

THE EFFECT OF INITIAL MEAN VELOCITY PROFILE ON THE EVOLUTION OF INSTABILITIES IN TWO-DIMENSIONAL WAKES

H.M.R.Barker, M.J.Maghrebi and J.Soria

Department of Mechanical Engineering

Monash University

Clayton, Victoria

AUSTRALIA

ABSTRACT

The evolution of instability modes for two examples of a family of plane wake mean velocity profiles has been studied using direct numerical simulation (DNS) with a two-dimensional spectral method. Comparisons are made with growth rates predicted by linear stability theory, and the effect of evolving mean velocity profiles on instability mode amplification is studied.

INTRODUCTION

The near wake of a two-dimensional bluff trailing edged plate contains thin mixing layers extending downstream from the separation points. Mixing layers such as these may be unstable to perturbations with wavenumbers or frequencies other than the Von-Karman vortex shedding instability (see Sheridan *et al* [1992]).

Bloor [1964] first observed the formation of small vortices in the shear layer separating from a circular cylinder. These vortices and their similarity to boundary layer instability waves was suggested as a possible factor in the development of three-dimensionality and turbulence in wake flows. Monke-witz & Nguyen [1987] studied the stability characteristics of a family of mean velocity profiles typical of 2-dimensional near wakes of bluff bodies, given by

$$U = U_0 \left(1 - \frac{1 + R}{1 + \sinh [y \sinh^{-1}(1)]^{2n}} \right) \quad (1)$$

where the parameter R controls the magnitude of reverse flow at the wake centerline, and n is a pa-

rameter which varies the thickness of the mixing layers. Their analysis examined the effect of variation of these two parameters on the nature of the instability using the Rayleigh equation as the model governing the linear stability of two dimensional plane wake flows. An absolutely unstable region in two dimensional R - n space was bounded by positive (*i.e.* reverse flow) and negative values of R . Their linear analysis naturally assumes *infinitesimal* disturbances and does not predict the behaviour of the nonlinear interactions between disturbances of finite magnitude. The bounding of the absolutely unstable region by variation of R was investigated by Leu and Ho [1992,1993]. They found that vortex shedding in the wake of a blunt trailing edged splitter plate can be controlled by modification of the wake mean velocity profile using suction at the trailing edge.

This paper presents results of DNS calculations describing the interaction of temporally growing perturbations for initial mean velocity profiles typical of both streamlined and blunt trailing edged body wakes. The use of DNS permits calculation of the nonlinear interaction of multiple instability modes up to large perturbation amplitudes.

MATHEMATICAL FORMULATION

The numerical calculations used to obtain the temporal evolution of the wake flows and instabilities are performed by solving the incompressible vorticity equation

$$\frac{\partial \omega_i}{\partial t} + u_j \frac{\partial \omega_i}{\partial x_j} = \omega_j S_{ij} + \frac{1}{Re} \frac{\partial^2 \omega_i}{\partial x_j \partial x_j} \quad (2)$$

with periodic boundary conditions in the streamwise

(x) direction and a doubly infinite domain in the cross-stream direction (y). The length of the computational box in the streamwise direction is set to an integer multiple of the longest possible perturbation wavelength, i.e. $L_x = \lambda_x = 2\pi/k_x$. The Reynolds number is defined as $Re = U_0\delta_0^0/\nu$, where U_0 is the initial centerline deficit velocity, δ_0^0 is the initial half-width and ν is the kinematic viscosity. All independent and dependent variables are non-dimensionalised by U_0 and δ_0^0 (e.g. $t' = tU_0/\delta_0^0$).

Numerical method

A spectral Galerkin method developed by Rogers & Moser [1990] was adapted for the two-dimensional time-developing plane wake DNS calculation. Details of this numerical method are described in Spalart *et al.* [1991]. A Galilean transformation of the mean velocity profile is used to satisfy homogeneous boundary conditions in the cross stream direction. Jacobi polynomials are used as expansion functions in the cross-stream direction. Fourier expansion functions are used in the streamwise direction. The numerical calculation was performed with 256×128 Fourier/Jacobi modes.

Linear stability theory has been used to calculate the temporal amplification rates ω_i and mode shapes of various wavenumber disturbances for each initial mean velocity profile. These disturbances are given by the eigenvalues and eigenfunctions (mode shapes) of vorticity perturbations of the form

$$\omega'_z = \Re(\tilde{\omega}_z(y)e^{-i\omega t}) \quad (3)$$

These eigenfunctions and eigenvalues are obtained by solving the vorticity form of the Orr-Sommerfeld equation using a similar Galerkin method as for the temporal DNS calculations.

Initial conditions

Three initial mean velocity profiles were used in this investigation. One of these was a Gaussian mean velocity profile of the form

$$U = -U_0 \exp(-c_0 y^2) \quad (4)$$

with $U_0 = 1$ and $c_0 = 0.69315$. This value of c_0 gives a half-width of one. The initial perturbation field consisted of the eigenmode obtained from the solution of the Orr-Sommerfeld equation for $k_x = 0.8$. This is the most amplified wavenumber for this mean velocity profile at a Reynolds number of 700. For the Gaussian wake $L_x = 2\lambda_x = 7.853\delta_0^0$

A mean velocity profile of the form given in (1) was used with $U_0 = 1$, $n = 1$ and 3 to model the two-dimensional near wake of a bluff body. The initial disturbance field for the simulations consisted of all discrete unstable two dimensional eigenmodes introduced with random phase. The initial cross-stream integrated energies of each mode were equal. For

the $n = 1$ case, the unstable modes range from $k_x = 0.1 - 1.6$. Similarly for the $n = 3$ case, the unstable modes range from $k_x = 0.1 - 3.2$. The computational domain was thus set to $L_x = 20\pi\delta_0^0$ in both cases. The initial Reynolds number was 700.

RESULTS AND DISCUSSION

Two initial mean velocity profiles of the form (1) have been used to investigate the effect of variation of the ratio of mixing layer thickness to wake half-width. The parameter n in (1) controls this effect. For $n = 1$ the mean velocity profile is similar to a Gaussian type profile, whereas for $n = 3$ the center of the wake has a flat region with narrower mixing layers on each side. The evolution of the mean velocity profiles is plotted in Figs. 1 and 2.

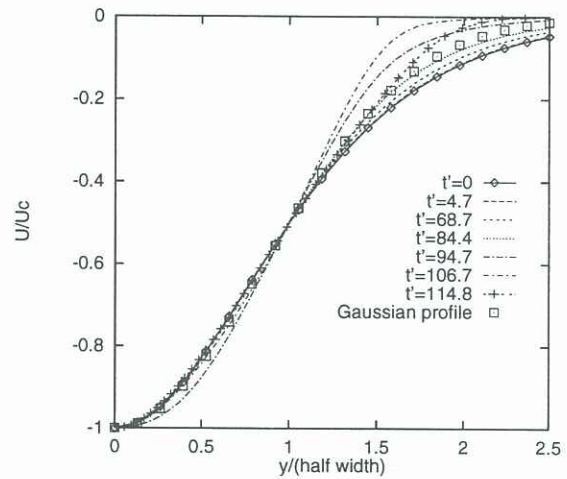


Figure 1: MEAN VELOCITY PROFILE EVOLUTION FOR $U = \frac{U_0}{1 + \sinh(c_1 y)^2}$

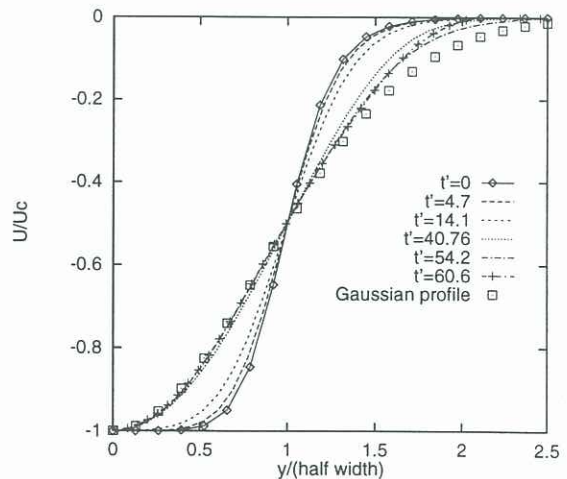


Figure 2: MEAN VELOCITY PROFILE EVOLUTION FOR $U = \frac{U_0}{1 + \sinh(c_1 y)^6}$

The temporal stability curves in Fig. 3 show a higher temporal amplification rate $\omega_i = 0.3434$ at

$k_x = 1.6$ for the $n = 3$ case compared to $\omega_i = 0.1401$ at $k_x = 0.8$ for the $n = 1$ case.

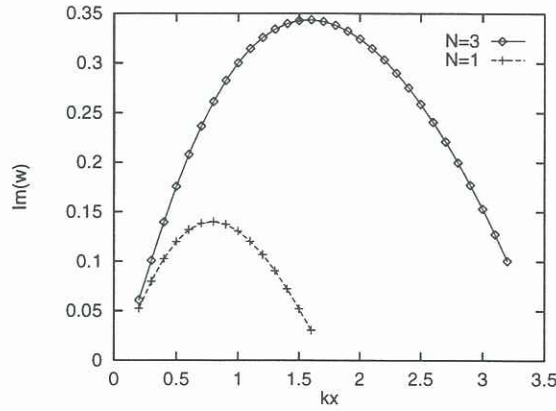


Figure 3: LINEAR STABILITY THEORY TEMPORAL GROWTH RATES FOR $U = \frac{U_0}{1 + \sinh(c_1 y)^{2n}}$

Examination of the fluctuating spanwise vorticity fields in Figs. 4 and 5 shows that in the case of the higher wavenumber instability there are two pairs of opposite sign vorticity regions arranged symmetrically about the wake centerline, whereas in the lower wavenumber instability there is only one pair of like signed vortices on either side of a single central vorticity region.

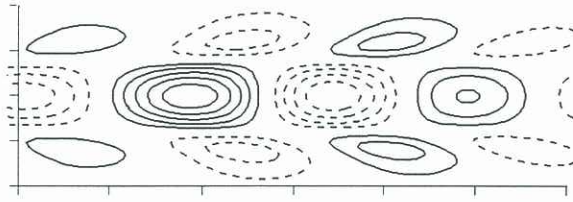


Figure 4: FLUCTUATING VORTICITY FIELD FOR INITIAL MEAN VELOCITY $U = \frac{U_0}{1 + \sinh(c_1 y)^2}$, $t' = 23.9$, CONTOUR LEVELS ARE FOR $-1.5 \times 10^{-3} \leq \omega_z \leq 1.5 \times 10^{-3}$ WITH CONTOUR STEP SIZE OF 3.0×10^{-4} . X-GRADUATIONS = $2\delta_0^0$, Y-GRADUATIONS = δ_0^0

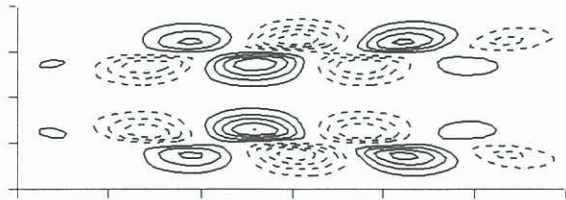


Figure 5: FLUCTUATING VORTICITY FIELD FOR INITIAL MEAN VELOCITY $U = \frac{U_0}{1 + \sinh(c_1 y)^6}$, $t' = 23.1$ CONTOUR LEVELS ARE FOR $-0.1 \leq \omega_z \leq 0.1$ WITH CONTOUR STEP SIZE OF 2.0×10^{-2} . X-GRADUATIONS = $2\delta_0^0$, Y-GRADUATIONS = δ_0^0

Comparison of the square root of cross-stream integrated energy at the most unstable wavenumbers for small (linear) amplitudes between the DNS data and the exponential growth rates predicted from linear stability theory (shown in Fig. 6) indicates a good agreement after an initial transient.

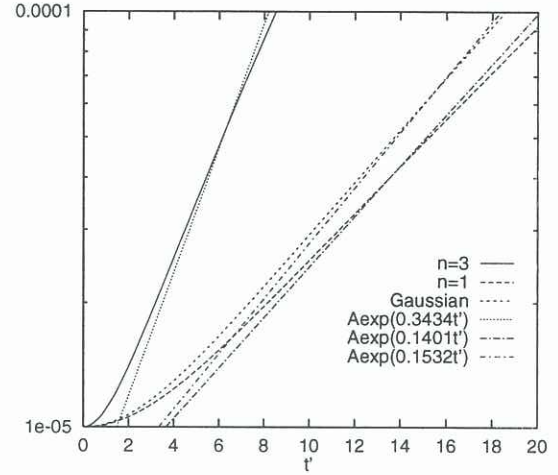


Figure 6: SQUARE ROOT OF CROSS-STREAM INTEGRATED ENERGY OF MOST UNSTABLE MODES

As the perturbations grow to levels greater than 0.05, the mean velocity profiles evolve to a shape where the centerline deficit velocity U_c is filled and the wake half-width δ_0 increases. Figures 1 and 2 show the evolution of the mean velocity profiles where U is normalised by U_c and the cross-stream direction is normalised by δ_0 . Only one side of the symmetric profiles has been plotted. Note that the inner wake of the former changes relatively little, and that both profiles evolve to a state for which the inner wake is similar to a Gaussian wake profile.

As the mean streamwise velocity for the $n = 3$ case evolves from its initial state, the vorticity energy spectrum shown as a surface in wavenumber-time-energy space in Fig. 7 changes from a single initial peak corresponding to the high wavenumber instability to two peaks at lower wavenumbers. The spectrum evolution for the $n = 1$ initial mean velocity profile is shown in Fig. 8. This graph indicates that for this case, only a single dominant mode which corresponds to the most unstable mode in the initial disturbance is amplified.

CONCLUSION

The effect of changing the initial mean velocity profile in the direct numerical simulation of a two-dimensional plane wake from a smooth profile with broad mixing layers to a profile with thinner mixing layers has been studied. The bluff wake (thin mixing layers) has an initial instability at higher wavenumber, corresponding to an instability of the thin mixing layers. As the wake evolves to a Gaussian-like mean velocity profile, this instability is damped and insta-

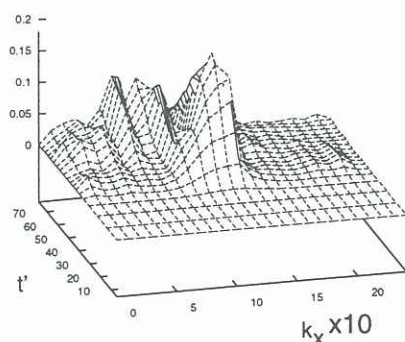


Figure 7: EVOLUTION OF VORTICITY ENERGY SPECTRUM FOR $N=3$ IN THE INITIAL STREAMWISE MEAN VELOCITY PROFILE GIVEN BY EQ.(1)

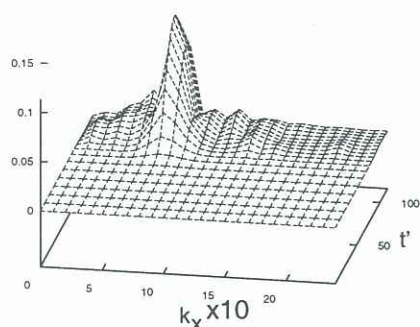


Figure 8: EVOLUTION OF VORTICITY ENERGY SPECTRUM FOR $N=1$ IN THE INITIAL STREAMWISE MEAN VELOCITY PROFILE GIVEN BY EQ.(1)

bilities at lower wavenumbers corresponding to the new mean velocity profile grow.

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