11th Australasian Fluid Mechanics Conference
University of Tasmania, Hobart, Australia
14-18 December 1992

THE MEASUREMENT OF ∂u/∂y IN THE WALL REGION OF A TURBULENT CHANNEL FLOW

Y. ZHU and R.A. ANTONIA

Department of Mechanical Engineering
University of Newcastle, NSW 2308, AUSTRALIA

ABSTRACT

Estimates of $\overline{(\partial u/\partial y)^2}$ have been obtained in the wall region of a fully developed turbulent channel flow using a pair of parallel hot wires. Both a spectral correction procedure, similar to that of Wyngaard (1969) and a correlation method were used. The results from the correlation method are in close agreement with those from direct numerical simulations. The spectral correction method yields incorrect results very close to the wall where the anisotropy is quite large. It does however yield satisfactory results away from the wall, even though the departures from isotropy are not small.

INTRODUCTION

The quantity $\overline{(\partial u/\partial y)^2}$ represents a significant contribution to the average energy dissipation ϵ and the spanwise mean square vorticity $\overline{\omega_z^2}$ in the wall region ($y^+ \leq 40$, the superscript + denoting normalisation by wall variables) of turbulent shear flows (Antonia et al., 1991). However, the measurement of $\partial u/\partial y$ in this region is not straightforward. When parallel hot wires are used (aligned in a spanwise direction and separated by a distance Δy in a direction normal to the wall), the main difficulty consists in selecting an appropriate value of Δy or alternatively in correcting for the effect of Δy. Wyngaard's (1969) spectral correction analysis (see also Antonia et al., 1987; Hussein and George, 1990) assumes isotropy. While this assumption appears reasonable at sufficiently high wavenumbers when the magnitude of the mean strain rate $\partial \overline{U}/\partial y$ [or perhaps more pertinently the non-dimensional parameter $(\partial \overline{U}/\partial y)\overline{q^2}/\epsilon$, where $\overline{q^2}$ is the average turbulent energy] is small, it becomes tenuous when the wall is approached. This region is also strongly inhomogeneous in the y direction so that it would not be appropriate to relax the assumption of isotropy to one of homogeneity.

In this paper we compare estimates of $\overline{(\partial u/\partial y)^2}$ obtained from the spectral method with those inferred from a two-point correlation analysis of the same data and from calculations of $\overline{(\partial u/\partial y)^2}$ obtained from direct numerical simulations of this flow.

EXPERIMENTAL ARRANGEMENT

Measurements were made in a fully developed turbulent channel flow (Figure 1) at a Reynolds number of 3300 ($Re = U_0 h/\nu$, where h is the half-width of the duct and U_0 is the velocity at the centreline), which is comparable to that used in the simulations. The Reynolds number is small enough for the magnitude of the Kolmogorov length scale η to be sufficiently large near the wall ($\eta \approx 0.2$ mm at $y^+ = 2$). A complete description of the duct and instrumentation is given in Zhu and Antonia (1992).

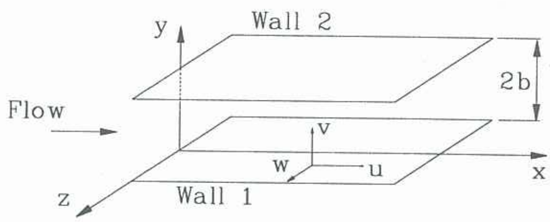

Figure 1 Schematic experimental arrangement of fully developed channel flow.

Measurements were made with two parallel hot wires. Each wire (Pt-10% Rh) was 1.3 μm in diameter and about 0.3 mm in length. The wires were traversed across the duct with a mechanism with a least count of 0.01 mm. The initial distance of the wires from the wall was determined using the reflection method and a theodolite. The spacing between the wires could be controlled with a separate mechanism (least count 0.01 mm). The wires were aligned in the z (spanwise) direction and separated in the y direction. For the correlation measurements, one wire was fixed. For the $\Delta u/\Delta y$ measurements, both wires were moved an equal distance so that the midpoint between the wires remained at the same location.

The hot wires were operated with in-house constant temperature circuits at an overheat ratio of 0.5. Output voltages from the anemometers were passed through buck and gain circuits and low-pass filtered (the cut-off frequency f_c was in the range 400–1250 Hz). To determine f_c, the signals were differentiated and their spectra were examined on a two-channel real-time spectrum analyser (HP3582A). The signals were next digitised (12 bit AD converter) into an IBM-compatible personal computer at a sampling frequency of (2 to 3) f_c and subsequently transferred to a VAX 780 computer via an optical cable link for further analysis.

CORRELATION METHOD

The two-point velocity correlation $R_{uu}(\Delta y)$ is defined as

$$R_{uu}(\Delta y) = \frac{\overline{u(y)u(y+\Delta y)}}{\overline{u^2}(y)^{\frac{1}{2}}\overline{u^2}(y+\Delta y)^{\frac{1}{2}}} \quad . \qquad (1)$$

Using a Taylor series expansion, $R_{uu}(\Delta y)$ can be approximated as (e.g. Townsend, 1956)

$$R_{uu}(\Delta y) = 1 - (\Delta y)^2 \left\{ \frac{\overline{\left(\frac{\partial u}{\partial y}\right)^2}}{2\overline{u^2}} - \frac{1}{8(\overline{u^2})^2}\left(\frac{\partial \overline{u^2}}{\partial y}\right)^2 \right\} \qquad (2)$$

to order $(\Delta y)^3$. Since Eq. (2) does not assume isotropy or homogeneity, it can in principle be used in the wall region. Measurements and direct numerical simulation (DNS) data for $\overline{u^2}(y)$ suggest that the second term inside the brackets becomes negligible for $y^+ \gtrsim 10$. In this case, Eq. (2) reduces to the homogeneous form (e.g. Batchelor, 1953)

$$R_{uu}(\Delta y) = 1 - \frac{(\Delta y)^2}{2\overline{u^2}}\overline{\left(\frac{\partial u}{\partial y}\right)^2} \quad . \qquad (3)$$

For $y^+ \lesssim 10$, the second term cannot be neglected, especially when $y^+ \leq 2$.

Figures 2a and 2b show measured and DNS values of $(1 - R_{uu})$, as a function of Δy^{*2} for $y^+ = 5$ and 40 respectively (the asterisk denotes normalisation by Kolmogorov scales). The DNS data indicate that the correlation for $\Delta y > 0$ is different from that for $\Delta y < 0$, especially at smaller y^+ values, reflecting the strong inhomogeneity in the direction of the mean shear. At small separations ($1 < \Delta y^* < 3$), the measurements ($\Delta y > 0$) are slightly lower than the DNS data ($\Delta y > 0$). The slope at the origin gives an estimate of the second term of Eq. (3).

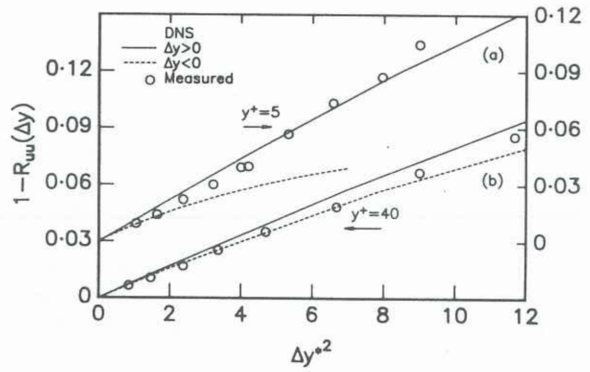

Figure 2 Variation of $(1 - R_{uu})$ as a function of Δy^{*2}. (a) $y^+ = 5$; (b) $y^+ = 40$. Measured : o. DNS : ——, $(\Delta y > 0)$; ·····, $(\Delta y < 0)$.

In Figures 3a and 3b, $(1 - R_{uu})$ is plotted as a function of Δy^*, using a logarithmic scale. Both measured and DNS data exhibit a slope of 2, although this is limited to a rather narrow range of Δy^* for the measurements at $y^+ = 5$. The intersection of the line of slope 2 with $\Delta y^* = 1$ also provides an estimate for the second term of Eq. (3). Because of the strong inhomogeneity for $y^+ \leq 10$, the estimation of $\overline{(du/dy)^2}$ via Eq. (3) is not reliable.

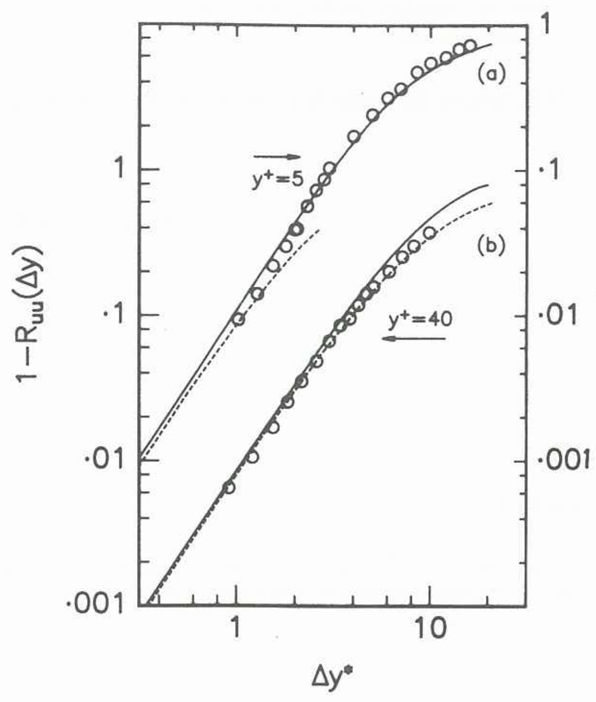

Figure 3 Variation of $(1 - R_{uu})$ as a function of Δy^*. (a) $y^+ = 5$; (b) $y^+ = 40$. Symbols as in Figure 2.

SPECTRAL CORRECTION METHOD

The true value of $\overline{(\partial u_i/\partial x_j)^2}$ may be written (Wyngaard, 1969)

$$\overline{\left(\frac{\partial u_i}{\partial x_j}\right)^2} = \iiint_{-\infty}^{+\infty} k_j^2 \phi_{ii}(\boldsymbol{k})dk_1dk_2dk_3 \quad , \qquad (4)$$

where $\boldsymbol{k}$ is the wavenumber vector with components k_1, k_2 and k_3 and $\phi_{ij}(\boldsymbol{k})$ is the energy spectrum tensor. For isotropic turbulence (e.g. Batchelor, 1953)

$$\phi_{ij}(\boldsymbol{k}) = \phi_{ij}(k) = \frac{E(k)}{4\pi k^4}(k^2\delta_{ij} - k_ik_j) \quad , \qquad (5)$$

where $k = (k_1^2 + k_2^2 + k_3^2)^{\frac{1}{2}}$ is the magnitude of $\boldsymbol{k}$. The measured velocity derivative $\overline{(\partial u_i/\partial x_j)_m^2}$ may be assumed to be given by (Wyngaard, 1969)

$$\overline{\left(\frac{\partial u_i}{\partial x_j}\right)_m^2} = \frac{4}{\Delta x_j^2}\iiint_{-\infty}^{+\infty} \sin^2\left(\frac{k_j \Delta x_j}{2}\right)\phi_{ii}(\boldsymbol{k})dk_1dk_2dk_3 \quad (6)$$

where Δx_j is the wire separation. The effect of wire length is neglected here. Values of r, defined by

$$r = \frac{\overline{(\partial u_i/\partial x_j)_m^2}}{\overline{(\partial u_i/\partial x_j)^2}} \qquad (7)$$

can be found from Eqs. (4) and (6) for a given $E(k)$. In theory, $r \rightarrow 1$ as $\Delta x_j \rightarrow 0$.

Figures 4a and 4b show the values of $\overline{(\partial u^+/\partial y^+)_m^2}$ and the "corrected" values of $\overline{(\partial u^+/\partial y^+)^2}$ as a function of Δy^* for $y^+ = 5$ and 40 respectively. The "corrected" values were obtained by dividing $\overline{(\partial u^+/\partial y^+)_m^2}$ by the ratio r. The latter quantity is evaluated from Eqs. (4), (6) and (7), using the isotropic form of $\phi_{ii}(k)$, viz. Eq. (5).

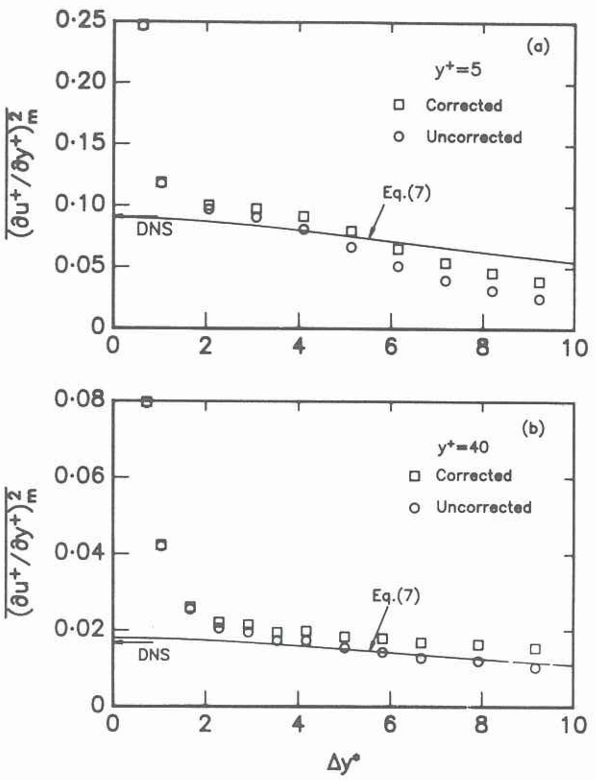

Figure 4 Finite difference approximation to $\overline{(\partial u^+/\partial y^+)^2}$ as a function of the separation Δy^*. (a) $y^+ = 5$; (b) $y^+ = 40$. Measured value : □. Spectrally corrected value : ○. ——, Eq.(7).

There is a systematic and rapid increase of $\overline{(\partial u^+/\partial y^+)^2_m}$ as $\Delta y \to 0$, which is mainly due to contamination by electronic noise. There are several other sources of errors; for example, errors in wire calibration, possible flow interference due to the wires and/or their supports and the uncertainty in measuring Δy (e.g. Mestayer and Chambaud, 1979; Antonia et al., 1984). When Δy increases, the major error is the attenuation in the high wavenumber part of the $\Delta u/\Delta y$ spectrum. Because of the large (systematic and random) errors in the range $\Delta y^* \lesssim 3$, the data in this range are not reliable for the purpose of estimating $\overline{(\partial u/\partial y)^2}$. Klewicki and Falco (1990) and Hussein and George (1990) have suggested that Δy^* should be about 1 to provide a good estimation of $\overline{(\partial u/\partial y)^2}$. Figures 4a and 4b indicate that this selection would lead to $\overline{(\partial u/\partial y)^2}$ being overestimated by at least 50%. For $y^+ \gtrsim 3$, the values of $\overline{(\partial u^+/\partial y^+)^2_m}$ at $y^+ = 5$ show a large departure from Eq. (7), where the denominator, or true value of $\overline{(\partial u/\partial y)^2}$, is identified with the value obtained from the correlation method. The "corrected" values of $\overline{(\partial u^+/\partial y^+)^2}$ decrease with Δy^*, suggesting that the method cannot yield an unambiguous estimate of $\overline{(\partial u/\partial y)^2}$. However at $y^+ = 40$, $\overline{(\partial u^+/\partial y^+)^2_m}$ is in reasonable agreement with the calculation; accordingly, $\overline{(\partial u^+/\partial y^+)^2}$ decreases relatively slowly as Δy^* increases.

It is clear from the above discussion that only the correlation method can provide reliable data for $\overline{(\partial u/\partial y)^2}$ in the near-wall region. Figure 5 shows that the correlation results agree (to within 7%) with the DNS data. However, if statistics of $\partial u/\partial y$ are required, then a signal is needed which mimics the properties of $\partial u/\partial y$ as closely as possible. It is experimentally convenient to approximate $\partial u/\partial y$ by $\Delta u/\Delta y$, where Δu is the difference between the signals from the parallel wires and the separation Δy is fixed. Figure 5 indicates that $\overline{(\Delta u^+/\Delta y^+)^2}$ is in good agreement (in the range 5–10%) with the DNS data with an appropriate choice of separation. Here, a value of 0.7 mm was used for Δy; this corresponds to $\Delta y^* = 3.6$ at $y^+ = 5$ and 2.9 at $y^+ = 40$. For this choice of Δy, it would appear that there is no need to correct the measurements, at least for the purpose of estimating the mean square value of $\partial u/\partial y$. Obviously, this choice needs to be justified, both in terms of other statistics of $\partial u/\partial y$ and higher values of the Reynolds number; these aspects require further investigation.

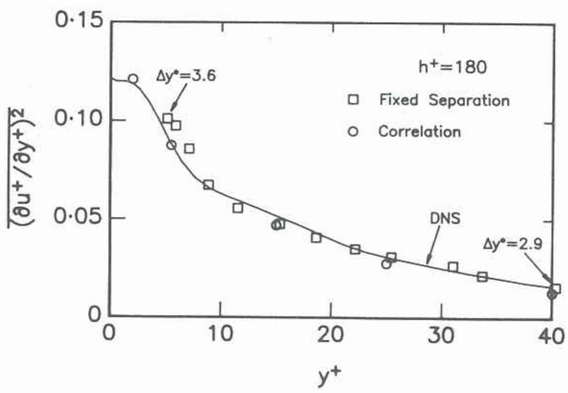

Figure 5 Variation across the channel of $\overline{(\partial u^+/\partial y^+)^2}$. Correlation : ○. Fixed separation : □. DNS : ——.

CONCLUSIONS

The quantity $\overline{(\partial u/\partial y)^2}$ was measured in the wall region of a fully developed channel flow using two parallel hot wires. It is estimated that the separation between the wires should be greater than about three Kolmogorov length scales to avoid noise contamination. A correlation method and a spectral correction method were used to measure $\overline{(\partial u/\partial y)^2}$. Results from the correlation method agree to within 7% with the DNS data for the same flow and Reynolds number. Results from the spectral correction method are reasonable for $y^+ \gtrsim 40$. Very close to the wall, this latter method is unsuitable due to the strong departure from isotropy. It appears that reasonable results for $\overline{(\partial u/\partial y)^2}$ can be obtained throughout the wall region when the separation between the hot wires is chosen to be in the range 3–4 Kolmogorov length scales.

ACKNOWLEDGEMENTS

The support of the Australian Research Council is gratefully acknowledged. We would also like to thank Dr J. Kim for the DNS correlation data.

REFERENCES

ANTONIA, R. A., BROWNE, L. W. B. and CHAMBERS, A. J. (1984) On the Spectrum of the Transverse Derivative of Streamwise Velocity in a Turbulent Flow. *Phys. Fluids*, **27**, 2628-2631.

ANTONIA, R. A., KIM, J. and BROWNE, L. W. B. (1991) Some Characteristics of Small Scale Turbulence in a Turbulent Duct Flow. *J. Fluid Mech.*, **233**, 369-388.

ANTONIA, R. A. and MI, J. (1991) Corrections for Velocity and Temperature Derivatives in Turbulent Flows. *Expts. in Fluids* (to appear).

ANTONIA, R. A., SHAH, D. A. and BROWNE, L. W. B. (1987) Spectra of Velocity Derivatives in a Turbulent Wake, *Phys. Fluids*, **30**, 3455-3462.

BATCHELOR, G. K. (1953) *The Theory of Homogeneous Turbulence*, Cambridge, Cambridge University Press.

HUSSEIN, H. J. and GEORGE, W. K. (1990) Influence of Wire Spacing on Derivative Measurement with Parallel Hot-Wire Probes, in W. M. Bower, M. J. Morris and M. Samimy (eds.) *Forum on Turbulent Flows – 1990*, ASME FED-Vol. 94, 121-124.

KLEWICKI, J. C. and FALCO, R. E. (1990) On Accurately Measuring Statistics Associated with Small-Scale Structure in Turbulent boundary Layers Using Hot-Wire Probe, *J. Fluid Mech.*, **209**, 119-142.

MESTAYER, P. and CHAMBAUD, P. (1979) Some Limitations to Measurements of Turbulence Micro-Structure with Hot and Cold Wires. *Boundary-Layer Meteorol.*, **16**, 311-329.

TOWNSEND, A. A. (1956) *The Structure of Turbulent Shear Flow*, Cambridge, Cambridge University Press.

WYNGAARD, J. C. (1969) Spatial Resolution of the Vorticity Meter and Other Hot-Wire Arrays, *J. Sci. Instrum.*, **2**, 983-987.

ZHU, Y. and ANTONIA, R. A. : 1992. Temperature Dissipation Measurements in a Fully Developed Turbulent Channel Flow, *Expts. in Fluids* (submitted)