leeway angles for a sailboat hull during the steadily turning part of a tack. The centre of mass is at C and the centre of the turn is at O. R is the radius of the turn and L and D are net forces resolved along the radial direction OC and along the trajectory of C, respectively.

but will be best handled by a perturbation analysis.

Before proceeding, it is as well to consider whether the situation so far described is satisfactory for analysis. What about angular acceleration? Isn't that what a rudder is for? It is found that the time, t_a, taken to accelerate to θ at the beginning of the tack and to decelerate again at the end of turning part of the tack is often small compared with the time required to make the turn. The latter can be estimated as $t_l = 3/(2\theta) = 3R/(2V_B)$ since the turn is about 1.5 radians (heading angle change plus 2 × leeway angle). The time for angular acceleration, t_a, can be estimated from the rudder area, S_R, its lift coefficient during angular acceleration (here assumed 0.5) its moment arm, l_R, the moment of inertia about the vertical axis ($\rho_w \Delta r_{zz}^2$, where Δ is the displacement, ρ_w the density of the water, and r_{zz} is the radius of gyration about the vertical axis), and the angular velocity required, V_B/R. This results in the following estimate:

$$t_a/t_l = 2.7\Delta r_{zz}^2/(l_R S_R R^2) \tag{2}$$

This ratio is found to be of order 0.1 for many sailboats. (Note that the radius of gyration r_{zz} should include allowance for "added inertia" effects associated with angularly accelerating the water near the hull: this can typically increase r_{zz} by about 50%.) This estimate is confirmed by simulations of sailboats in which the equations of motion are integrated for 4 degrees of freedom: see Masuyama et al (1993), Mattiske (1993) and Wellham (1994).

To accelerate angularly in yaw at the start of the turn, the force on the rudder needs to be opposite to that shown in Figure 1. Most sailboats are designed so that the centre of mass C is very near the centre of effort of the keel so that during the steady turning part of the turn there is little side force contributed by the rudder. (During this part of the turn the side force contributed by the rudder may be determined by requiring the moments about C to be in equilibrium.) There is little incidence angle on the rudder and the rudder angle is then given by the requirement that the normal to its blade goes through O. At the end of the turn the rudder force needs to be in the direction shown and large enough to give a negative moment about C. If the rudder carries significant side force during the turn this extra force could stall the rudder. During the steady part of the turn the side force of the keel and the rudder are active in changing the linear momentum of the boat. These side forces result in added drag in the form of induced drag. The boat is slowed during the turning part of the tack and a loss in distance made good to windward is associated with this reduced speed during the (linear) acceleration phase of the tack. We now return to the analysis of the steady turning situation.

PERFORMANCE ANALYSIS

In a steady turn there is no acceleration along the direction OC in the trajectory coordinates adopted. Accordingly, there is no "added mass" term associated with the force balance in this direction. We have then that

$$L - \rho_w \Delta\, V_B \dot{\theta} = 0,$$

or

$$V_B \dot{\theta} = L/(\rho_w \Delta), \tag{3}$$

The added mass for acceleration in the longitudinal direction for a sailboat is quite small and neglecting this we obtain:

$$\rho_w\, \Delta\, \dot{V}_B = -D$$

or

$$\dot{V}_B = -D/(\rho_w\, \Delta). \tag{4}$$

Dividing Eqn. (4) by Eqn. (3) yields:

$$\frac{d(\ln V_b)}{d\theta} = -D/L$$

This may be integrated to give

$$V_{Bf} = V_{Bi}\, \exp\{-\Delta\theta/(L/D)_{av}\} \tag{5}$$

where V_{Bi} is the boat speed in to the tack, V_{Bf} is the boat speed at the end of the turning part of the tack, and $\Delta\theta = \theta_f - \theta_i$ is the total angle of turn of the trajectory (change in heading plus 2 × leeway angle). Also the "average" L/D, $(L/D)_{av}$ is really that of its reciprocal since

$$(L/D)_{av} \equiv \left\{ \int_{\theta_i}^{\theta_f} (D/L)\, d\theta \right\}^{-1} \qquad (6)$$

Although the turning is assumed to be steady, the wave-drag component of D is such a strong function of V_B that such averaging is usually necessary.

It is seen from Eqn. (5) that for minimum loss of boat speed during the turning part of the tack the best possible ratio, $(L/D)_{av}$, must be used. Here D is the total drag on the hull and includes the effects of the induced drag of the appendages. The induced drag coefficients increase with the square of the lift coefficients so that a maximum for L/D is obtained. For optimum tacking performance it is necessary to steer the boat at a turning radius, R_{opt}, given by

$$R_{opt} = 2\Delta/(SC_L)_{opt} \qquad (7)$$

where the "lift area" SC_L is approximately

$$SC_L \approx S_K C_{LK} + S_R C_{LR} \qquad (8)$$

where S_R and S_K are the plan form areas and C_{LR} and C_{LK} the lift coefficients of the rudder and keel respectively. The optimum value of SC_L is at the maximum for $(L/D)_{av}$. Equation (7) is obtained by substituting $\dot{\theta} = V_B/R$ and $L = ½\rho_w V_B^2\, SC_L$ into Eqn. (3), and setting for optimum lift coefficient. (The formula for turning radius is valid at other SC_Ls, also.)

DISCUSSION

If the turning part of the tack is taken at too tight a radius, R, the (L/D) attained will be lower than the maximum that is available and the loss of boat speed during the turn will be greater than necessary. This will mean that more time will be spent at low speed during re-acceleration back to the equilibrium speed. The loss in distance made good will be higher than for an optimum tack. Indeed, if the radius is much too tight, SC_L will exceed its stall value and the resulting flow separation causes the drag to shoot up to very high levels so that the speed out of the tack, V_{Bf} is greatly reduced and the loss in distance made good becomes huge. On the other hand, if the turn is too wide (L/D) will also be less than optimum with increased losses of boat speed and distance made good. The margin from stall will be larger, however, and this may be a good strategy in rough seas where the seakeeping motions of the boat may give temporarily high incidences on the keel and/or rudder and cause stall.

Good sailhandling during the tack can help to reduce the loss in boatspeed. The driving force gained can be viewed as subtracting from D and thus improving (L/D). The sail forces, however, require an increased value of SC_L and this increases the induced drag. Simulations with the 4 degree of freedom code (Wellham, 1994) take into account these effects and the effect of yaw rate and roll in heel on the sail lift: it is found that the Simple Tacking Theory result of Eqn. (5) still gives good estimates for the loss in boat speed. It is also found that there is little loss in distance made good during the turning part of the tack due, in main, to the shooting up effect of turning up into the wind. A formula for the loss in distance made good thus only needs to consider the acceleration phase of the tack. Such a formula has yet to be derived.

In general terms, the lift comes from the keel and rudder and much of the drag from the hull resistance. If the total appendage area, S, is too small the maximum L/D attainable will be low and the boat will lose a lot of speed in the tack as shown by Eqn. (5), and will need to make wide turns as shown by Eqn. (7). If S is too large the tacking performance will be excellent but the sailboat will have reduced straightline sailing speeds due to the increased wetted area.

The contribution of the induced drag from the keel and rudder is significant, and measures taken to reduce this will improve the tacking performance. Sharing the side force more equally between the keel and rudder should help in this regard. For this to happen during the turn the centre of gravity C should lie partway between the keel quarter-chord line and that for the rudder. The best position for the CG will depend on the effective spans of the keel and rudder. In a companion paper (Bilger, 1995) it is shown that the induced drag is minimised when the forces on the keel and rudder are in proportion to the square of their effective spans. Current practice is to have the CG near the centre of the keel so that there will be little side force on the rudder during the turn.

Conventional test tank data can be used to get a first estimate of the attainable maximum for L/D. The data can be corrected for the differences in loading of the keel and rudder that will occur during the turn due to the effects discussed in the preceding paragraph. A question of major concern will be about whether there is significant separation caused by the change in flow pattern about the hull. Large negative leeway can be generated near the bow and this could cause flow separation. Data from turning tank tests would be needed to find the magnitude of this effect. The effect can be reduced by judicious use of the trim tab or steerable front foil if such is available. This is discussed next.

BOW SEPARATION PROBLEMS

The drag in the turning part of the tack can be seriously affected if the angle of incidence near the bow is large where the cross-sections are narrow and deep so that separation occurs. The incident flow direction at any point on the hull is given by the tangent to the circle through the point centred at O. For any point X on the boat centreline the leeway angle, λ, is related to the leeway angle at C, λ_C, by

$$\lambda = \tan^{-1}\{\tan\lambda_C - XC/(R\cos\lambda_C)\} \quad (9)$$

where XC is positive if X is ahead of C and the sine rule has been used for the triangle XOC. At the bow the leeway is usually negative so that the incidence angle is $-\lambda_B$ given by

$$-\lambda_B = \tan^{-1}\{BC/(R\cos\lambda_C) - \tan\lambda_C\} \quad (10)$$

It can be seen that with some boats it would pay to increase the leeway at the centre of mass, λ_C. This could be done by swapping the trimtab over before the tack or steering the keel as indicated in Fig. 1. Early swapping over the trimtab will decrease the stall margin for the keel at optimum turning radius, so a wider turn may be necessary.

SUMMARY AND CONCLUSIONS

In summary, the Simple Tacking Theory presented here divides a tack into two main parts: the turning part and a linear acceleration part. In the turning part only a small fraction of the time, Eqn. (2), is spent in angularly accelerating the boat in yaw, and the sail-force contribution to the accelerations is in net small and is neglected to first order. The boat speed drops due to the drag on the hull and appendages, some of which arises from the induced drag coming from the side force needed to change the direction of the linear momentum (i.e. the centripetal force needed going around the turn). This drop in boat speed is inevitable and is given by Eqn. (5). To minimise it the boat should be turned at a radius, Eqn. (7), giving the lift coefficient on the appendages corresponding to optimum (L/D). Loading the rudder to assist the keel in the turn can help here. To reduce drag arising from separated flow on deep-sectioned bows, the leeway at the centre of mass can be increased by putting the trim tab over early or by steering the front foil appropriately where one is fitted. Assuming that the flow remains attached on the hull the hull drag should not be too far different from that in straight-line sailing. Simulations making this assumption (Mattiske, 1993; Wellham, 1994) show good agreement with Eqn. (5). Time-resolved data from sailboats is needed to confirm this modelling. Most of the loss in distance made good occurs during the second part of the tack while the boat is being accelerated to overcome the loss made in the turn. There will be an optimum angle β for this acceleration. Work on this is proceeding but it depends sensitively on the sail coefficients used.

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REFERENCES

Bilger, R.W., 1995, "Sail force coefficients and optimum appendages for a sailboat", *12th Australasian Fluid Mechanics Conference*, The University of Sydney, NSW, Australia

Masuyama, Y., Nakamura, I., Tatano, H., and Takagi, K., 1993, "Dynamic performance of sailing cruiser by full-scale sea tests", *11th Chesapeake Sailing Yacht Symposium, SNAME*, Maryland, USA, pp. 161-180.

Mattiske, A., 1993, "Computer modelling to optimise the dynamic performance of sailing yachts", BE Thesis, Department of Mechanical and Mechatronic Engineering, The University of Sydney, NSW, Australia, 134 pp.

Wellham, S., 1994, "America's Cup Yacht Simulation and Steering Strategies in Tacking", BE Thesis, Department of Mechanical and Mechatronic Engineering, University of Sydney, NSW, Australia, 173 pp.