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A DESINGULARIZED BOUNDARY INTEGRAL METHOD FOR FULLY NONLINEAR WATER WAVE PROBLEMS

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ABSTRACT

Fully nonlinear water wave problems are solved using Euler-Lagrange time stepping methods. The mixed boundary value problem that arises at each time step is solved using a desingularized approach. In the desingularized approach, the singularities generating the flow field are outside the fluid domain. This allows the singularity distribution to be replaced by isolated Rankine sources with the corresponding reduction in computational complexity and computer time.

Examples are given for sloshing in a two-dimensional container and the three-dimensional diffraction of incident waves by a vertical cylinder.

INTRODUCTION

With the recent increases in computational power, it has become more practical to solve directly the nonlinear free surface hydrodynamic problems associated with ships and offshore structures. Fully nonlinear free surface computations can be performed by a variety of methods. Longuet-Higgins and Cokelet (1976) first introduced the mixed Euler-Lagrange method for solving fully nonlinear, two-dimensional water wave problems in the time domain by a time-stepping procedure. In this procedure, a boundary value problem with a Dirichlet condition on the free surface and a Neumann condition on the hull surface is solved at each time step. The kinematic and dynamic free surface boundary conditions are used to time march the value of the free surface potential and elevation. The hull position and surface normal velocities are updated from the equations of motion of the vessel. Variations of this method have been applied by many researchers to a wide variety of two- and three-dimensional problems (cf. Beck 1994).

To successfully implement an Euler-Lagrange algorithm requires a stable time stepping scheme and a fast and accurate method to solve the mixed boundary value problem that results at each time step. We use a fourth order Runga-Kutta method for the time stepping; the mixed boundary value problem is solved using a desingularized boundary integral method (cf. Webster 1975, Cao et al. 1991, Beck 1994, or Beck et al. 1993, 1994). In this method, the velocity potential is constructed by singularities distributed on auxiliary surfaces separated from the problem boundaries and outside the flow domain. For water wave problems, the auxiliary surfaces are above the free surface, inside the hull, and outside the appropriate boundaries at infinity. The strengths of the singularities are determined so that the boundary conditions are satisfied at chosen collocation points. To ensure the convergence of the method, the desingularization distance decreases as the computational grid becomes finer. Because of the desingularization, the resulting kernel in the integral equation is nonsingular (or desingularized) and special care is not required to evaluate integrals over the panels. Simple numerical quadratures can be used to greatly reduce the computational effort, particularly by avoiding transcendental functions. In fact, we have found that for the source distribution method, the distributed sources may be replaced by simple isolated Rankine sources. Isolated Rankine sources also allow the direct computation of the induced velocities in the fluid and on its boundaries without further numerical integration or differentiation. The resulting code does not require any special logic and is easily vectorized. At present, the method is $O(N^2)$, but we are working on using multipole expansions to reduce the computational effort to $O(N)$. The method has been successfully applied to problems involving nonlinear shallow water waves (Cao et al. 1993); a submerged spheroid (Bertram et al. 1991); two and three dimensional stationary floating bodies (Beck 1994, Beck et al. 1993); and the wave resistance, added mass, and damping for a simplified mathematical hull form at forward speed (Beck et al. 1994).

FULLY NONLINEAR PROBLEM FORMULATION

An ideal, incompressible fluid is assumed and surface tension is neglected. The problem is started from rest so that the flow remains irrotational. A coordinate system Oxyz translating in the negative x direction relative to a space fixed frame is used. The time dependent velocity of translation is given by $U_o(t)$. The Oxyz axis system is chosen such that the $z = 0$ plane corresponds to the calm water level and z is positive upwards. The x-z plane is coincident with the centerplane of the vessel. The total velocity potential of the flow can then be expressed as

$$\Phi = U_o(t)x + \phi(x,y,z,t) \tag{1}$$

where $\phi(x,y,z,t)$ is the perturbation potential. Both Φ and ϕ satisfy the Laplace equation:

$$\nabla^2\Phi = 0 \tag{2}$$

Boundary conditions must be applied on all surfaces surrounding the fluid domain: the free surface (S_F), the body surface (S_H), the bottom (S_B) and the surrounding surface at infinity (S_∞). A kinematic body boundary condition is applied on the instantaneous position of the body wetted surface:

$$\frac{\partial\phi}{\partial n} = -U_o(t)n_1 + \mathbf{V}_H \cdot \mathbf{n} \quad \text{on} \quad S_H \tag{3}$$

where $\mathbf{n} = (n_1, n_2, n_3)$ is the unit normal vector into the surface (out of the fluid domain) and $\mathbf{V}_H$ is the velocity of a point on the body surface including rotational effects relative to the Oxyz coordinate system. The subscripts 1,2,3 refer to the x, y, and z axis directions respectively. The kinematic condition is also applied on the bottom:

$$\frac{\partial\phi}{\partial n} = -U_o(t)n_1 + \mathbf{V}_B \cdot \mathbf{n} \quad \text{on} \quad S_B \tag{4}$$

where $\mathbf{V}_B$ is the velocity of the bottom relative to the Oxyz system. For an infinitely deep ocean equation (4) reduces to

$$\nabla\phi \to 0 \quad \text{as} \quad z \to -\infty \tag{5}$$

Finite depth will increase the computational time because of the additional unknowns necessary to meet the bottom boundary condition but there is no increase in computational difficulty. In fact, the flatness of the bottom is immaterial. The only overhead relative to a flat bottom is an increase in the required number of nodes to represent the nonflat bottom.

On the instantaneous free surface both the kinematic and dynamic conditions must be satisfied. The kinematic condition is

$$\frac{\partial\eta}{\partial t} = -\nabla\phi \cdot \nabla\eta + \frac{\partial\phi}{\partial z} - U_o(t)\frac{\partial\eta}{\partial x} \quad \text{on} \quad S_F \tag{6}$$

where $z = \eta\ (x, y, t)$ is the free surface elevation. Using Bernoulli's equation, the dynamic condition is

$$\frac{\partial\phi}{\partial t} = -g\eta - \frac{1}{2}\nabla\phi \cdot \nabla\phi - U_o(t) \cdot \frac{\partial\phi}{\partial x} - \frac{P_a}{\rho} \quad \text{on SF} \tag{7}$$

where ρ is the fluid density, g the gravitational acceleration, and P_a the ambient pressure which may be a function of space and time.

Appropriate conditions are also necessary on the far field boundaries. These may include walls, absorbing boundaries, and/or radiation conditions. Incident waves are introduced into the problem domain by a wavemaker on the upstream boundary. Depending on the water depth and wave frequencies, we have used a piston, a paddle, a plunger, and the equivalent of a pneumatic wavemaker. For the calculations in this paper a piston type was used. In addition, the initial values of the potential and free surface elevation must be specified such that

$$\begin{aligned} \phi &= 0 \quad t \le 0 \quad \text{in fluid domain} \\ \eta &= 0 \quad t \le 0 \end{aligned} \tag{8}$$

The primary difficulty associated with fully nonlinear water wave calculations is the updating and numerical stability of the free surface. As shown in Beck et al. (1994), the kinematic and dynamic free surface boundary conditions (6) and (7) that are used to time step the free surface elevation and potential may be put in the form:

$$\frac{\delta\eta}{\delta t} = \frac{\partial\phi}{\partial z} - (\nabla\phi - \mathbf{v}) \cdot \nabla\eta - U_o(t)\frac{\partial\eta}{\partial x} \quad \text{on } S_F \tag{9}$$

and

$$\frac{\delta\phi}{\delta t} = -g\eta - \frac{1}{2}\nabla\phi \cdot \nabla\phi + \mathbf{v} \cdot \nabla\phi - \frac{P_a}{\rho} - U_o(t)\frac{\partial\phi}{\partial x} \quad \text{on } S_F \tag{10}$$

where $\frac{\delta}{\delta t} \equiv \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla$ is the time derivative following a generalized collocation point moving along a prescribed path with a given velocity $\mathbf{v}$. If the prescribed velocity is set equal to the fluid velocity (i.e. $\mathbf{v} = U_o(t) \cdot \mathbf{i} + \nabla\phi$) then the collocation points become material points and follow the fluid particles. In this case, the Euler-Lagrange method developed by Longuet-Higgins and Cokelet (1976) is recovered and spatial derivatives of the free surface elevation, η, are not required as can be seen from equation (9). However, for floating bodies at forward speed the collocation points tend to pile up in the bow and stern regions near the stagnation points. This difficulty can be overcome by using generalized collocation points. The collocation points can be fixed relative to the ship in which case $\mathbf{v} = \left(0, 0, \frac{\delta\eta}{\delta t}\right)$. This method tends to have time stepping stability problems because of the lack of downstream convection. The collocation points can also be given a prescribed path around the ship hull $\left(\mathbf{v} = \left(U(t), V(t), \frac{\delta\eta}{\delta t}\right)\right)$ that avoids the pile up around the bow and stern and still has the natural downstream convection.

RESULTS AND DISCUSSION

Nestegård (1994) administered two example problems as part of a Det Norske Veritas survey of nonlinear inviscid water wave codes. The first problem was the simulation of free surface sloshing in a two-dimensional wave tank. The second was the simulation of wave diffraction by a vertical cylinder. Results were obtained using various computational methods for the solution of the Laplace equation including the Desingularized Euler-Lagrange Time-Domain Approach or DELTA method presented here, the Boundary Integral Method (BIM), the Finite Element Method (FEM), the Finite Difference Method (FDM), the Finite Volume Method (FVM), the Spectral Method (SM) and the Spectral/Splines Method (SSM).

Free Surface Sloshing

In a two-dimensional tank, the free surface is given an initial known elevation. The wave tank is 160 meters long and 70 meters deep. The initial free surface elevation is,

$$\eta(x,t=0)=12\left(1-\left(\frac{x}{53}\right)^2\right)e^{-\left(\frac{x}{76}\right)^2}$$

The free surface is released at time = 0 and allowed to move under the influence of gravity. Figure 1 shows a series of free surface elevations at discrete times as computed by the desingularized method. This sequence of elevation plots illustrates the time history of the nonlinear slosh modes. For the survey, the free surface elevation and velocity vector are sampled at t = 9.2 seconds and x = 60 meters. Table 1 shows the results obtained by the nine participants and the method they used computing the sloshing problem. The names of the participants are listed alphabetically and do not correlate to the ordering in the table. The results obtained using the DELTA method are clearly consistent with predictions from other codes. The consistency of the results show that fully nonlinear two-dimensional problems can be accurately solved by a variety of methods.

Wave Diffraction by a Vertical Cylinder

In a three-dimensional tank, incident waves are diffracted by a vertical cylinder. The problem parameters are, A/H=0.1, H/R=1.16, kR=1.324. Where A is the incident wave amplitude, H is the water depth, k is the wave number, and R is the cylinder radius. The quantities reported in the survey were the amplitude of the non-dimensional horizontal force on the cylinder defined as, $F/\rho g R^2 A$ and the non-dimensional wave run-up on the front (or incident wave side) of the cylinder defined as, η_{max}/A. Figure 2 shows the time histories of the non-dimensional force and wave run-up. Two time histories are overlaid here to illustrate the spatial convergence of the calculations under grid refinement. The constant amplitude of the oscillations indicates temporal convergence. Figure 3 shows the free surface elevation near the cylinder at the moment of maximum wave run-up. This figure illustrates the significance of the run-up relative to the amplitude of the incident wave. Table 2 shows the results obtained by the six participants for this problem. Again, the names are listed alphabetically and do not correlate to the ordering in the table. An experimental result (Chakrabarti 1975) for the horizontal force is also shown for comparison. The table shows that there is more scatter in the results indicating that more research is required for three-dimensional problems. The DELTA method result is close to the experiment and has a larger wave run-up than the other reported computations. The large run-up may be due to our treatment of the intersection of the body and the free surface. Conventional panel methods have collocation points at the center of the panel and consequently do not have collocation points on the intersection line. The DELTA method uses simple sources instead of panels and the collocation points can be placed directly on the intersection line producing a more accurate simulation of the body/free surface intersection.

CONCLUSIONS

Computations and comparisons with predictions from other numerical methods have shown that the desingularized method is a fast and accurate technique to solve fully nonlinear water wave problems. The desingularization allows the use of isolated Rankine sources rather than the more complex panel distributions. This leads to computationally fast algorithms. In addition, the free surface and body surface are not discretized with flat panels, thus avoiding the difficulties associated with compound curvature.

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PARTICIPANTS		METHOD
K.J. Bai	Seoul National University	FEM
R.F. Beck	University of Michigan	BIM
H.S. Choi	Seoul National University	BIM
A. Clement	Ecole Centrale de Nantes	BIM
R. Cointe & L. Boudet	Bassin d'Essais des Carènes	BIM
C. Greated	University of Edinburgh	FDM
E. Mehlum	SINTEF SI	SSM
R. Eatock Taylor	University of Oxford	FEM
P.J. Zandbergen	University of Twente	BIM

RESULTS

Surface Elevation η(m)	Horizontal velocity(m/s)	Vertical velocity(m/s)
-3.803	-2.456	-0.363
-3.860	-2.480	-0.560
-3.815	-2.423	-0.577
-3.759	-2.411	-0.602
-3.820	-2.417	-0.580
-3.803	-2.417	-0.572
-3.720	-2.480	-0.690
-3.811	**-2.411**	**-0.550 – U. of Mich.**
-3.810	-2.240	-0.560

Table 1: Survey results for the 2-D sloshing problem (cf. Nestegård 1994)

PARTICIPANTS		METHOD
K.J. Bai	Seoul National University	FEM
R.F. Beck	University of Michigan	BIM
H.S. Choi	Seoul National University	BIM
P. Ferrant	SIREHNA	BIM
C.H. Kim	Texas A&M University	BIM
P.J. Zandbergen	University of Twente	BIM

RESULTS

Horizontal Force $F/\rho g R^2 A$	Wave run-up η_{max}/A
2.53	1.83
2.82	1.79
3.10	1.80
2.88	1.82
2.45	1.58
2.95	**2.23 – U. of Mich.**
3.10	---- – Experiment

Table 2: Survey results for the wave diffraction by a vertical cylinder problem (cf. Nestegård 1994)

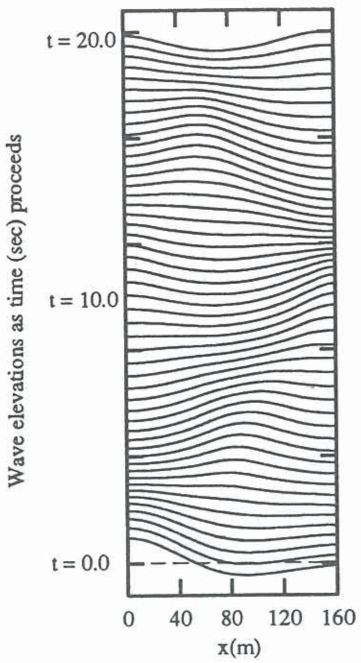

Figure 1: Free surface elevations for the 2-D sloshing problem

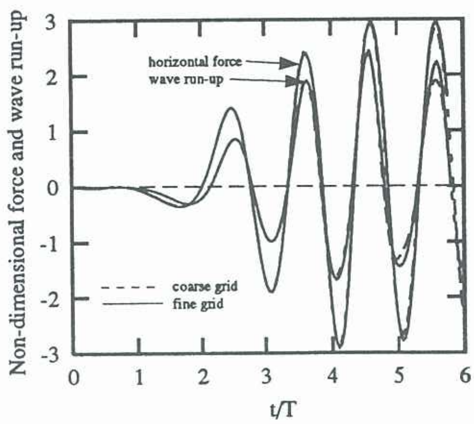

Figure 2: Time histories of non-dimensional force and wave run-up

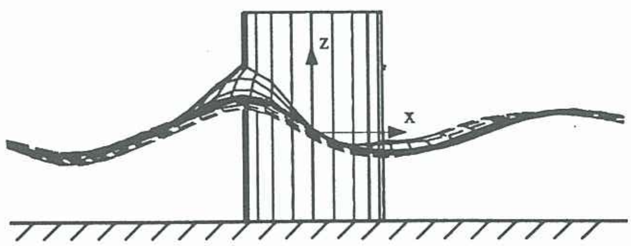

Figure 3: Wave run-up on the incident wave side of a vertical cylinder