

A SURGE WAVE SCALING FOR THE PURGING OF DENSITY STABILISED PONDS

S.W. Armfield

School of Civil Engineering
University of New South Wales
Sydney, New South Wales
Australia

W. Debler

Dept. of Mechanical Engineering and Applied Mechanics
University of Michigan
Ann-Arbor, Michigan
USA

T. Asaeda

Dept. of Civil and Environmental Engineering
Saitama University
Saitama 338
Japan

ABSTRACT

Laboratory experiments and numerical simulations have been carried out for the purging of density stabilised saline ponds by less dense overflows. It is observed that at flow initiation a surge wave is formed at the interface of the dense and less dense fluid. This surge wave can carry a significant quantity of dense fluid out of the pond resulting in a time rate of efflux of saline fluid several orders of magnitude greater than that associated with turbulence diffusion, which is the dominant mechanism of efflux for most of the remainder of the flow. It is shown that the amplitude of this surge wave is dependent on the internal densimetric Froude number, that the nature of this relationship undergoes a distinct change at about Froude number $Fr = 1$, and that both below, and for a region above, this critical Froude number the relationship is, at least approximately, linear.

INTRODUCTION

Saline pools, formed by ground water intrusion through the river bed during periods of low flow, are known to occur in many of Australia's rivers, and have resulted in major environmental problems. For instance a number of studies have been carried out in Victoria in which the adverse effects of high salinities have been identified (Anderson and Morrison 1989). When the pools are subjected to a large enough fresh-water overflow, as a result of a rainfall event, or the release of water from upstream storage, the resulting mixing and transport will purge the pool of saline water. In order to maintain the integrity of the river system it is necessary to understand the flow dynamics of the water within the pools during the purging flows.

To assist in the development of an understanding of this process laboratory experiments and numerical

simulations have been conducted for the purging of density stabilised ponds by less dense overflows. The purging process, which follows initiation of the fresh overflow, comprises at least three distinct mechanisms (Armfield and Debler 1993).

(1) An initial surge wave is pushed from the pond at the onset of the overflow.

(2) Shear forces acting between the overflow and the denser fluid can lead to unmixed fluid being carried out by tractive forces.

(3) Turbulent effects at the interface can lead to the mixing of dense fluid into the overflow via interfacial instability. The resulting mixed fluid of intermediate density is then carried away by the overflow.

We will address only the first of these mechanisms. At initiation of the flow it is observed that a large wave is set in motion at the interface of the fresh and saline water. This leads to a surge of dense fluid over the downstream lip of the basin, which is then advected away by the overflow. This surge wave can carry with it a significant fraction of the saline fluid in the cavity in a time scale of the order of one thousandth of the total purging time of the cavity. The surge wave thus has an efflux rate several orders of magnitude greater than the subsequent mechanisms, (2) and (3) above.

In this paper we will report the results of a series of laboratory experiments and numerical simulations for this flow in a square cavity with Froude numbers in the range $0.1 < Fr < 10$. It will be shown that the amplitude of the surge wave is Froude number dependent, with a change in the relationship occurring at a Froude number of approximately $Fr = 1$.

NUMERICAL AND EXPERIMENTAL METHODS

The numerical scheme integrates the full unsteady two dimensional Navier-Stokes equations with appropriate initial and boundary conditions. The equations

are discretised on a non-uniform cartesian mesh using a finite volume approach. The viscous, pressure gradient and continuity terms are approximated using centred second order discretisations, while the non-linear terms are discretised using the third order limited SMART approach (Gaskell and Lau 1988). Time integration is carried out using a second order Crank-Nicolson scheme. Pressure is obtained and continuity enforced using a standard Poisson pressure correction equation. The equations are discretised on a non-staggered mesh and second-order elliptic correction terms are included in the continuity equation to prevent the occurrence of the well known grid-scale pressure oscillation (Armfield 1991, 1994).

For the laboratory experiments a flume 11m long was modified with a false floor to create a duct of cross-section 10cm high and 29cm wide. A square cavity of dimension 11.6cm×11.6cm was set into the false floor and filled with salt water at a predetermined density with food coloring at a concentration of about 2 parts per thousand mixed in as a marker, while the duct was filled with fresh water. The experiment was then left to stand for 2h to allow any circulation to diffuse away.

At time $t = 0$, water was admitted to the flume through a flow straightener and turbulence damper, impulsively setting in motion the fluid in the combined duct/basin system. After initiation the surge wave is generated, followed by the other features described in the introduction, and ultimately the basin is fully purged of saline fluid. Typical purging times for the parameters considered are of the order of hours. During the development of the flow photographs were taken at regular intervals, with the marker added to the dense fluid acting to make the purging process visible.

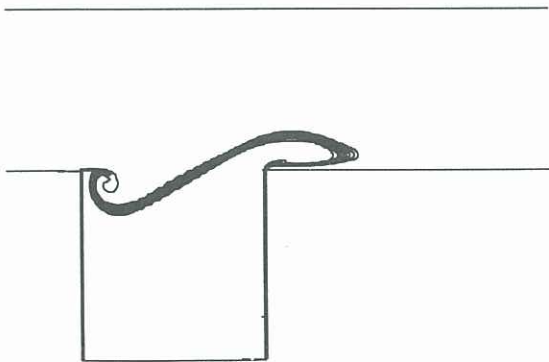


Figure 1: Density contours for the $Fr = 1.76$ flow at $t = 1.5$ showing the surge wave.

RESULTS

Density contours for the development of the surge wave, obtained by numerical simulation, are presented here. At initiation of the flow the fluid is quiescent with the interface between the fresh and saline

fluid horizontal and level with the channel floor. A parabolic velocity profile is then switched on at the inlet and the flow is allowed to develop. It should be noted that the full computational domain is not shown here. The inlet boundary is located approximately two channel depths upstream of the basin entrance, and the outlet boundary is four channel depths downstream. The top boundary is stress free, the bottom and basin side boundaries are non slip and all quantities at the exit have zero streamwise variation. The results shown here were obtained with a non-uniform 100×100 grid, with approximately 70×70 nodes in the basin.

Figure 1 contains density contours for the surge wave obtained using the numerical scheme. The density difference for this flow is $\Delta\rho = 0.0008$, and the Reynolds number of the overflow is $Re = 5000$ based on the channel depth and mean channel velocity. This gives an internal densimetric Froude number, defined as

$$Fr = \frac{U}{\sqrt{g\Delta\rho h_b}}$$

of $Fr = 1.76$, where h_b is the depth of the basin containing the saline fluid, $\Delta\rho = \frac{\rho' - \rho}{\rho}$ is the density difference non-dimensionalised by the density of the overflow and U is the mean channel velocity.

The result shown in Figure 1 was obtained at $t = 1.5$ in non-dimensional time units based on mean channel velocity and channel depth, or $t = 1$ s for a channel 10cm deep, corresponding to the experimental apparatus.

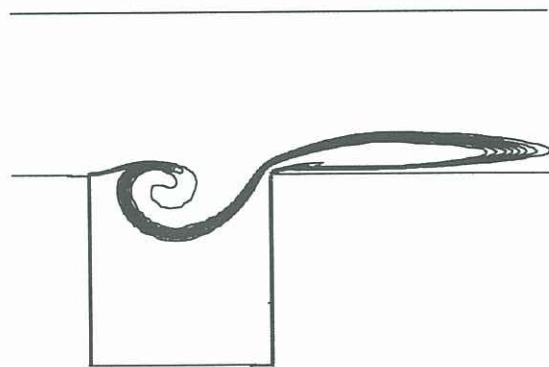


Figure 2: Density contours for the $Fr = 1.76$ flow at $t = 4.5$.

Figures 2 and 3 show the subsequent development of the surge wave. As can be seen the wave pushes a splash of saline fluid out of the basin into the channel, which is then advected away by the overflow. At the same time, for this flow, a strong overturning with an associated roller occurs at the interface. This roller becomes trapped within the basin and leads to a strong mixing action and is closely associated with the later transition to turbulence.

Figure 4 shows the surge wave for $Fr = 0.44$. The depression of the interface and splash of denser fluid

being advected away by the overflow is again apparent. However for this Froude number no strong overturning of the basin fluid is generated, and clearly the flow has a different character to that of the larger Froude number flow shown above. The overturning has been found to only occur in Froude number $Fr > 1$ flows. The overturning is a result of the formation of a recirculation as the overflow separates from the upstream lip of the basin. For the larger Froude number flows this recirculation is strong enough to entrain unmixed saline fluid from the basin, forming the observed roller. For the lower Froude number flows, provided the Reynolds number is high enough, a separation will still form and entrain some mixed fluid from the interface region, as seen here, however the recirculation is not strong enough to entrain unmixed fluid, and no roller and overturn occurs.

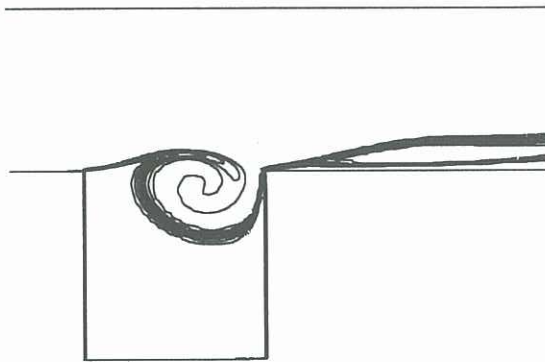


Figure 3: Density contours for the $Fr = 1.76$ flow at $t = 6.75$.

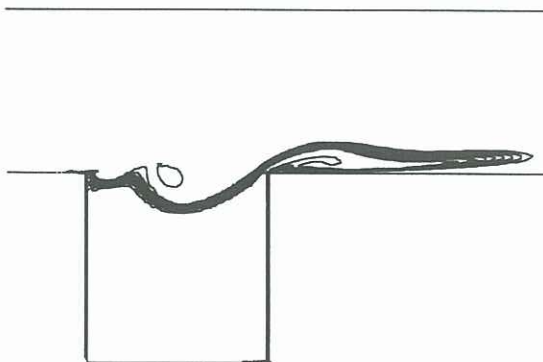


Figure 4: Density contours for the $Fr = 0.44$ flow.

A plot of the amplitude of the surge wave versus the Froude number is shown in Figure 5. The amplitude of the surge wave is measured as the depression

of the interface, which, non-dimensionalised by the basin depth, is represented on the plot as δ . For the low Froude number flows in which no roller forms, the interface has an easily identified and measured maximum, which was used to obtain δ . For the larger Froude number flows it was difficult to identify a maximum depression associated with the passage of the wave, and for these flows the depth was measured at the time when the splash had just fully exited the basin, corresponding to Figure 2 above for the $Fr = 1.76$ flow for instance. It should be noted that the δ presented in Armfield and Debler (1993) was obtained by estimating the drop of an equivalent horizontal interface after the passage of the surge wave, a method which produced different results to those shown here, partly because of the additional efflux associated with internal waves that occurred before the interface height was obtained. The method used in this paper has been adopted because it is far easier to apply in a consistent way with the experimental results.

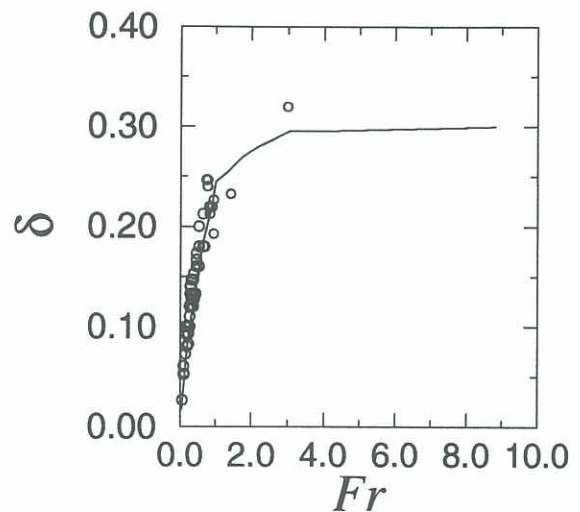


Figure 5: Plot of δ against Froude number for the numerical results (solid line) and the experimental results (circles).

From Figure 5 it is clear not only that δ is a monotonically increasing function of the Froude number, but also that the functional relation undergoes a change at about Froude number $Fr = 1$, and that the Froude number dependence ceases almost entirely at about Froude number $Fr = 3.5$. Also shown on this plot are a series of experimental data points, as can be seen the correspondence between the numerical and available experimental results is good, although it is clear that additional experimental data points are required in the high Froude number region for full verification.

DISCUSSION AND CONCLUSIONS

The surge wave that forms at initiation of the purging flow has been studied both numerically and experimentally. It has been found that the numerical solution is able to predict the occurrence and characteristics of this feature, based on comparison with the available experimental data. An extensive numer-

ical investigation of the feature has been carried out and it has been observed that the amplitude of the surge wave is a monotonically increasing function of the Froude number, with the relationship undergoing a sudden variation at about Froude number $Fr = 1$ and reaches a maximum at about $Fr = 3.5$.

It has also been observed that the character of the wave changes at a Froude number of approximately $Fr = 1$. For the large Froude number case a strong roller is formed at the interface of the two fluids. This roller entrains unmixed saline fluid from the basin, leading to a strong overturning and subsequent mixing. The roller is a result of the recirculation formed by the separation of the overflow from the up-stream lip of the basin. For the low Froude number flow a recirculation region can also form, but it cannot entrain unmixed saline fluid from the basin and no overturning with associated mixing results.

A scaling argument may be developed to predict the relation between the amplitude of the surge wave and the Froude number as follows. At initiation the flow in the combined flume/basin is potential as a result of the difference in time scale for the flow start-up and the establishment of viscous boundary layers. As a result the saline fluid in the basin is impulsively set in motion and the surge-wave results. The initial motion of the fluid in the basin will lead to a tilting of the interface, with an associated increase in the potential energy of the dense fluid of the form,

$$\Delta E_p \sim g d \rho a^2 L^3,$$

where ΔE_p is the change in potential energy, a is the slope of the interface, L is the length of the basin and $d\rho$ is $(\rho' - \rho)$. The drop in interface height associated with this tilting will then be of the form $l \sim aL$, and therefore

$$\Delta E_p \sim g d \rho l^2 L.$$

If we assume that the change in potential energy is balanced by the kinetic energy of the flow then,

$$g d \rho l^2 L \sim L h_b \rho' U^2.$$

Rearranging this gives an expression for l of the form,

$$l^2 \sim \frac{h_b U^2}{g d \rho / \rho'},$$

dividing both sides by h_b^2 and taking the square root then gives

$$\delta \sim \frac{U}{\sqrt{g d \rho / \rho' h_b}},$$

and thus we have the scaling

$$\delta \sim k Fr,$$

where k is the scaling constant.

Analysis of the results given in Figure 5 show that for $Fr < 1$ $k \sim 0.25$, while for $1 < Fr < 4$, $k \sim 0.025$. Again it should be noted that the k value for $Fr > 1$ obtained here is different from that obtained in Armfield and Debler (1993), as a result of the different method used to obtain δ .

The sudden change in the relation between δ and Fr at $Fr = 1$ is likely to be a result of the formation

of the overturning. It may be postulated that once overturning occurs and a roller is formed a considerable part of the kinetic energy that would otherwise be generating the surge wave is being utilised to drive the overturn. Ultimately this energy is dissipated in mixing, or heat, and is therefore not transferred to the surge wave.

It should also be noted that for higher aspect ratio (longer) cavities one shot purging events, in which all of the saline fluid is carried out of the basin by the surge wave, have been achieved experimentally, although these have not as yet been simulated numerically. Additionally surge waves have also now been identified in data collected in the field for the Valdivia estuary in Chile (Debler and Imberger 1995). It is believed that in this location the surge wave has a significant influence on the flushing of effluent from the estuary.

ACKNOWLEDGEMENT

The authors are indebted to Professor Jorg Imberger, Director of the Centre for Water Research at the University of Western Australia for arranging that some of these experiments could be performed in the Hydraulics Laboratory. The remainder of the work was carried out at the University of New South Wales Water Research Laboratory. This work was supported by the Australian Research Council's small grants scheme and the Victorian Environmental Protection Authority.

REFERENCES

- Anderson J.R. and Morison A.K., 1989, 'Environmental consequences of saline groundwater intrusion into the Wimmera river,' *BMR J. Aust. Geol. Geophys.*, Vol. 11, pp 233-235.
- Armfield S.W., 1991, 'Finite difference solutions of the Navier-Stokes equations on staggered and non-staggered grids,' *Computers and Fluids*, Vol. 20, 1-17.
- Armfield S.W., 1994, 'Ellipticity accuracy and convergence of the discrete Navier-Stoke equations,' *J. Computational Physics*, Vol. 114, pp 176-184.
- Armfield S.W. and Debler W., 1993, 'Purging of density stabilised basins,' *Int. J. Heat and Mass Transfer*, Vol. 36, pp 519-530.
- Debler W. and Imberger J., 1995, 'Criteria for Flushing Mechanisms in Estuarine and Laboratory Experiments,' *ASCE J. Hyd. Engr.*, submitted.
- Gaskell P.H. and Lau K.C., 1988, 'Curvature compensated convective transport: smart, a new boundedness preserving transport algorithm,' *Int. J. Numerical Methods in Fluids*, Vol. 8, pp 617-641.