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EVALUATION OF DEPTH-INTEGRATED TURBULENCE MODELS FOR UNIDIRECTIONAL OPEN-CHANNEL FLOW

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ABSTRACT

Different turbulence models are investigated for unidirectional depth-integrated flow in simple and compound open-channels. A one-equation turbulence model is proposed that accounts for the influence of bed friction, the spanwise velocity gradient and their interaction. The turbulent length scale is directly estimated as a function of local values. The model can be used to estimate turbulent intensities, velocity distribution and local bed shear in straight river channels with variable bed roughness.

INTRODUCTION

Many free-surface flow problems, e.g. flows in rivers and estuaries, can be simplified by integrating the flow quantities over the flow depth. Assuming the vertical acceleration of the water to be small, the shallow-water equations can be applied to describe the spatial variation of the flow. The depth-integration also simplifies the treatment of internal boundaries (i.e. the boundary between wet and dry domains) and reduces the calculation time thereby allowing the simulation of large water bodies over long time periods. The results can be used for ecological studies or the simulation of convection and mixing of pollutants or sediments where cross-section averaged values are not sufficient but local values and their spatial variation are of importance.

In a depth-integrated model the fluxes due to turbulence and secondary currents have to be modelled. For a wide river the horizontal shear is small and may only be significant near the banks. Thus, bed friction is the primary contribution for the production of turbulence. Bed shear stress can be estimated by a logarithmic friction law or, for hydraulically rough beds, by the Manning formula. For the turbulent stresses in spanwise direction the eddy viscosity concept can be applied. To estimate the eddy viscosity, different models have been applied (constant eddy viscosity, mixing length model, one- and two-equation models). Since the equations describing the turbulence characteristic are highly non-linear it is somewhat arbitrary how the integration over the depth is carried out. Thus, the more sophisticated turbulence models may not give better results. In the following, a turbulence model for the depth-averaged equations is proposed that is suitable for application in open-channels.

FLOW EQUATION

The momentum equation for unidirectional depth-averaged flow in open-channels with arbitrary cross-section reads

$$\frac{\partial}{\partial y}\left(h\bar{\tau}_{yx}\right) + \rho g h S_e - \rho u_*^2 = 0 \qquad (1)$$

where x and y are the longitudinal and lateral dimensions respectively and h is the depth of flow, ρ is the density of the fluid, S_e is the energy slope and u_* is the local friction velocity. This equation is a first approximation for flows in straight river-channels with variable bed roughness (Goring *et al.*, 1995). Applying the eddy-viscosity concept to describe the momentum transfer due to turbulence, the Reynolds stresses can be expressed as

$$\bar{\tau}_{yx} = \rho\bar{\nu}_t \frac{\partial \bar{u}}{\partial y} \qquad (2)$$

with the depth-averaged values of eddy viscosity $\bar{\nu}_t$ and flow velocity $\bar{u}$. The conservation form of the momentum equation is solved with a finite volume scheme. Special attention has been paid to the evaluation of turbulent stress near water's edge, treating vertical walls like steep side slopes (Beffa, 1994).

k-L MODEL

For local values the eddy viscosity can be related to the turbulent energy k and a turbulent length-scale L by the Kolmogorov-Prandtl expression and it is assumed that this still holds after integration over the depth of flow. Taking depth-averaged values for the eddy viscosity and the turbulent energy yields

$$\bar{\nu}_t \sim \sqrt{\bar{k}}\tilde{L} \tag{3}$$

where $\tilde{L}$ can be considered as a mean turbulent length scale for depth-averaged flow and, therefore, is not a depth-averaged value itself. For planar flow, which occurs in hydraulically wide sections, the depth-averaged value of the eddy viscosity is derived from a logarithmic velocity profile after integration over the depth to

$$\bar{\nu}_t = c_v u_* h \tag{4}$$

with the friction velocity u_* and the coefficient $c_v = \kappa/6$ where κ denotes the von Karman constant (=0.40). The turbulent energy is usually normalised with the friction velocity

$$\bar{k} = c_k u_*^2 \tag{5}$$

with the empirical coefficient c_k=2.0 for planar flow over smooth and rough beds (Nezu & Nakagawa, 1993). Combining (3), (4) and (5) we get a relationship between the local depth of flow and the turbulent length scale

$$\tilde{L} = \frac{c_v h}{\sqrt{c_k}} \tag{6}$$

A one-equation model is derived using a transport equation for the turbulent energy. For unidirectional depth-averaged flow the equation for the turbulent energy can be written as

$$\frac{1}{h}\frac{\partial}{\partial y}\left(h\bar{\nu}_t\frac{\partial \bar{k}}{\partial y}\right) + \bar{\nu}_t\left(\frac{\partial \bar{u}}{\partial y}\right)^2 + P_f - c_d\frac{\bar{k}^{3/2}}{\tilde{L}} = 0 \tag{7}$$

with the empirical coefficient c_d=0.08 (Rodi, 1984). The production term P_f arises from bed friction, as in the depth-averaged version of the k-ϵ model (Rastogi & Rodi, 1978). For planar flow the derivatives in (7) are zero and the production P_f and the dissipation are in local equilibrium. With (5) and (6) this yields

$$P_f = \frac{c_d c_k^2 u_*^3}{c_v h} \tag{8}$$

For non-planar flow, lateral shear also affects the bed shear. In this case Eq.(3) still applies but

$$\frac{\bar{\nu}_t}{c_v u_* h} \neq 1 \tag{9}$$

Assuming the eddy viscosity is isotropic and for constant depth, we can write

$$\frac{c_v u_*}{c_{vo} u_{*o}} = \frac{\bar{\nu}_t}{c_v u_* h} \tag{10}$$

where c_{vo} and u_{*o} denote the values for planar flow. In order to get an additional equation for the unknowns we remember that the logarithmic velocity law is based on a linear relationship between the friction velocity and von Karman's constant. Assuming this holds also in the presence of lateral shear gives

$$\frac{u_*}{c_v} = \frac{u_{*o}}{c_{vo}} \tag{11}$$

Substituting Eq.(11) into (10) we finally get

$$\frac{c_v}{c_{vo}} = \left(\frac{\bar{\nu}_t}{c_v u_* h}\right)^{1/2} \tag{12}$$

for the viscosity coefficient due to bed friction. The effect of this correction is the following: Due to lateral shear the turbulent energy and the eddy viscosity are increased. As a consequence, the value of c_v is increased and hence the turbulent length scale $\tilde{L}$, the production of turbulent energy P_f and the friction velocity u_*.

VALIDATION

The derivation of the k-L model is not rigorous since depth-averaged values are used to describe turbulence parameters which vary over the depth. To determine the validity of this approach we compared the results of the depth-averaged model with a two-dimensional vertical model for unidirectional flow. In this model the standard k-ϵ formulation is used (Rodi, 1984) and no correction for the free surface is applied. The calculated values of the 2d model were integrated over the depth and considered as exact values for this kind of flow; then four different depth-integrated models were assessed: (i) turbulent stresses neglected, (ii) only turbulence due to bed friction included (Eq.4), (iii) depth-averaged k-ϵ model (Rastogi & Rodi, 1978), (iv) present k-L model.

Figure 1a shows the results for a smooth triangular section with side slope 1:1. The model *fric* neglects the influence of the lateral shear. The predicted values for the turbulent energy are therefore too low near the borders. The k-ϵ model gives too high values for $\bar{k}$. The same order of discrepancy has also be noted for calculation in meandering channels (Wenka, 1994) and is based on the fact that in the derivation of the model, $\bar{k}$ is not taken as a strictly depth-averaged value (Rastogi & Rodi, 1978). The k-L model, however, estimates the distribution of $\bar{k}$ reasonably well and it gives the best result for the Reynolds stresses. For the eddy viscosity the values are slightly overestimated in the centre of the channel.

For application of the model in practice we need to know how it handles steep side slopes. Therefore, in figure 1b the same comparison is made for a rectangular cross-section (width=200mm,

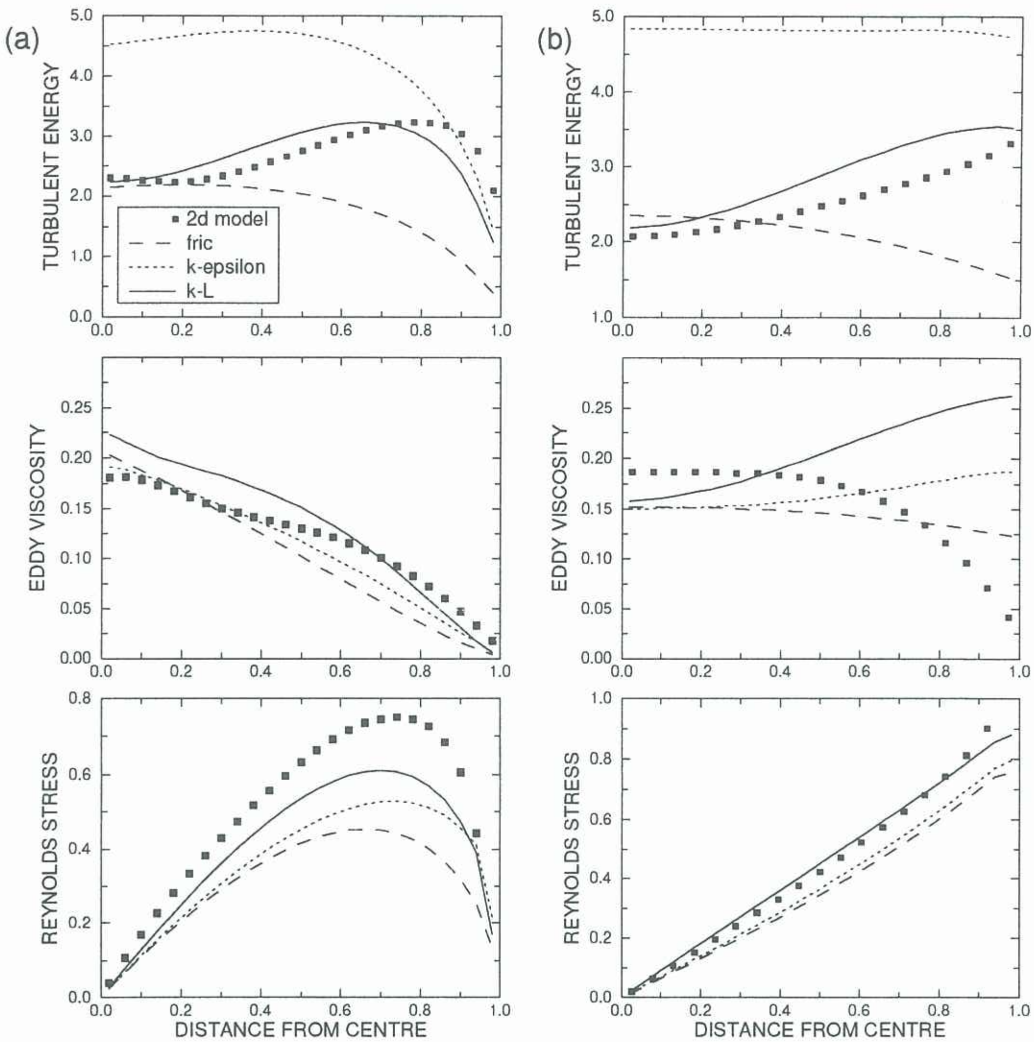

Fig. 1: Computed depth-averaged values of turbulent energy $\bar{k}/u_*^2$, eddy viscosity $\bar{\nu}_t/(u_* R)$ and Reynolds stress $\bar{\tau}_{yx}/(gRS_e)$ for unidirectional flow: (a) triangular section; (b) rectangular section.

depth=100mm). Again, the k-L model gives the best result for the turbulent energy and the Reynolds stresses. However, the reduction of the turbulent length scale near the side wall is not accounted for. As a consequence, the model overestimates the eddy viscosity near the wall. Taking the reduction of the length scale into account would improve the results but it would also make the application of the model more complicated.

Figure 2 illustrates the calculated distribution of flow velocity and bed shear for three different cross-sections and we have also included observed bed shear data. For the triangular shape the 2d model correctly predicts the decrease of the bed shear towards the centre of the channel. For the trapezoidal section the observed reduction of bed shear at the foot of the side slope is smaller than predicted with the 2d model. And for the compound channel higher bed shear is observed at the side wall between main channel and flood plain. Secondary flows are not taken into account in the 2d model which might be the explanation for the difference between calculated and measured values. All depth-averaged models that consider Reynolds stresses give very similar results, but the k-L model is closest to the exact values for the distribution of the bed shear. It correctly estimates the maximum bed shear for the triangular and the trapezoidal section and is close to the 2d results for the compound channel.

CONCLUSIONS

Reynolds stresses are important for the distribution of velocity and bed shear in simple and compound channels. The results in figure 2 show that even a rough estimate of the eddy viscosity (Eq.4) allows estimation of velocity and bed shear distribution near side walls. The depth-averaged version of the k-ϵ model does not improve the results and overpredicts the values for turbulent energy. The k-L model, however, gives better results for bed shear, Reynolds stresses and turbulent energy.

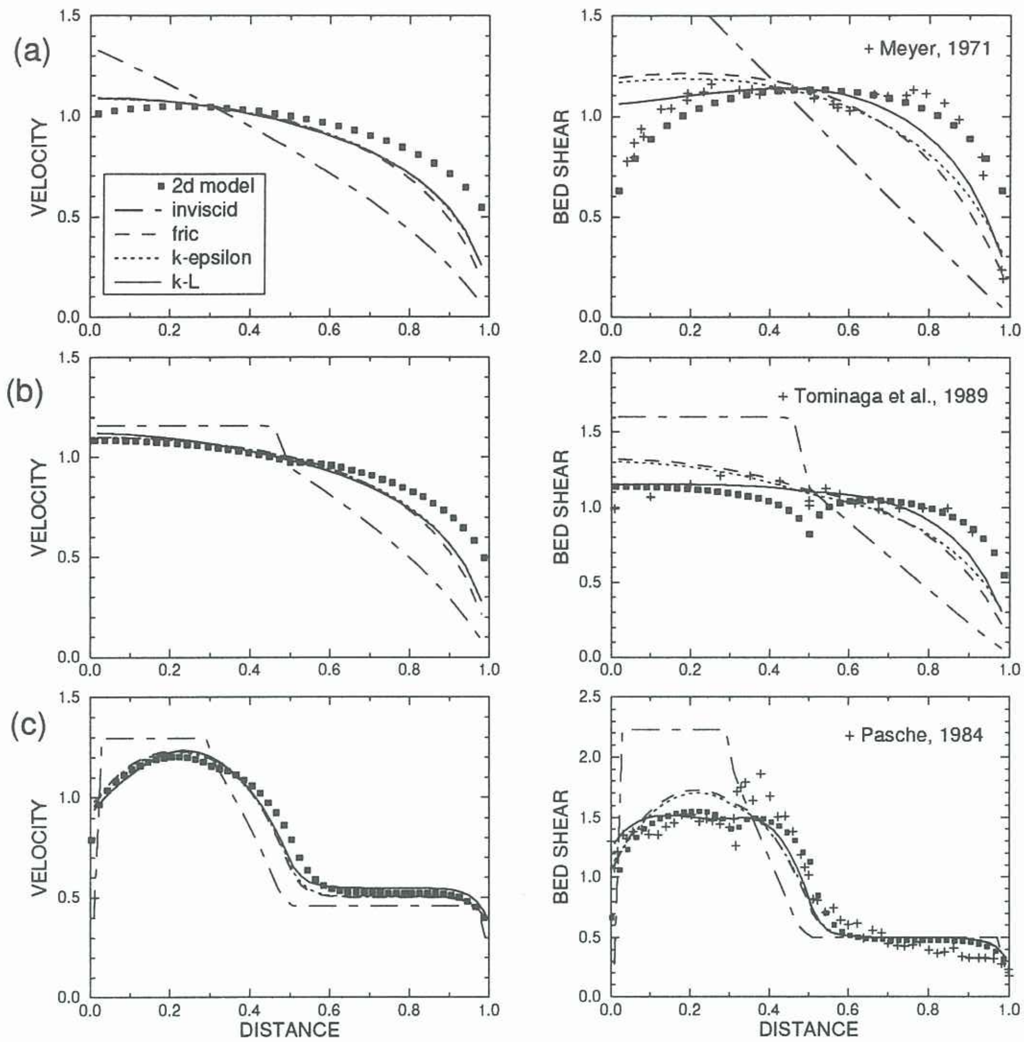

Fig. 2: Computed values of depth-averaged velocity and bed shear stress: (a) triangular section; (b) trapezoidal section; (c) compound section.

We conclude that the proposed k-L model can be used to predict unidirectional flows in open channels with side slopes of up to 45 degrees. For steeper side slopes, bed shear and turbulent mixing are overestimated towards the slope. Applications of the model to 2d plane flows over rough beds such as in river bends are currently under way. The question to be answered is whether this simple model can be used as an alternative to more complicated turbulence models.

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