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A STUDY OF THE ROLL-UP OF A VORTEX SHEET AT THE HEAD OF A RECTANGULAR PISTON

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ABSTRACT

An experimental investigation has been carried out on the roll-up of a vortex sheet at the head of a rectangular piston. This flow is of interest for two reasons. One is that it has obvious practical implications in the design of internal combustion engines and reciprocating pumps etc. The other is that it is an important fundamental flow which can provide information as to the behaviour of vortex sheets in closed systems. The choice of a rectangular piston was made in the hope of simplifying the system somewhat for comparison with results for the roll-up of a vortex sheet in front of a round piston which has been reported in Allen & Chong (1993,1995). In the case of the square piston it is hoped that the complicating effects of vortex stretching (at least at small times) and the effects of complex axisymmetric images will be avoided. The flow should be nominally two-dimensional (planar) at least for some initial time. This also makes the modeling and computation of the flow simpler. In this paper some careful flow visualisation results are presented.

EXPERIMENTAL APPARATUS

In order to generate the nominally two dimensional vortex an experimental apparatus was constructed that consisted of a 10 by 8 inch rectangular piston moving through a duct filled with water. A schematic of the experimental apparatus is shown in figure 1.

Fluid returns to the rear of the piston via a return circuit, similar to a closed-circuit wind tunnel. Honeycombs and turning vanes are included in the circuit to ensure that the flow in the working section is straight and parallel. The piston is driven by a stepper motor which drives a linear traversing mechanism which, in turn, acts on a hollow shaft connected to the piston. The drive mechanism is also connected to a sled, running on linear bearings, which carries a CCD camera. The camera moves with the piston thus giving a frame of reference in which the piston appears stationary. The piston is fitted with dye injectors around its outer edge. The dye injection tubes run through the hollow drive shaft to the dye reservoirs. The flow is visualised by injecting fluoriscein dye through 1mm holes in the piston face, close to the piston/wall junction. The vortex cross-section is illuminated with a collimated, parallel laser sheet which originates from the downstream end of the duct.

RESULTS AND DISCUSSION

Dimensional analysis suggests that the position of a vortex filament on the sheet, $z = x + iy$ as defined in figure 2, is given by

$$z = f(U_p, t, \Gamma, \nu, D)$$

where U_p is the piston velocity, t is the time from the start of the motion, ν is the kinematic viscosity, Γ is the amount of circulation on the sheet between the core and the point z and D is the length scale of the apparatus.

Applying Buckingham's theorem we have the following functional relationship

$$\frac{z}{U_p t} = G\left(\frac{\Gamma}{tU_p^2}, \frac{U_p^2 t}{\nu}, \frac{D}{U_p t}\right)$$

Replacing $U_p t$ with L_w where L_w is the distance the wall has moved from rest the above equation

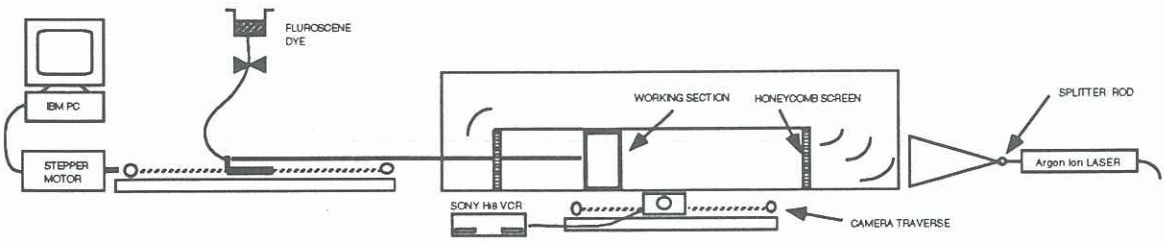


Figure 1: Experimental apparatus

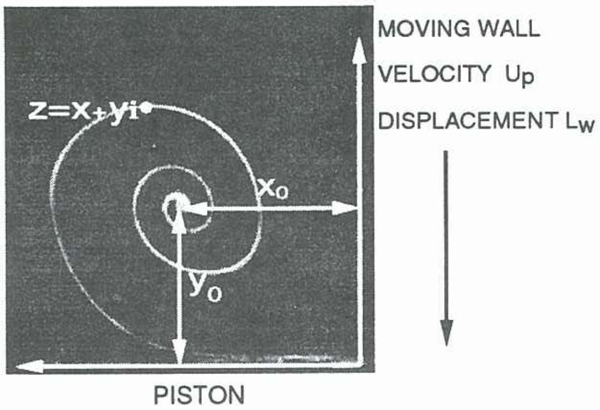


Figure 2: Definition of spiral parameters

becomes

$$\frac{z}{L_w} = G\left(\frac{\Gamma}{L_w U_p}, \frac{L_w U_p}{\nu}, \frac{D}{L_w}\right)$$

The Reynolds number for this experiment, $L_w U_p/\nu = Re$, develops during the experiment. At early stages of development if the size of the spiral is much smaller than the experimental length scale of the apparatus then we may be able to neglect D as a dependant variable. Also if the Reynolds number is sufficiently high it may be possible to neglect the effect of viscosity. This leads to a functional relation involving only two non-dimensional groups.

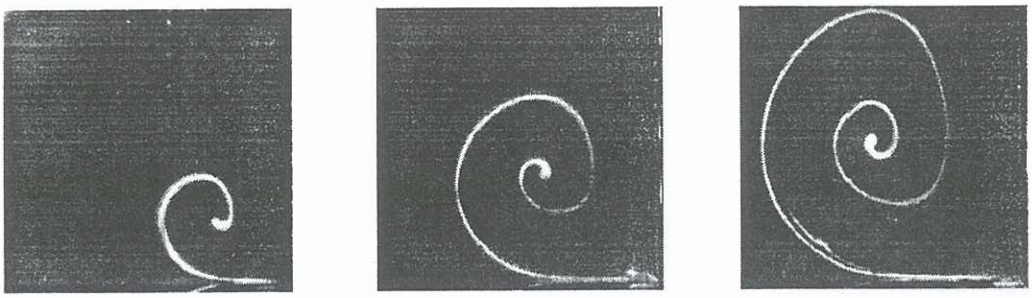
$$\frac{z}{L_w} = G\left(\frac{\Gamma}{L_w U_p}\right)$$

This equation predicts that the sheet scales in a dimensionally similar fashion. In order to check this similarity solution we need to track a filament on the sheet. The only filament which we can track reliably using flow visualization is the core of the spiral, $z_o = x_o + iy_o$, where Γ and hence $\Gamma/(L_w U_p)$ is equal zero.

The experiment consisted of running the piston at three different constant velocities, 7.195, 12.51 and 19.00 mm/s. Results from these experiments are shown below. Figures 3 and 4 show typical flow visualisation photographs of the vortex for the slowest and fastest cases respectively. The behaviour for all three cases is broadly similar with the spiral roll-up growing with time. In order to examine the structure of the spirals more closely measurements of the trajectory of the core have been made from a video of the developing spiral.

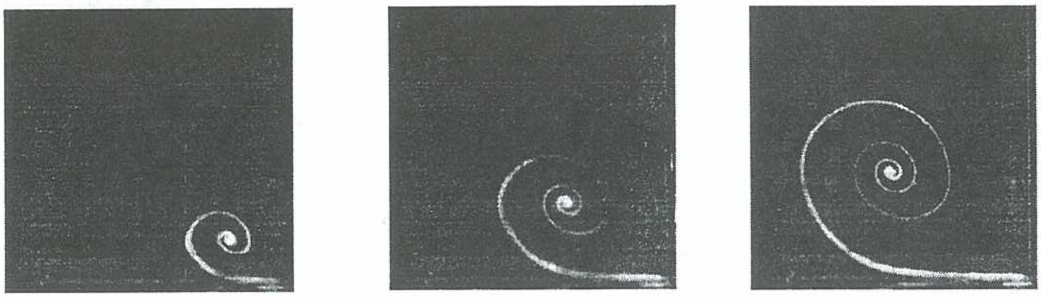
Figure 5 shows the non-dimensional core co-ordinates plotted versus Reynolds number. If the shape of the spiral is independent of Reynolds number and D then all cases should collapse onto a single horizontal line. The results are very interesting. There does appear to be some collapse for the y_o co-ordinate suggesting a region where both Reynolds number and D are unimportant. The x_o co-ordinate shows broadly similar behaviour (in terms of the shape of the graph) however the region where the graph could be said to be independent of Reynolds number and D is smaller.

The peel-off at large Reynolds numbers can then be seen to be due to the effect of the length scale of the apparatus i.e. when x_o/D and y_o/D become significant.



L_w/D=0.453 L_w/D=0.906 $L_w/D = 1.360$

Figure 3: Sequence of photos for $U_w = 7.195mm/sec$



$L_w/D = 0.374$ $L_w/D = 0.748$ $L_w/D = 1.122$

Figure 4: Sequence of photos for $U_w = 19.00mm/sec$

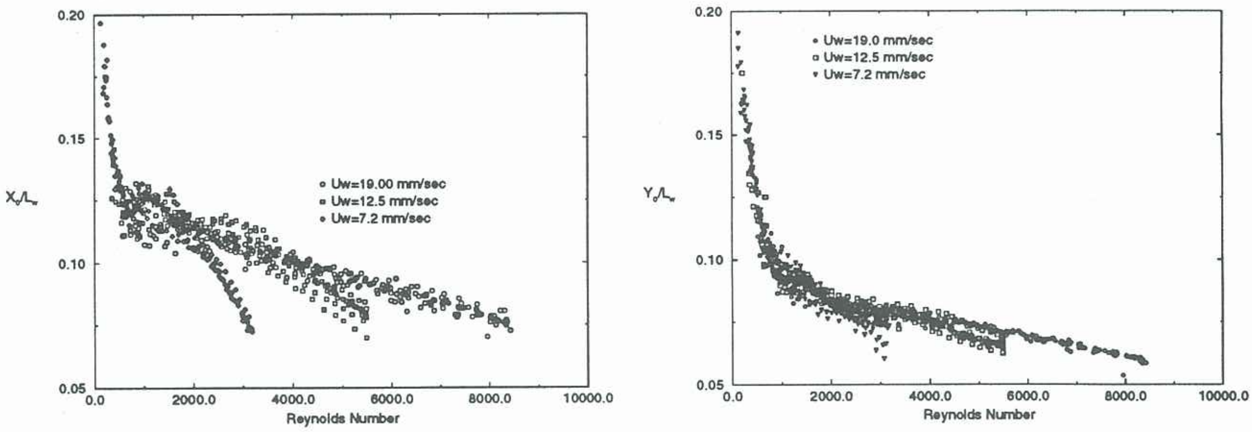


Figure 5: Co-ordinates of centre of spiral vs Re

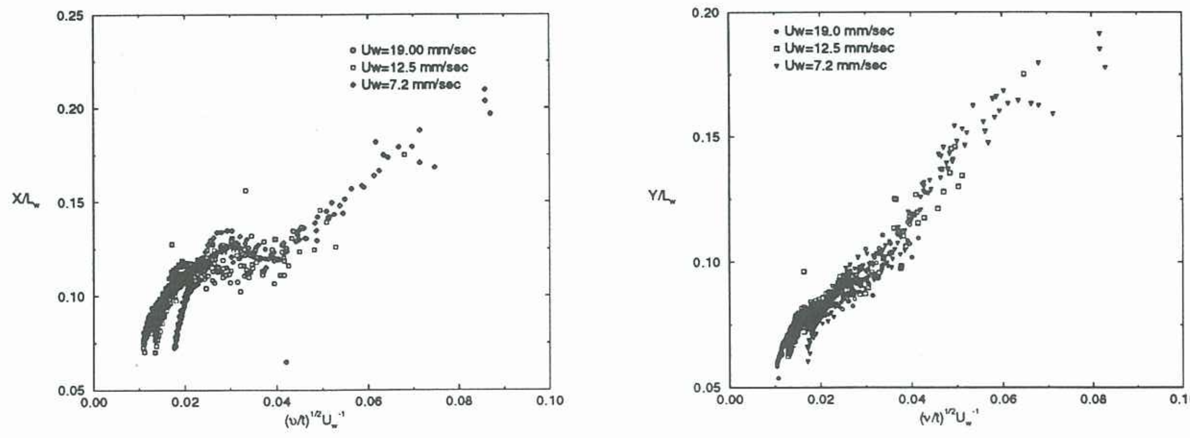


Figure 6: Co-ordinates of centre of spiral vs $Re^{-1/2}$

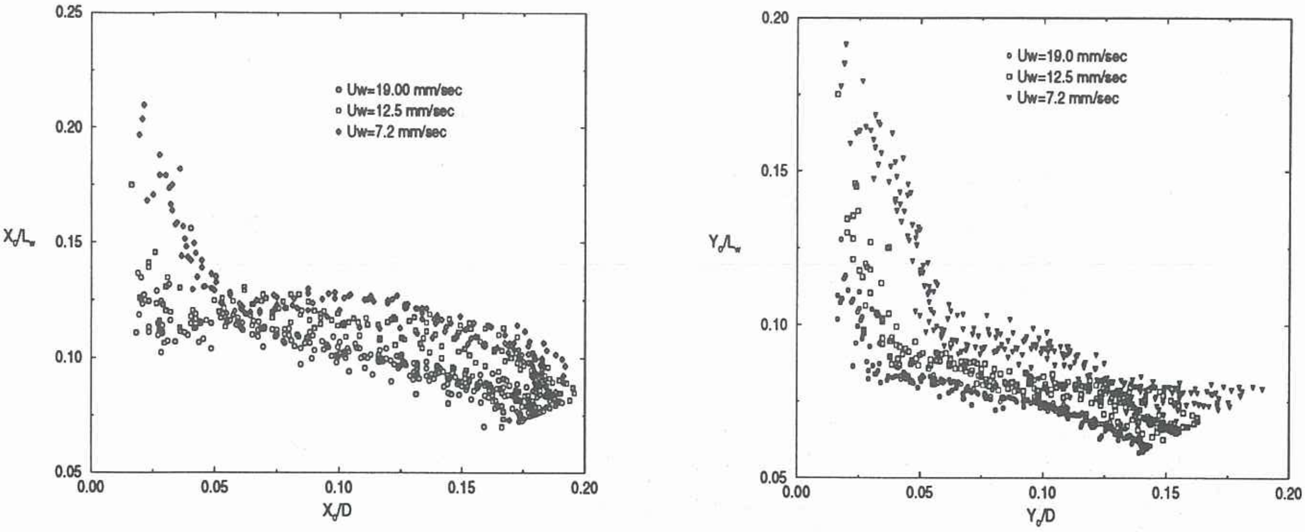


Figure 7: Co-ordinates of centre of spiral non-dimensionalised by L_w and D

The low Reynolds number region of both graphs corresponds to the development when the spiral is small and it is interesting to note that in this region it appears that x_o/L_w and y_o/L_w are only functions of Reynolds number only since for the different piston speeds all the data collapses onto a single curve. This is not surprising since it suggests that the length scale of the apparatus is not important when the spiral is small and viscous forces dominate. At low Reynolds numbers we would expect the location of the core of the spiral to be a function of viscosity and time. Expressing this as a non-dimensiomal equation we have

$$\frac{z_o}{L_w} \propto \nu^{1/2}/(t^{1/2}U_p) = Re^{-1/2}$$

Figure 6 shows plots of y_o/L_w and x_o/L_w vs $\nu^{1/2}/(t^{1/2}U_p)$. These plots show the apprximate linear relationship between z_o/L_w and $Re^{-1/2}$ at low Reynolds numbers.

The peel-off at large Reynolds numbers can then be attributed to the effect of the length scale of the apparatus i.e. when x_o/D and y_o/D become significant. In order to examine this point further figure 7 shows the x_o and y_o co-ordinates non-dimensionalised by L_w vs the same co-ordinate non-dimensionalised by D. From these plots it can be seen that the peel-offs shown on figure 5 for the x_o co-ordinate are now clustered around an approximately constant value of x_o/D. For the y_o co-ordinate there is little or no obvious cluster of points at a constant y_o/D value that is consistent with the peel-off shown in figure 5. It appears that the y_o co-ordinate is more dependent on Reynolds number than the x_o co-ordinate as the value of z_o/D increases.

CONCLUSIONS

The trajectory of the vortex core can be classified into three zones. The first is during its early development when Reynolds number is small and viscous forces are significant. In this region $z_o/L_w \simeq 0.5Re^{-1/2}$. A second relatively brief stage when the trajectory is independant of Reynolds number but the structure is small enough to be also independant of external length scales. In this region the structure can be classified as "self-similar" , as discussed in Pullin (1979), and a final stage where the external length scales operate in a complex fashion to effect the core trajectory. It should also be noted that three-dimensional effects will occur as the size of the spiral becomes large in comparison to the apparatus length-scale.

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